Modelling of Angle of Shearing Resistance using Support Vector Machines

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***Abstract*-The determination of the angle of shearing resistance of any soil is an important task in in geotechnical engineering practice. This paper examines the potential of support vector machine (SVM) for predicting the angle of shearing resistance from triaxial test data set. SVM is a statistical learning theory based on a structural risk minimization that minimizes both error and weight terms. The four input variables used for the prediction of angle of shearing resistance are the %fine grained soil, %coarse grained soil, liquid limit and bulk density. Sensitivity analysis has been carried out to investigate the relative importance of each of the input parameters. The sensitivity analysis cleared that liquid limit [LL] influenced angle of shearing resistance the most. Comparison between SVM and some other models is also presented. The result of the study has shown that the SVM approach has the potential to be a practical tool for determination of angle of shearing resistance**.

Keywords - Angle of shearing resistance, SVM, Sensitivity analysis.

I. INTRODUCTION

For a reliable design of any geotechnical structure, the primary requirement is precise determination of angle of shearing resistance (ϕ’).Angle of shearing resistance is a shear strength parameter and known as the interlocking among the soil particles. It is used to determine the bearing capacity of foundation systems, earth pressure acting on retaining walls and to analyse the stability of natural slopes against slope failures and landslides. It depends mainly on three parameters i.e. soil type, density of soil and plasticity of soil. Clay soils having high plasticity exhibit lower angle of shearing resistance. On the other hand the value of angle of shearing resistance increases as the grain size of soil increases. This important parameter can be computed using laboratory or field tests. Triaxial compression and direct shear tests are the two most common tests for determining the (ϕ’) in the laboratory. The testing procedures of triaxial compression and direct shear tests have been standardized by (ASTM WK3821; ASTM-6528-00) [1,2], respectively and the tests are most suitable for clayey and sandy soils respectively. However, they are laborious, time taking and costly methods. The triaxial test is more desirable for clayey soil and takes a long time to complete. For the sandy soils, the direct box shear test is frequently used and it has simple test procedure than the triaxial test. Since the determination of (ϕ’) by laboratory methods is a time-taking, cumbersome and costly process, empirical equation based on soil parameters which are determined by basic laboratory tests can be preferred to determine the angle of shearing resistance. However the most of the empirical equations are based on limited information and do not provide precise results. The other drawbacks of these equations are the equations are developed by using only one parameter of soil to determine the (ϕ’) [3-6].Whereas the soil has complex structure, inaccurate physical properties and heterogeneities associated with formation of them [7]. In late years, new soft computing methods such as artificial neural networks (ANNs) have been successfully applied to modelling of various geotechnical engineering problems [8]. The insufficiency of ANNs to produce simplified prediction equation can create difficulty in different circumstances. To overcome these problems, an alternative approach has come called support vector machine (SVM). In SVM, high generalization performance is achieved by minimizing the sum of training set error and a term that depends on the Vapnik- Chervonenkis(VC) dimension. There are three distinct characteristics of SVM when it is used to estimate the regression function SVM estimates the regression using a set of linear functions that are defined in a high dimensional space initially then SVM carries out regression estimation by risk minimization where the risk is measured using Vapnik’s ε-insensitive loss function. At last, SVM uses a risk function consisting of the empirical error and regularization term which is derived from SRM principal [9,10]. In the content of this paper, new approach based on support vector machine (SVM) are presented for the determination of (ϕ’) value of soils. The datasets for training and testing were obtained from different geotechnical applications in Turkey and literature study performed herein [11]. Four basic soil parameters, the percentage of fine grained (FG), the percentage of coarse grained (CG), liquid limit (LL) and bulk density (BD) were used to the SVM model as input parameters. The result obtained is also compared with other present models

