

Modelling And Multi-Response Optimization Of Hard Milling Process Based On RSM And GRA Approach

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Abstract

The study of metal removal rate and cutting temperature is most significant among the others like features of tools and work materials. Since these are the determinant factors of the production rate and cost-efficiency of the tools. Milling of hardened tool steels became a highly expensive for the manufacturing industries today as these are being widely used in many applications like automobile, structural, etc. A significant improvement in the efficiency of this process may be obtained with the development of mathematical relations between the set of input and output parameters of a machining process. The models reveal the level of significance of the process parameter on response. Therefore, the constituencies of critical process control factors leading to desired responses with acceptable variations ensuring a lower cost of manufacturing can be identified. In this investigation, milling experiments are conducted to machine hardened EN 31 tool steel with carbide cutting inserters. Initially, the design of experiments was conducted to plan the experimentation by considering the machining variables of depth of cut, feed and spindle speed. Metal removal rate and cutting temperature were measured for each experimental run. Response surface methodology is used to build up the mathematical surface model for the measured values of responses. The ANOVA technique has been used to verify the adequacy of the models at 95% confidence interval. Since the influence of machining parameters on the metal removal rate and cutting temperature are with conflicting nature, the problem is considered as multi-objective optimization problem. Hence, Gray relational analysis (GRA) was adapted to the response values to obtain the optimal set of input parameters.

Keywords: Hard milling, empirical modelling, RSM, optimization, GRA.

1. Introduction

Hardened steels are being used in a variety of industrial applications like automotive, aerospace etc. These materials are often classified as difficult-to-machine materials due to high strength and low thermal conductivity. This drives to severe cutting forces and cutting temperatures and hence a shorter the tool life. Tool life is the significant economic factor, particularly for milling and turning of heat resistant alloys [1]. Agawal et al. [2] assessed the relative performance of coated and uncoated carbide tools (inserts) in the machining of three cast austenitic stainless-steels. Uhlmann et al. [3] stated that, the harder diamond tools cannot be used to machine the steels due to reactive nature and the secondary harder tools like cubic boron nitride (CBN) and PCBN are efficient in place of former but are highly expensive. Szymon et al. [4] presented a comparison of tool life of sintered carbide and CBN ball end mills. This investigation revealed that the tool life of sintered carbide is higher than the CBN up to a certain range of cutting speed. Also, the cutting speed was observed as an independent dominating factor on abrasive wear of CBN cutter. Pinaki Chakraborty et al. [5] developed the a mathematical model for tool wear during end-milling of AISI 4340 steel with multi-layer physical vapor deposition (PVD) coated carbide inserts under semi-dry and dry cutting conditions. From this research, it is also observed that cutting speed has the most comprehensive effect on tool wear progression. Aslan et al. [6] performed a comparative study on cutting tool performance in end milling of AISI D3 tool steel with coated carbide, coated cermet, alumina (Al₂O₃) based mixed ceramic and cubic boron nitride (CBN) cutting tools.

In the present work, two important performance measures of hard milling responses,

viz., metal removal rate (MRR) and cutting temperature (T) were considered for investigation. The empirical models of the chosen responses were developed in terms of the prominent process control variables of depth of cut, feed and cutting speed using a well known statistical technique called response surface methodology. Analysis of variance (ANOVA) is then adapted to check the adequacy of the developed models at 95% confidence interval. The measured response values are carried to find the optimal machining conditions. A multi-response optimization technique, Gray Relational Analysis was implemented to find the optimal machining conditions.

2. Response Surface Methodology

Response surface methodology is a widely used tool for design and analysis of experiments [7]. It is a collection of statistical and mathematical techniques useful for developing, improving and optimizing process [8]. In its process, a suitable relationship is developed between output of interest 'y' and a set of controllable variables $\{x_1, x_2, \dots, x_n\}$. A second-order nonlinear response function usually utilized [13] in the form:

$$y = b_0 + \sum_{i=1}^n b_i x_i + \sum_{i=1}^n b_{ii} x_i^2 + \sum_{i < j} b_{ij} x_i x_j + \varepsilon \quad (1)$$

Where, ε represents the noise or error observed in the response y such that the expected response is $(y-\varepsilon)$ and b 's are the regression coefficients to be estimated.

In the present work, development of the mathematical models and analysis has done with the use of a statistical tool called Stat-Ease Design Expert [9].

