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Modelling and Control of Coupled Tank Liquid Level System using Backstepping Method

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Abstract- **The level and flow control in tanks are the heart of all chemical engineering system. The control of liquid level in tanks and flow between tanks is a basic problem in the process industries. Many times the liquids will be processed by chemical or mixing treatment in the tanks, but always the level of fluid in the tanks must be controlled and the flow between tanks must be regulated in presence of non-linearity, disturbance and time varying system parameters. This work introduce the approach of modelling and compute a level backstepping control strategy with pure feedback form for non-linear modelled coupled tanks system.The goal of the control algorithm is to track the desired level of liquid in second tank by using flow rate of liquid into first tank as the manipulated variable. The designed non-linear controller is capable of tracking the desired water level for all set points with high degree of accuracy, maximally fast and without significant overshoot.**

Keywords---Backstepping, Coupled tank, Non-linear model, Process control

I. INTRODUCTION

The liquid level control in Coupled Tank System is a classical benchmark control problem. Level control is one of the control system variable which are more important in process industries. The process industries requires liquid to be pumped as well as stored in tanks and then repumped to another tank. Many times the liquids will be processed by chemical or mixing treatment in the tanks, but always the level of fluid in the tanks must be controlled and the flow between tanks must be regulated. The quality of the product of the mixture depends on the level of the reactants in the mixing tank. Tank level control systems are used frequently in different processes. All of the pharmaceutical industries, petrochemical plants, food/beverage industries and nuclear power plants depend upon tank level control systems. It is essential for control system engineers to understand how tank control systems work and how the level control problem is solved. The liquid level system has time varying system parameters and non-linear characteristics in the complex industrial process. Most of the control performances in the actual design are usually defined by overshoots, rising time, settling time, steady state error etc.

Various attempts in controlling liquid level of coupled tank system were proposed. The design of PI controller using Characteristics Ratio Assignment method for linear modelled coupled tank SISO process was proposed by M. Senthilkumar et al [1].The mathematical modelling and designing of Sliding Mode Control for a liquid level control system when tanks are coupled by using baffles was proposed by Hur Abbas et al [2] and a fuzzy logic controller for liquid level control introduced by Abdelelah et al [3].Muhammad Nasiruddin Mahyuddin et al proposed a Direct Model Reference Adaptive Control for Coupled Tank System [4] and Comparison between PI and MRAC on coupled tank system done by M. Saad et al [5].

This paper presents the mathematical modelling of non-linear coupled tank system andintroduce a level backstepping control strategy with pure feedback form for coupled tank liquid level system and the results are compared with conventional PID controller.

The structure of this paper is as follows. Section 2 deals with the system description.The non-linear modelling of the coupled tank system is explained in section 3. Section 4 highlights the Backstepping control designs. Simulation results with PID and Backstepping method is given in section 5. Conclusion is discussedin section 6.

II. SYSTEM DESCRIPTION

The coupled tank system includes two tanks mounted above a reservoir, which function as a storage for liquid. It has an independent pump to pump liquid from reservoir to tanks. The two tanks are connected in an interactive manner. When two tanks are coupled, the liquid in two tanks interact and exhibit a non-linear behavior. The liquid meets resistance when flowing through a conduit such as a pipeline. If a liquid flow through the pipe is under turbulent flow condition, the outlet flow rate being function of the square root of the tank height. The discharge coefficient of liquid flowing out of tank can vary by using valves.