II. SUPPORT VECTOR MACHINE (SVM)

The theoretical foundation of support vector machine has been developed by Vapnik [9].SVM is an emerging machine learning technology in which model complexity and prediction error can be minimized simultaneously. This study uses the SVM as a regression technique by introducing ε-insensitive loss function. In this section, a brief introduction on the construction process of SVM is presented. More points can be found in many publications. [9-17] The ε - insensitive loss function can be defined in a following way:

$$L_{\varepsilon}(y) = 0 \quad \text{for } |f(x) - y| < \varepsilon$$

otherwise $L_\varepsilon(y) = |f(x) - y| - \varepsilon$ (1)

Consider the training dataset $\{(x_1, y_1)\ldots(x_l, y_l)\}$, x is the input and y is equal to the output R^n = n-dimensional vector space; and r = one dimensional vector space; and ε–error insensitive zone. The four inputs variables used for the SVM model in this study are the [FG, CG, LL and BD]. The output of this model is angle of shearing resistance (ϕ'). So for this model x= [FG, CG, LL and BD] and y= angle of shearing resistance (ϕ').

The main aim of the SVM is to determine a function f(x) that can approximate the future values precisely. The general support vector regression for estimating linear takes the form f(x)=(wx)+b (2)

Where, $w \in R^n$ and $b \in r$; w = adjustable weight vector; and b = scalar threshold.
The main objective of the SVM is to discover a function that gives a deviation ε from the real output (y), which is, at the same time as flat as possible. Flatness is the measure of w in the equation. So the value of w should be minimized as much as possible. One way of obtaining this is, by minimizing the Euclidean norm i.e. $\|w\|^2$= (w,w). It can be written as a convex optimization problem [18].

Minimize: $\frac{1}{2}\|w\|^2$

Subjected to:

$$y_i - (\langle w.x_i \rangle + b) \le \varepsilon \quad i=1,2,\ldots,l \qquad (3)$$

$$(\langle w.x_i \rangle + b) - y_i \le \varepsilon, \quad i = 1, 2,\ldots,l \qquad (4)$$

The most adept regression line is defined by minimizing the following cost function

Minimize: $\frac{1}{2}\|w\|^2 + C\sum_{i=1}^{l}(\xi_i + \xi_i^*)$

Subjected to:

$$y_i - (\langle w.x_i \rangle + b) \le \varepsilon + \xi_i, \quad i=1,2,\ldots,l \qquad (5)$$

$$(\langle w.x_i \rangle + b) - y_i \le \varepsilon + \xi_i^*, \quad i=1,2,\ldots,l \qquad (6)$$

$$\xi_i \ge 0 \;\&\; \xi_i^* \ge 0, \quad i=1,2,\ldots,l \qquad (7)$$

Slack variables ($\xi_i \ge 0$ and $\xi_i^* \ge 0$) find the degree to which samples with error more than ε can be penalized. The capacity factor (C) ranges from 0 to infinity determines the trade-off between the flatness of function f(x) and the amount up to which deviations larger than ε are tolerated [19]. In practise, the capacity factor (C) is chosen by trial and error only. Optimization problem is resolved by Lagrangian multipliers (α_i, α_i^*) and its answer is given by [8]

$$f(x) = \sum_{\text{support vectors}} (\alpha_i - \alpha_i^*)(x_i.x) + b \qquad (8)$$

Where $b = -\left(\frac{1}{2}\right) w.[x_r + x_s]$;

An important prospect is some Lagrange multipliers will be zero, then these training objects are considered to be irrelevant for the final solution. The training objects with nonzero Lagrange multipliers are called as support vectors

III. SVM IMPLEMENTATION FOR ANGLE OF SHEARING RESISTANCE (□') PREDICTION

In SVM, First of all, each of the input variables (FG, CG, LL and BD) is normalized to their respective maximum value. The output variable, angle of shearing resistance (ϕ') was also normalized with respect to maximum (ϕ') value.

To implement the SVM the dataset has been divided into two subsets;

1. A training data set: This data set is required to construct the model. In this study, 46 out of a total of 66 data sets are considered for training.

2. A testing data set: This is required to estimate the model's performance. In this study the remaining 20 out of are used as a testing data set.