The adequacy of the predicted models was checked by Analysis of variance (ANOVA). It calculates the F-ratio, which is the ratio between the regression mean square and the mean square error. If the calculated value of F-ratio is higher than the tabulated value of F-ratio for roughness, then the model is adequate at desired significance level α to represent the relationship between machining response and the machining parameters.

3. Gray Relational Analysis

Gray relational analysis (GRA) proposed by Deng is a method of measuring the degree of approximation among sequences according to the grey relational grade [10]. GRA analyzes uncertain relations between one main factor and all the

other factors in a given system between the sequences with less data [11]. The processing steps are listed below [13].

1. Normalize the response matrix from zero to one by using Eq. (2) and (3).

Lower-the-better (LB) is the criterion:

$$x_i(k) = \frac{\max y_i(k) - y_i(k)}{\max y_i(k) - \min y_i(k)} \quad (2)$$

Higher-the-better (HB) is the criterion:

$$x_i(k) = \frac{y_i(k) - \min y_i(k)}{\max y_i(k) - \min y_i(k)} \quad (3)$$

where, $x_i(k)$ is the normalised value of k^{th} response, $\min y_i(k)$ is the smallest value of $y_i(k)$ for k^{th} response and $\max y_i(k)$ is the largest value of $y_i(k)$ for k^{th} response. x is the normalised array.

2. Calculation of grey relational coefficient from the normalised matrix.

$$\xi_i(k) = \frac{\Delta_{\min} + \zeta \Delta_{\max}}{\Delta_{0i}(k) + \zeta \Delta_{\max}} \quad (4)$$

Where, $\Delta_{0i} = \|x_0(k) - x_i(k)\|$: is the deviation of absolute value $x_0(k)$ and $x_i(k)$. ζ is the distinguishing coefficient $0 \leq \zeta \leq 1$.

$$\Delta_{\min} = \min_{\forall j \in i} \min_{\forall k} \|x_0(k) - x_j(k)\| \quad (5)$$

$$\Delta_{\max} = \max_{\forall j \in i} \max_{\forall k} \|x_0(k) - x_j(k)\| \quad (6)$$

3. Determination of overall grey relational grade.

The overall gray relational grade represents as the overall performance characteristic of multiple responses of the process. This is calculated as the average of individual gray relational grades of the responses at i^{th} experimental run.

$$\gamma_i = \frac{1}{n} \sum_{k=1}^n \xi_i(k) \quad (7)$$

It means, the overall gray relational grade converts the multi-response (multi-gray relational grades) optimization problem into a single response (overall gray relational grade) optimization problem, with the objective function as

maximization of overall grey relational grade. Hence, the overall grey relational grades rank the experimental runs as; the experimental run having higher grey relational grade refers as that corresponding combination of variables is closer to the optimal values. The optimal parametric combination is then evaluated by maximizing the overall grey relational grade.

4. Experimental Details

In this work, depth of cut, feed and cutting speed are considered as the control variables and MRR and cutting temperature as the output responses. In order to reduce the number of experimental runs, experiments are planned based on design of experiments (DOE). Central composite design with 27 experiments was selected. Table 1 lists the feasible values of each process variable. Experiments are conducted on a precision CNC milling machine model BFW AGNI 45. Hardened steel EN31 plate of size 150x100x10 mm with ≈ 60 HRC is considered as the work piece material and TaeguTec make M9810048402 carbide milling turning inserts and with SCRM90TP45016R18DTGNL milling cutter with 4 cutting inserts was used in machining. For each experimental run, the metal removal rate is calculated by the weight loss method. Each experiment is run for a fixed length of 75 mm length. During each experiment the cutting temperature was measured by a IR Thermometer by maintaining 1.5 meter distance between the thermometer and cutting tool edge. Each experiment was repeated for three times and the average of the measures values were considered as the final response values. Table 2 represents the matrix of experimental values. The Fig.1 shows the experimental setup. The Figs.2 and 3 show the cutting tools & cutter and the IR Thermometer for temperature measurement used in experimentations.

