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Modeling the Response of a Memcapacitor for Impulse, Step, Ramp, and Sinusoidal Inputs

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***Abstract* - Micro-Electro-Mechanical Systems, or MEMS, is a technology of very small scale devices. The dimensions of MEMS can vary from below one micron to several millimeters. MEMS have some mechanical functionalities such as the moving plate of a parallel plate capacitor (memcapacitor). MEMS researchers and developers have demonstrated an extremely large number of microsensors for almost every possible sensing modality including temperature, pressure, inertial forces, and chemical species. The equation of motion of the moving plate of a memcapacitor is governed by a non-linear differential equation with no known exact solution. Most research into determining the theoretical response of a memcapacitor to a time varying voltage, was done for the steady-state case. Non-linearity of the displacement of the plate in a memcapacitor presents a challenge in determining the plate's position and capacitive detection. This paper presents an analytical closed form solution to the non-linear differential equation, without the steady state assumption for an Impulse input, and an approximate solution for a step, ramp, and sinusoidal inputs.**

Keywords— ApproximateSolution, CapacitiveDetection, Memcapacitor.

I. INTRODUCTION

Many MEMS devices are electrostatic-driven such as capacitive pressure sensors[1], comb drives [2], RF switches [3], and inkjet printer head [4]. Electrostatic actuators are prevalent in Micro-Electro-Mechanical systems (MEMS) since they are compatible with microfabrication technology, have a low power consumption, and the electrostatic forces are large enough to drive a micro-motor. Electro - static actuatorscan be produced using the same micromachining technology that was developed for producing microelectronic systems [5], [6].

Due to the non-linear nature of some electrostatic forces, the electromechanical response of many electrostatic actuators is non-linear and their stable range may be limited by the pull-in-stability. The motion of the flexible plate of a MEMS device is governed by asecond order non-linear differential equation. There is no known general solution for that equation. However, with some approximations we can get anapproximate analytical solution that will reveal the characteristics of the solution for different inputs such as pulse, step, ramp and sinusoidal.

This paper has four parts, one for each input, the pulse $\delta(t)$, the step $u(t)$, the ramp t, and the sinusoid $\sin(wt)$. In each part, we discuss the analytical approximation of the characteristic differential equation for each input and then compare it with its numerical (MATLAB) solution.

II. CHARACTERISTIC DIFFERENTIAL EQUATION OF A NON-FRINGING MEMS AND AN IMPULSE RESPONSE

In this section the impulse response of a MEMS is analyzed. As a simple model, consider the MEMS shown in Fig.1.

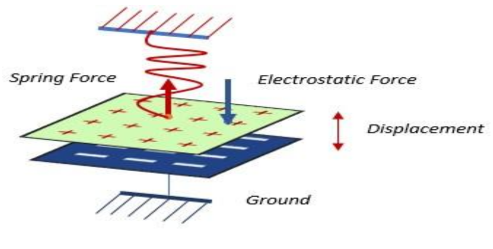

Fig.1. MEMS model.

The MEMS device under examination consists of a top electrode of mass m and area A suspended from a linear spring with stiffness k and a bottom fixed electrically grounded electrode. The initial separation of the electrodes is d. When a voltage is applied across the plates, the electric force applied on the top electrode pulls it towards the bottom electrode, once the electrode is displaced, an elastic recovery force by the spring tends to pull the upper plate back towards its original position as shown in Fig.2. [7].

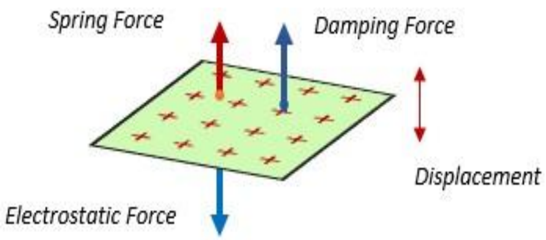

Fig.2. Free body diagram of the top electrode.