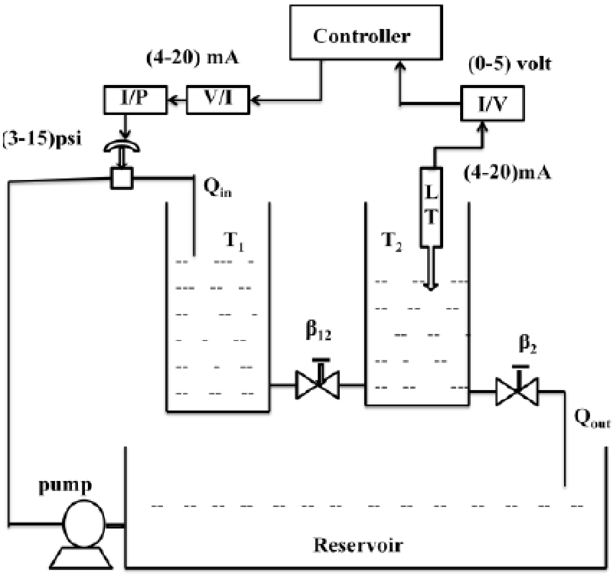

Figure 1: Block Diagram of Coupled Tank System

Both tanks are identical in cross section and is represented as A(cm^2). The inlet flow Q_{in} is given to the tank 1 and the outlet flow Q_{out} is taken from tank 2. A manual valve is available between tank1 and tank2 which can be used to change the interaction between the tanks. The change in water level h_1(cm) in tank1 affects the water level h_2(cm) in tank 2. The water level variation in tank1 and tank2 depends on the inlet and outlet flows. The liquid level in second tank ie, h_2 (cm)is maintained at some desired value by using flow rate of the liquid into first tank Q_{in}(cm^3/sec) as the manipulated variable.

The control of liquid level in tanks presents a challenging problem due to its non-linear behavior which is due to the interacting characteristics. In interacting process, dynamics of tank1 affects the dynamics of tank2 and vice versa because flow rate depends on the difference between the liquid levels.

III. MATHEMATICAL MODELLING OF COUPLED TANK SYSTEM

Let,

h_1 and $h_2$be the height of liquid in tank 1 and tank 2 respectively (cm)

A_1 and A_2 be the cross-sectional area of tank 1 and tank 2 respectively (cm^2)

Q_{in} be flow rate of liquid into tank 1(cm^3/sec)

Q_{out} be flow rate of liquid out of tank 2(cm^3/sec)

a_2 be the cross sectional area of outlet pipe in tank 2 (cm^2)

a_{12} be the cross sectional area of interaction pipe between tank 1 and tank 2 (cm^2)

β_{12}be the valve ratio of interaction pipe between tank 1 and tank 2

β_2 be the valve ratio of outlet pipe of tank 2

g be the acceleration due to gravity

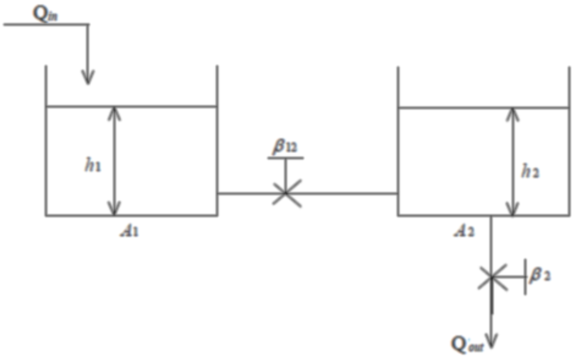

Figure 2: The Coupled tank SISO Process

It is assumed that the liquid used is non-viscous, incompressible. The nonlinear equation of the coupled tank system can be obtained by mass balance equation and it is given by,

Rate of change of mass in the tank = Mass flow in - Mass flow out

ie,$\frac{Adh}{dt} = Q_{in} - Q_{out}$

The dynamic equations for tank 1:

$$A_1 \frac{dh_1(t)}{dt} = Q_{in} - \beta_{12} a_{12}\sqrt{2g[h_1(t) - h_2}(t)]$$

U(t) = Q_{in}

$$\frac{dh_1(t)}{dt} = \frac{u(t)}{A_1} - \frac{\beta_{12} a_{12}}{A_1}\sqrt{2g[h_1(t) - h_2}(t)] \qquad (1)$$

The dynamic equations for tank 2:

$$A_2 \frac{dh_2(t)}{dt} = \beta_{12} a_{12}\sqrt{2g[h_1(t) - h_2}(t)$$

$$-\beta_2 a_2\sqrt{2gh_2}(t)$$

$$\frac{dh_2(t)}{dt} = \frac{\beta_{12} a_{12}}{A_2}\sqrt{2g[h_1(t) - h_2}(t)]$$

$$-\frac{\beta_2\, a_2}{A_2}\sqrt{2gh_2} \qquad (2)$$

At equilibrium, for constant water level setpoint, the derivatives must be zero ie, $\dot{h}_1$= $\dot{h}_2$= 0. In addition, for the case when $h_1 = h_2$, the system model is decoupled. So $h_1 > h_2$.