Table 1 Machining Variables and their Levels

S. No	Variables	Units	Notation	Range		
				-1	0	1
1.	Depth of cut	mm	DOC	0.1	0.2	0.3
2.	Feed Rate	mm/tooth F		0.1	0.3	0.5
3.	Cutting Speed	m/min	V	120	180	240

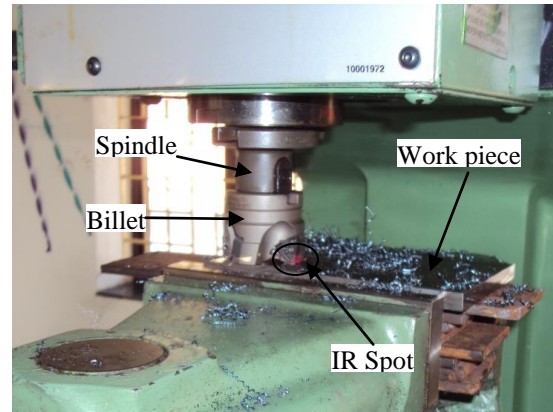


Fig. 1 Experimental setup



Fig. 2 Cutting inserts and the milling cutter

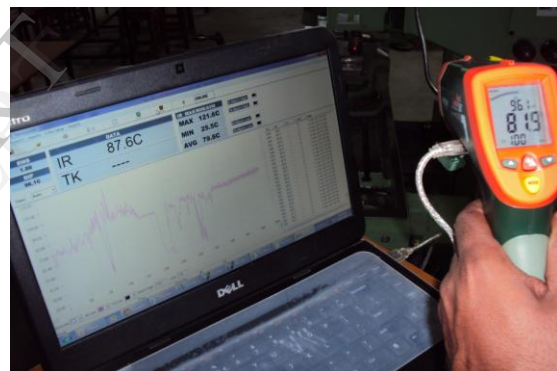


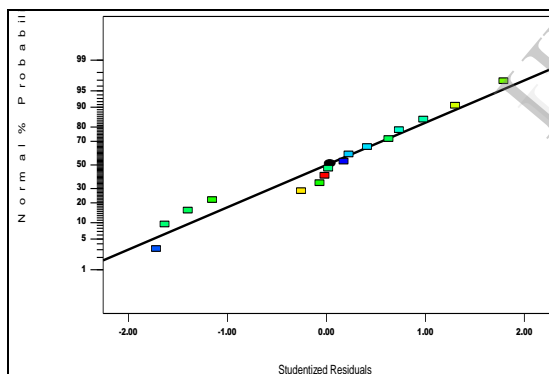
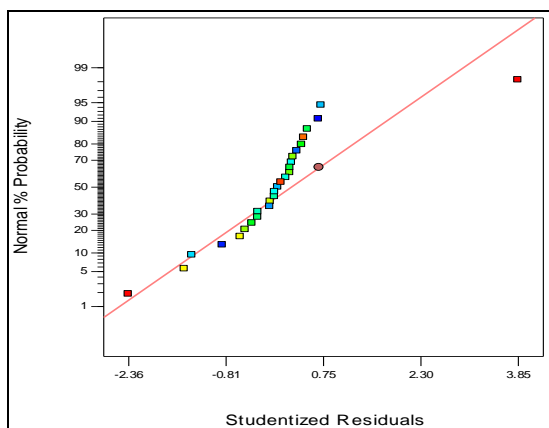
Fig. 3 IR Thermometer

5. Development of Empirical Model

In the present study, mathematical relationship between control variables and the responses was developed using the response surface methodology. Design Expert 8 is used to analyze the variance and to compute the regression coefficients for the proposed models. For the present case study, the second order model has been postulated because of its more accuracy. This model is checked for adequacy by using analysis of variance (ANOVA). Tables 3 and 4 are the ANOVA of MRR and cutting temperature respectively. From the Table 3 and 4, the model F-values of 95.72 and 201.02 implies that the models are significant and the p-values less than 0.05 indicate the model terms are significant.

Table2. Experimentally measured values

Exp. No.	D mm	F mm/tooth	V m/min	MRR grm/min	Temp. °C
1.	0.1	0.1	120	0.00545	169.05
2.	0.1	0.1	180	0.00854	181.29
3.	0.1	0.1	240	0.01055	245.94
4.	0.1	0.3	120	0.00848	247.29
5.	0.1	0.3	180	0.01154	278.44
6.	0.1	0.3	240	0.01358	344.44
7.	0.1	0.5	120	0.02645	419.28
8.	0.1	0.5	180	0.02954	460.62
9.	0.1	0.5	240	0.03152	539.59
10.	0.2	0.1	120	0.02345	209.12
11.	0.2	0.1	180	0.02654	231.36
12.	0.2	0.1	240	0.02855	286.01
13.	0.2	0.3	120	0.02645	276.39
14.	0.2	0.3	180	0.02954	307.54
15.	0.2	0.3	240	0.03156	373.54
16.	0.2	0.5	120	0.04445	435.84
17.	0.2	0.5	180	0.04754	477.17
18.	0.2	0.5	240	0.04951	556.14
19.	0.3	0.1	120	0.04845	277.42
20.	0.3	0.1	180	0.05154	299.66
21.	0.3	0.1	240	0.05353	354.31
22.	0.3	0.3	120	0.05145	333.71
23.	0.3	0.3	180	0.05454	364.86
24.	0.3	0.3	240	0.05652	430.86
25.	0.3	0.5	120	0.06945	480.62
26.	0.3	0.5	180	0.07254	591.95
27.	0.3	0.5	240	0.05454	580.92