The forces acting on the top electrode are:
The damping force,

$$F_d = d_c \frac{dy(t)}{dt} \quad (1)$$

the spring force,

$$F_s = ky(t) \quad (2)$$

and the electrostatic force,

$$F_e = \frac{A\varepsilon_0 V_s^2}{2(d-y(t))^2} \quad (3)$$

Where $y(t)$ is the displacement, A is the area of the upper plate, d_c is the damping coefficient, ε_0 is the permittivity of free space, and V_s is the input voltage. Applying the first principle of dynamics,

$$\sum F = ma$$

we get

$$m\frac{d^2y(t)}{dt^2} + d_c\frac{dy(t)}{dt} + ky(t) = \frac{A\varepsilon_0 V_s^2}{2\left(d-y(t)\right)^2} \quad (4)$$

The displacement $y(t)$ depends on the value of the applied voltage. To avoid collapsing the two electrodes, the maximum distance that the upper plate can travel is[7].

$$y = \frac{d}{3} \quad (5)$$

and that corresponds to the pull-in-voltage

$$V_s = \sqrt{\frac{8kd^3}{27A\varepsilon_0}} \quad (6)$$

Any voltage above the pull-in-value will cause the collapse of the two electrodes.

If V_s is assumed to be a very narrow pulse,i.e. $V_s = \delta(t)$, equation (4) becomes:

$$m\frac{d^2y(t)}{dt^2} + d_c\frac{dy(t)}{dt} + ky(t) = \frac{A\varepsilon_0\left(\delta(t)\right)^2}{2(d-y(t))^2} \quad (7)$$

which has the exact solution

$$y(t) = \left(\frac{A\varepsilon_0}{2d^2c_1m}\right)\exp\left(-\frac{d_c}{2m}t\right)\sinh\left(c_1t\right) \quad (8)$$

where $c_1 = \sqrt{\left(\frac{d_c}{2m}\right)^2 - \frac{k}{m}}$.

With parameters for the MEMS device provided by the Universidad Autonoma de Ciudad Juarez, MX, which are parameters used in a design using SUMMIT-V technology and considering a vacuum level of 1×10^{-5} Torr, namely,

$$\varepsilon_0 = 8.854 \times 10^{-12} \ \mathrm{F/m}$$
$$d = 5 \times 10^{-6} \ \mathrm{m}$$
$$k = 0.3125 \ \mathrm{N/m}$$
$$d_c = 10^{-4} \ \mathrm{Kg/s}$$
$$m = 2 \times 10^{-6} \ \mathrm{Kg}$$
$$A = 9 \times 10^{-8} \ \mathrm{m^2}$$

the pull-in-voltage becomes 3.811 V.Two plots of equation (8) for different impulse inputs (very narrow pulses with peaks of 1 and 0.033) are shown in Figs3 and 4 respectively.

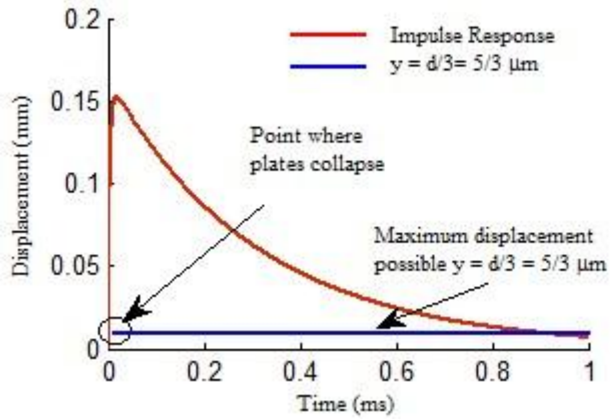

Fig.3. Impulse response with pulse $V_s = \delta(t)$ V.

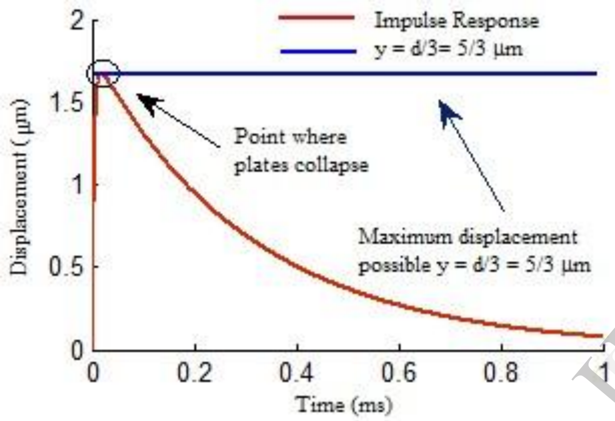

Fig.4.Impulse response with pulse $V_s = 0.033\delta(t)$ V.