Parameters	Value
$A_{1,}A_2\ (cm^2)$	154
$a_{2,}a_{12}\ (cm^2)$	0.5
β_{12}	1.5315195
β_2	0.6820043
g $\left(cm^2 / sec \right)$	981

Table 1: Parameters of Coupled Tank System

Let,

$z_1 = h_2 > 0$ and $z_2 = h_1 - h_2 > 0$

z = $\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$, u = q (t)

and

b $= \frac{\beta_{12} a_{12}}{A_2}\sqrt{2g}$, c = $\frac{\beta_2\ a_2}{A_2}\sqrt{2g}$, a = $\frac{1}{A_1}$

The output of the coupled tank system is taken to be the level of the second tank. Therefore, the dynamic model of coupled tank in eqs. (1) and (2) can be written as:

$\dot{z_1}$= b$\sqrt{z_2}$ - c$\sqrt{z_1}$ (3)

$\dot{z_2}$= au -2b$\sqrt{z_2}$ + c$\sqrt{z_1}$ (4)

y = z_1

The objective of the control scheme is to regulate the output y (t) = $z_1(t) = h_2(t)$ to a desired value $h_2(des)$. The dynamic model of the coupled tank system is highly non-linear. Therefore, we will define a transformation so that the dynamic model of the coupled tank system can be transformed into a form facilitates the control design.

Let,x= $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and define the transformation

x = T(z) such that

$x_1 = z_1$ (5)

x_2 =b$\sqrt{z_2}$ - c$\sqrt{z_1}$ (6)

The inverse transformation z = $T^{-1}(x)$ is such that

$z_1 = x_1$ (7)

$z_2 = \left(\frac{c\sqrt{x_1}+x_2}{b}\right)^2$ (8)

It can be checked that we can write the dynamic model of coupled tank system in eqn (5) and (6) can be written as:

$\dot{x_1} = x_2$ (9)

$\dot{x_2} = \frac{b\dot{z_2}}{2\sqrt{z_2}} - \frac{c\dot{z_1}}{2\sqrt{z_1}}$ (10)

Substitute the values of $\dot{z_1}$ and $\dot{z_2}$ in eqn (10), we get

$\dot{x_2} = \frac{bc}{2}\left[\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right] + \frac{c^2}{2} - b^2 + \frac{ab}{2\sqrt{z_2}}u$(11)

Where the values of z_1 and z_2 in above equation are function of $x_1$and x_2 as given in eqn (7) and (8).

Hence dynamic model of the system can be written as:

$\dot{x_1} = x_2$ (12)

$\dot{x_2} = f + \varnothing u$ (13)

y = x_1 (14)

Where, f = $\frac{bc}{2}\left[\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right] + \frac{c^2}{2} - b^2$

$$\varnothing = \frac{ab}{2\sqrt{z_2}}$$

The dynamic model in eqn (12), (13), (14) will be used to design backstepping control techniques for the coupled tank system.

IV. CONTROLLER DESIGN

In this section, we will define a controller design using backstepping method which is in pure feedback form for the coupled tank system.Backstepping control design is based on Lyapunov theory. The objective is to construct a control law that brings the system to some desired state. This is to say, we wish to make this state a stable equilibrium of the closed loopsystem.