**Fig. 4** Normal Probability plot of MRR**Fig. 5** Normal Probability plot of cutting temperature

The following equations are obtained for the output responses:

$$MRR = 0.00088 + 0.2325D - 0.00085F + 0.00037V - 0.00045DF - 0.00027DV - 0.000075FV \quad (8)$$

$$+ 0.2390D^2 + 0.000041F^2 - 0.000047V^2$$

$$Temp = 474.732 + 346.324D - 6.6175F - 0.4623V - 2.4651DF - 0.3716DV - 0.003676FV \quad (9)$$

$$+ 1466.8354D^2 + 0.0301F^2 + 0.001892V^2$$

Table 3 ANOVA of MRR

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	9.50E-03	9	1.06E-03	95.718	< 0.0001 significant
x_1	2.26E-03	1	2.26E-03	204.60	< 0.0001
x_2	5.86E-04	1	5.86E-04	53.070	< 0.0001
x_3	7.56E-06	1	7.56E-06	0.6851	0.4193
x_1x_2	3.46E-05	1	3.46E-05	3.1387	0.0944
x_1x_3	3.60E-05	1	3.60E-05	3.2626	0.0886
x_2x_3	3.77E-05	1	3.77E-05	3.4136	0.0821
x_1x_1	3.43E-05	1	3.43E-05	3.1077	0.0959
x_2x_2	1.98E-04	1	1.98E-04	17.931	0.0006
x_3x_3	1.99E-05	1	1.99E-05	1.8051	0.1967
Residual	9.20E+03	6	1.53E+03		
Lack of Fit	8.98E+03	5	1.80E+03	8.1417	0.2597 not significant
R-Squared			0.980648		
Adj R-Squared			0.970403		

To check whether the fitted model actually describe the experimental data, the multiple regression coefficient (R^2) has been calculated. The R^2 value for the MRR and cutting temperature has been found to be 0.9806 and 0.9907 and it shows that the second order model can explain the variation in the temperature up to the extent of 98.06% and 99.07%. Figs. 4 and 5 show the normal probability plots of the residuals for the output response.

Table 4 ANOVA of cutting temperature

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	3.88E+05	9	4.32E+04	201.01	< 0.0001 significant
x_1	9.08E+03	1	9.08E+03	42.291	< 0.0001
x_2	6.34E+04	1	6.34E+04	295.18	< 0.0001
x_3	2.17E+04	1	2.17E+04	101.09	< 0.0001
x_1x_2	1.03E+03	1	1.03E+03	4.7836	0.0430
x_1x_3	6.51E+01	1	6.51E+01	0.3030	0.5891
x_2x_3	8.96E+02	1	8.96E+02	4.1750	0.0568
x_1x_1	1.29E+03	1	1.29E+03	6.0130	0.0253
x_2x_2	1.07E+04	1	1.07E+04	49.784	< 0.0001

x_3x_3	3.17E+02	1	3.17E+02	1.4759	0.2410	
Residual	3.65E+03	17	2.15E+02			
Lack of Fit	7.98E+03	5	1.80E+03	8.1417	0.45	not significant
R-Squared			0.9907			
Adj R-Squared			0.985763			