The result in Fig.3 shows that there is overshooting caused by the electrostatic force due to the application of an impulse with magnitude 1 V which is less than the pull-in-voltage. That is, the system will overshoot if the voltage is ramped to its nominal value rapidly and the electrodes may collapse. Figure 4 shows that the system will not overshoot with a magnitude of 0.033V.To avoid overshooting caused by the electrostatic force, the voltage should ramp up to its nominal value slowly or the structure should be heavily damped [7].

III. STEP INPUT RESPONSE AND ANALYSIS

In this section the step response is analyzed and an approximate closed form solution for the displacement y isobtained. Let$V_s = Bu(t)$,the step input with a magnitude$B$, then (4) becomes

$$m\frac{d^2y(t)}{dt^2} + d_c\frac{dy(t)}{dt} + ky(t) = \frac{A\varepsilon_0 B^2}{2\left(d-y(t)\right)^2} \quad (9)$$

with $y(t) < d$ for all t, we have

$$\frac{A\varepsilon_0 B^2}{2\left(d-y(t)\right)^2} = \frac{A\varepsilon_0 B^2}{2d^2}\left(1 + \frac{2y}{d} + \sum_{n=2}^{\infty}(n+1)\frac{y^n}{d^n}\right)$$

$$\text{or} \sum_{n=2}^{\infty}(n+1)\frac{y^n}{d^n} = \left(\frac{1}{\left(1-\frac{y}{d}\right)^2} - 1 - \frac{2y}{d}\right) (10)$$

For a step input, $y(t)$ is an increasing function of time.The minimum value of $y(t)$ is 0, and the maximum value is the steady state value. The steady state value is obtained by solving (7) under steady state conditions that is solving [8]

$$ky(t) = \frac{A\varepsilon_0 B^2}{2(d-y(t))^2} \quad (11)$$

With thegiven parameters of the MEMS device,and$B$ =1,(11)yields three solutions, which are $y_1 = 4.465 \times 10^{-6}$, $y_2 = 5.207 \times 10^{-8}$ and $y_3 = 5.482 \times 10^{-6}$. Solution $y_3 > d$ is not a physical solution, and by (5) solution $y_1 > \frac{d}{3}$ is not a stable solution, since for that value of y the two plates collapse, thus the only stable solution is $y_2 = 5.207 \times 10^{-8}$.

Thus $y_{\min} = 0$and $y_{\max} = 5.207 \times 10^{-8}$ substitutingin(10)weget,

$$\left(\frac{1}{\left(1-\frac{y}{d}\right)^2} - 1 - \frac{2y}{d}\right) = \begin{cases} 0 \text{ for } y = 0 \\ 4.5613 \times 10^{-6} \text{ for } y = 5.207 \times 10^{-8} \end{cases}$$

In either case the value obtained is too small compared with

$$\left(1+\frac{2y(t)}{d}\right)=\begin{cases}1 \text{ for } y=0\\[2ex] 1.0208 \text{ for } y=5.207\,\text{x}\,10^{-8}\end{cases}$$

Therefore,

$$\frac{A\varepsilon_0 B^2}{2d^2}\left(1+\frac{2y}{d}+\sum_{n=2}^{\infty}(n+1)\frac{y^n}{d^n}\right)\approx\frac{A\varepsilon_0 B^2}{2d^2}\left(1+\frac{2y}{d}\right)$$

Thus (7) simplifies to

$$m\frac{d^2y}{dt^2}+d_c\frac{dy}{dt}+ky\approx\frac{A\varepsilon_0 B^2}{2d^2}\left(1+\frac{2y}{d}\right) \qquad (12)$$

With $\left(\frac{k}{m}-\frac{A\varepsilon_0 B^2}{md^3}\right)-\frac{d_c^{\,2}}{4m^2}<0$

and $y(0)=0, y'(0)=0$ and $y''(0) = \frac{A\varepsilon_0 B^2}{2md^2}$

the solution of (12) is

$$y(t)=N_1\exp(-(c_2+a)t)+N_2\exp((c_2-a)t)+c_3$$
(13)

where

$$c_3=\frac{A\varepsilon_0 B^2 d}{2\left(kd^3-A\varepsilon_0 B^2\right)},\ a=\frac{d_c}{2m}$$

$$c_2=\sqrt{\frac{d_c^{\,2}}{4m^2}+\frac{A\varepsilon_0 B^2}{md^3}-\frac{k}{m}},$$

$$N_1=-\frac{c_3h_1}{2c_2},\quad N_2=-\frac{c_3h_2}{2c_2}$$

Figs 5 and6 show the numerical (MATLAB) and analytical solutions respectively.