Consider the system affine in the control input:

$$\dot{x} = f(x) + g(x)u$$

Where x Ɛ R^n and u Ɛ R represent respectively the state variables and the control input of the system. Firstly, 'u' is regarded as the control input for the x-subsystem. 'u' can be chosen in any way to make the x-subsystem globally asymptotically stable. The choice is denoted $u^{des}(x)$ and is called a virtual control law. First we define the control error e_1 such that :

$$e_{1=} x_1 - x_1(des)$$

$x_1(des)$ is the desired set point and we select the following Lyapunov candidate function :

$V_1(e_1) = \frac{1}{2}{e_1}^2$, which is a positive definite function and its derivative must be a negative definite function.

ie, $\dot{V_1}(e) \leq -W(x) \leq 0$, [where $W(x)$ must be positive definite] then we can say that the system is globally bounded. So the control input 'u' ensures the objective of stability and asymptotic performance.The control objective is to regulate the output y (t) = $x_1(t) = h_2(t)$ to a desired value $h_2(des)$ [$h_2(des) = x_1(des)$] . The time derivative of above Lyapunov candidate function is given by :

$$\dot{V_1}(e_1) = e_1\dot{e_1}$$
$$= e_1\,\dot{x_1} = e_1\,x_2$$

Where x_2 chosen such that $\dot{V_1}(e)$ must be a negative definite.

$$x_2\,(new) = -c_1\,e_1$$

Due to $x_2\,(new)$ a second error generate which is given by,

$$e_2 = x_2(new) - x_2$$
$$\dot{e}_2 = x_2(\dot{new}) - \dot{x}_2$$

Let us now consider the second Lyapunov function to ensure that the system become globally asymptotically stable,

$$V_2(e_2) = V_1(e_1) + \frac{1}{2}{e_2}^2$$

$$= \frac{1}{2}[{e_1}^2 + {e_2}^2]$$

Which should a positive definite function and its derivative should be negative definite function.

$$V_2(\dot{e}_2) = e_2\dot{e}_2 = e_2[x_2(\dot{new}) - \dot{x}_2]$$
$$= e_2[x_2(\dot{new}) - \{ f + øu\}]$$

To make $V_2(\dot{e}_2)$ a negative definite function, the control law should be such that:

$$u(t) = \frac{x_2(\dot{new}) - f + C_2e_2}{ø} \text{ where } C_2 > 0$$

Where,

$$f = \frac{bc}{2}\left[\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right] + \frac{c^2}{2} - b^2$$

$$ø = \frac{ab}{2\sqrt{z_2}}$$

Where the values of z_1 and z_2 are function of x_1 and x_2 as given in eqn (7) and (8).The values of f and ø in terms of x_1 and x_2 is given by,

$$f = \frac{bc}{2}\left[\frac{b\sqrt{x_1}}{c\sqrt{x_1+x_2}} - \frac{c\sqrt{x_1+x_2}}{b\sqrt{x_1}}\right] + \frac{c^2}{2} - b^2$$

$$ø = \frac{ab^2}{2[c\sqrt{x_1 + x_2}]}$$

So the control law becomes,

u (t) =

$$\frac{x_2(\dot{new}) - \{\frac{bc}{2}\left[\frac{b\sqrt{x_1}}{c\sqrt{x_1+x_2}} - \frac{c\sqrt{x_1+x_2}}{b\sqrt{x_1}}\right] + \frac{c^2}{2} - b^2\} + C_2e_2}{\frac{ab^2}{2[c\sqrt{x_1+x_2}]}} \text{Where} C_2 > 0$$

V. RESULTS AND DISCUSSION

The response of the system for different operating level and set point tracking performance of the system with designed level backstepping control strategy in pure feedback form are observed. The obtained result with backstepping method is compared with conventional PID controller.