The training and testing data sets have been taken using a sorting technique to maintain the statistical consistency. The main aim of the application of SVM in this study is to get the proper values of design parameters (C&ε). Though identification of the optimal values of design parameters (C &ε) is a trial and error process, there are some guidelines that can be used for selecting the parameters. If C goes to infinitely large, SVM would not permit happening of any error and result in a complex model, whereas if C goes to zero, then the result would tolerate a large number of error and the model would be less complex. A large C allot higher penalties to errors so that the regression is trained to minimize the error with lower generalization, whereas a small C assigns higher penalties to errors, that allows the minimization of margin with errors thus higher generalization ability. With regards to selection of ε if ε is too small many support vectors are selected which leads to a risk of overfitting, whereas if ε is too large, a very few support vectors are selected, which leads to a reduction in the final prediction performance [20]. The programming of SVM has been done by using MATLAB and the optimum values of C and ε received in this study are presented in result and discussion section.

IV. SENSITIVITY ANALYSIS

A sensitivity analysis is being carried out on the constructed model to key out that input variable of data (FG, CG, LL and BD) which has the most significant impact on (ϕ') prediction. The sensitivity analysis is carried out by varying each of the input variables one at a time, at a constant rate of 30%. The percentage change of the output is calculated for the change of input parameter. The sensitivity (S) of each input parameter is calculated from the following formula:

$$S(\%) = \frac{1}{N}\sum_{j=1}^{N}\left(\frac{\%\ \text{change in ouput}}{\%\ \text{change in input}}\right)_j \times 100$$

Where N= number of data.

In the present study, training, testing and sensitivity analysis of SVM has been carried out by MATLAB.

V. RESULTS AND DISCUSSIONS

The four input variables used for the development of SVM model to predict angle of shearing resistance (ϕ') are FG, CG, LL and BD. The coefficient of correlation (R) of the predicted (ϕ') with respect to actual (ϕ'), determined using triaxial test on soil samples is the main creation that is used to evaluate the performance of the SVM model developed in this work.

The value of (R) should be close to one for a good model. The design values of C, ε and σ have been decided by trial and error approach, the design values of C, ε and σ are 100, 0.01 and 0.001 respectively. Figure 1 depicts the performance of training dataset. From figure 1, it is clear that the value of (R) is very close to one. Therefore, the developed SVM has successfully captured input and output relation for training dataset.

So, the developed SVM has capability for prediction of (ϕ') at any point. The following equation has been developed based on the developed SVM model.