Table 5 Normalized values and grey relational coefficients

Exp. No.	Normalized Values		Δ_{oi}	
	MRR grr/min	Temp. °C	MRR grr/min	Temp. °C
1.	0.0000	1.0000	1.0000	0.0000
2.	0.0461	0.9711	0.9539	0.0289
3.	0.0761	0.8182	0.9239	0.1818
4.	0.0452	0.8150	0.9548	0.1850
5.	0.0908	0.7413	0.9092	0.2587
6.	0.1213	0.5853	0.8787	0.4147
7.	0.3132	0.4083	0.6868	0.5917
8.	0.3593	0.3105	0.6407	0.6895
9.	0.3888	0.1238	0.6112	0.8762
10.	0.2685	0.9052	0.7315	0.0948
11.	0.3145	0.8527	0.6855	0.1473
12.	0.3445	0.7234	0.6555	0.2766
13.	0.3132	0.7462	0.6868	0.2538
14.	0.3593	0.6725	0.6407	0.3275
15.	0.3894	0.5165	0.6106	0.4835
16.	0.5817	0.3691	0.4183	0.6309
17.	0.6277	0.2714	0.3723	0.7286
18.	0.6571	0.0847	0.3429	0.9153
19.	0.6413	0.7437	0.3587	0.2563
20.	0.6874	0.6912	0.3126	0.3088
21.	0.7171	0.5619	0.2829	0.4381
22.	0.6861	0.6106	0.3139	0.3894
23.	0.7321	0.5370	0.2679	0.4630
24.	0.7617	0.3809	0.2383	0.6191
25.	0.9545	0.2633	0.0455	0.7367
26.	1.0006	0.0000	-0.0006	1.0000
27.	0.7321	0.0261	0.2679	0.9739

This plot reveals that the residuals are located on a straight line, which means that the errors are distributed normally on the regression model so that the model predicted is well fitted with the observed values.

6. Implementation of GRA

In the procedure of GRA, the responses are normalized as the first step using the equations 2 and 3 as shown in Table 5. As a part of the estimation of grey relational coefficients, the quality loss estimates of each individual has been calculated and listed in Table 5. Then the individual gray relational grades and the overall gray relational grade have been calculated by using Eq. 4 and Eq. 6 and are shown in Table 6. Here, the value of distinguishing coefficient is assumed as 0.5. The overall gray relational grade represents the quality index of multiple responses of the process; hence, the multi-objective optimization problem

has been converted in to single-objective optimization problem.

Table 6 Gray relational grade and Ranks

Exp. No.	$\xi_i(k)$			Rank
	MRR grr/min	Temp. °C	γ_i	
1.	0.3333	1.0000	0.6667	2
2.	0.3439	0.9453	0.6446	4
3.	0.3511	0.7333	0.5422	14
4.	0.3437	0.7299	0.5368	16
5.	0.3548	0.6591	0.5069	18
6.	0.3626	0.5466	0.4546	24
7.	0.4213	0.4580	0.4397	25
8.	0.4383	0.4204	0.4293	26
9.	0.4500	0.3633	0.4066	27
10.	0.4060	0.8407	0.6233	5
11.	0.4218	0.7724	0.5971	8
12.	0.4327	0.6439	0.5383	15
13.	0.4213	0.6633	0.5423	12
14.	0.4383	0.6042	0.5213	17
15.	0.4502	0.5084	0.4793	22
16.	0.5445	0.4421	0.4933	20
17.	0.5732	0.4070	0.4901	21
18.	0.5932	0.3533	0.4732	23
19.	0.5823	0.6612	0.6217	6
20.	0.6153	0.6182	0.6167	7
21.	0.6386	0.5330	0.5858	10
22.	0.6143	0.5622	0.5882	9
23.	0.6512	0.5192	0.5852	11
24.	0.6772	0.4468	0.5620	12
25.	0.9166	0.4043	0.6604	3
26.	1.0012	0.3333	0.6673	1
27.	0.6512	0.3392	0.4952	19

Therefore, the overall grey relational grades rank the experimental runs as; the experimental run having higher grey relational grade refers as that corresponding combination of variables is closer to the optimal values as listed in the Table 6. The optimal parametric combination is then evaluated by maximizing the overall grey relational grade. The optimal set of input parameters is DOC=0.3mm, feed 0.5 mm/tooth and speed 180 m/min and the optimal values of the out response obtained are 0.07254 grms/min metal removal rate and 591.95°C cutting temperature.

7. Conclusions

This paper aimed to develop the empirical models and investigate the optimal machinability parameters of milling process during machining EN 31 tool steel. In this consequence, milling experiments were conducted on vertical milling centre based on central composite design with 27 experiments. The response surface methodology was adopted to develop the mathematical models for the responses and

ANOVA is used to check the adequacy of the developed models and were found that the developed second order models can explain the variation in the temperature up to the extent of 98.06% and 99.07%. Then these experimentally measured values were carried to the optimization. GRA was successfully implemented to the measured experimental runs. The resulted optimal values of the milling process were listed. Hence, an operator can easily find out the optimal machining conditions without compromising at either metal removal rate or the cost of tooling with this investigation.

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