In figure 3, the response of the system for different operating level is shown. The system achieves consistent performance and maintains the desired transient response characteristic throughout all operating points [at 20 cm, 40 cm, 60 cm]. The set point tracking test consist of changing the set point consecutively during the operation, which is shown in figure 4. The set point change is done at 60 second by a magnitude of 30 cm height in water level. It can be seen that the level backstepping control strategy tracks the set point changes in water level accurately. The results obtained from backstepping method is compared with conventional PID controller. Zeigler– Nichols open loop method of tuning is used to obtain the parameters of PID controller. The PID controller exhibit inconsistent transient response performance with a peak overshoot and approximately 35 sec take to settle. The proposed backstepping control strategy with pure feedback form provide better transient response with no overshoot and the settling time reduces to 30 sec as compared to PID controller.

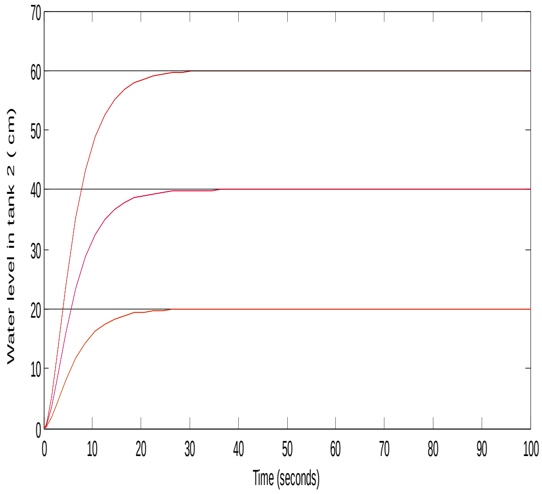

Figure 3: Response of the system for different operating level

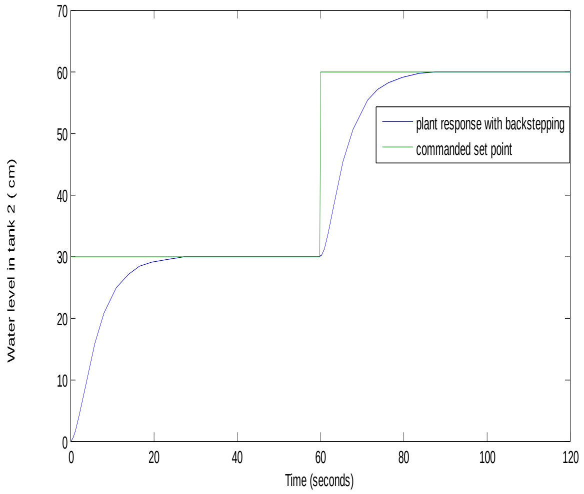

Figure 4: Set point tracking performance of the system

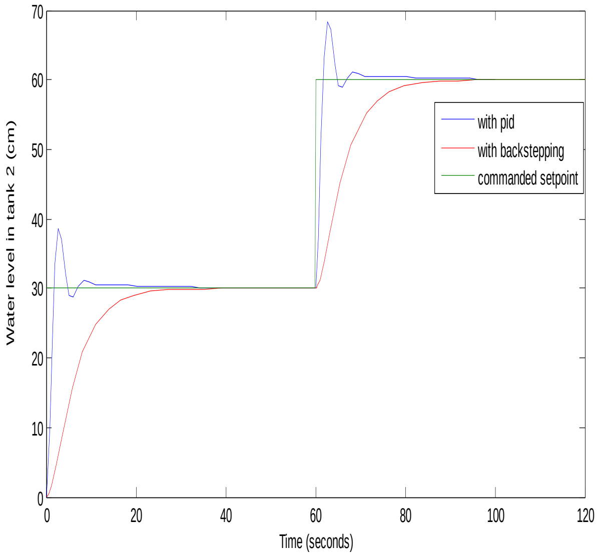

Figure 5: Comparison of Backstepping method with conventional PID

VII CONCLUSIONS

It can be shown that the level backstepping control strategy can cope with the coupled tank non-linear characteristics at all operating points (water level or height in tank 2). PID controller exhibit inconsistent transient performance at all set point changeand this shows that fixed controllers are not able to sustain the predefined transient response at all operating points. The designed non-linear controller is able to sustain the desired transient response throughout the set point changes without significant overshoot, maximally fast and with high degree of accuracy

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