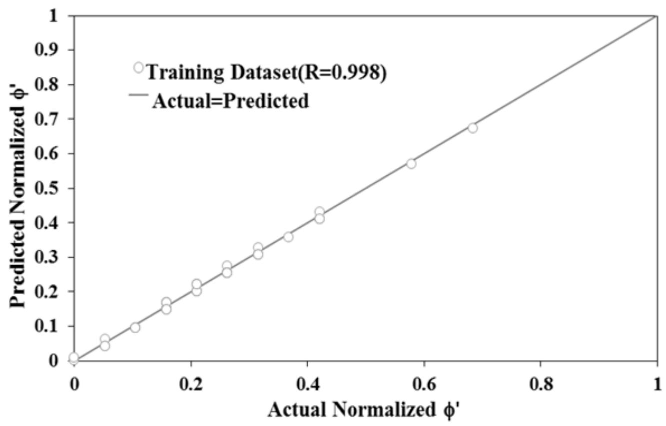

Fig. 1.perfomance of training dataset

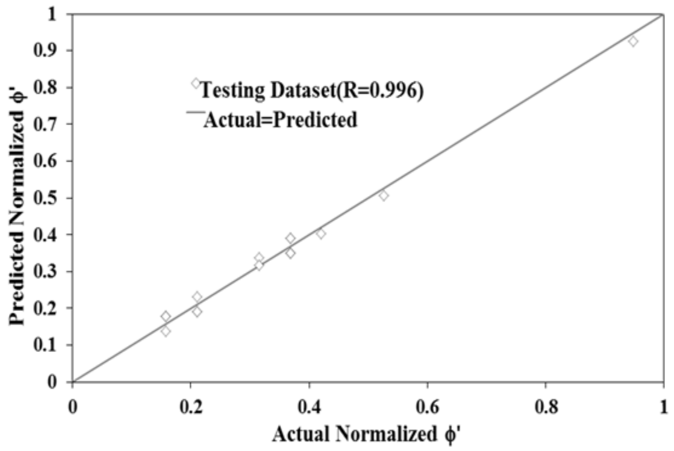

Fig. 2.perfomance of testing dataset

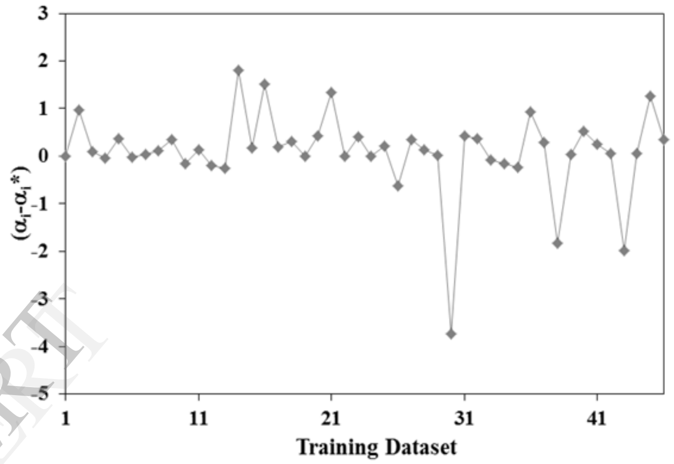

Fig. 3.values of $\left(\alpha_i - \alpha_i^*\right)$

FINAL EQUATION –

$$\phi' = \sum_{i=1}^{46}\left(\alpha_i - \alpha_i^*\right)\exp\left\{-\frac{\left(x_i - x\right)\left(x_i - x\right)^T}{0.02}\right\} \quad (9)$$

In this study, the developed SVM model produces 44 support vectors. These support vectors have been only used for the final prediction. So, there is real advantage attained in terms of sparseness. Sparseness means that a significant number of weights are zero, which has the consequences of producing compact, computationally effective models, which in addition are simple and therefore develop smooth function.

Many computing methods for predicting (ϕ') are presented in literature. Among these, three are chosen for the purpose of evaluating the performance of SVM model. These include the GEP model, ANFIS model and ANN [11]. Comparison of the results obtained from SVM model and other models for the training dataset are presented in terms of coefficient of relation (R) in table 1 .Table 1 shows that the SVM method performs better than the other models.

TABLE.I.

Values of R for different models				
	SVM Model	***ANFIS Model***	***ANN Model***	***GEP Model***
R	0.99	0.87	0.89	0.96

TABLE.II.

Input variables and sensitivity values	
Input Variables	***Sensitivity (S %)***
FG	7.0044
CG	4.3496
LL	12.544
BD	10.510

VI. CONCLUSION AND FUTURE WORK

- This study describes SVM for prediction of angle of shearing resistance (ϕ'). The developed equation was developed based on well- established and widely dispersed triaxial test results obtained from the literature.
- The performance of the SVM model was benchmarked against the ANN and other multiple regression based models.
- With the use of the developed equation, the □’ values can be estimated without carrying out the sophisticated and time- consuming laboratory or field tests.
- A finding from the sensitivity analysis results is that the most important parameter governing the (ϕ') behaviour is the soil liquid limit.
- The SVM model can be used for practical engineering purposes since it was developed based on tests conducted on clayey and sandy soils with wide range of properties. The proposed model is very simple. The predictive capability of the derived model is limited to the range of data used for its calibration. Despite this limitation, this model can be retrained

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