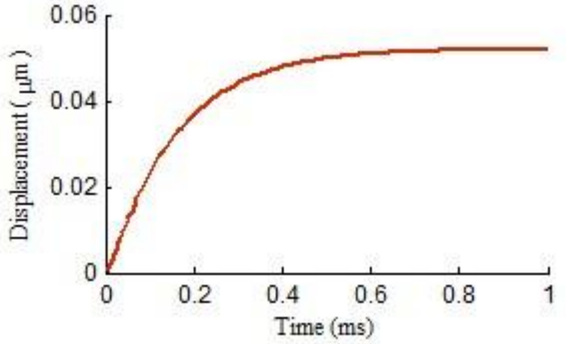

Fig.5. Numerical (MATLAB) solution with $V_s = u(t)$.

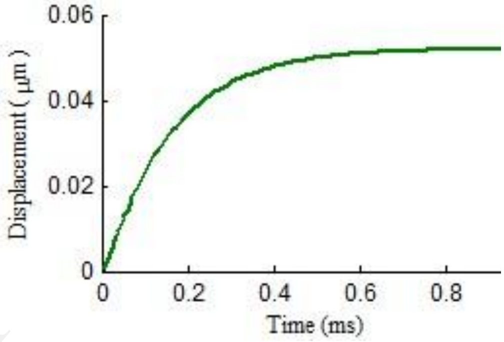

Fig.6. Analytical solution with $V_s = u(t)$.

The percentage error between the two analyses is $\approx 0\%$.

Figs7 and 8 show the comparison between the numerical (MATLAB) and analytical solutions for inputs $V_s = 2u(t)$ and $V_s = 3u(t)$ respectively. The percent errors obtained were 0.6% and 4.6%respectively.

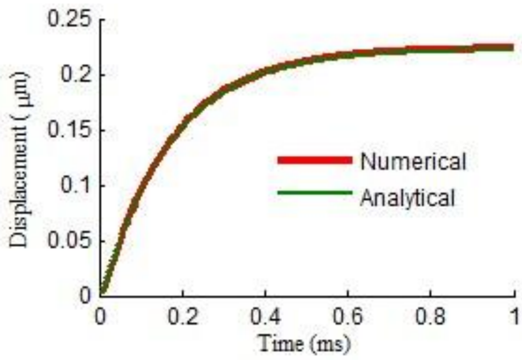

Fig.7. Numerical (MATLAB) and Analytical solutions with $V_s = 2u(t)$.

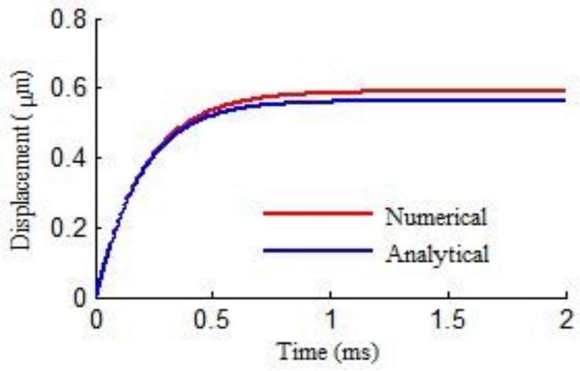

Fig.8. Numerical (MATLAB) and Analytical solutions with $V_s = 3u(t)$.

Since the electrostatic force is an increasing function of the voltage applied, the higher the voltage the larger the force and the longer the displacement of the plate. The error between the actual value of the displacement and the approximate one becomes larger but constant. At a voltage $V = V_{po}$, the pull-in voltage, the error found was 28%. However for all step inputs the transient part of the response is accurate enough to be used along with the steady state solution found by [8] and [9].

IV.RAMP INPUT RESPONSE AND ANALYSIS

In the previous section, we discussed the response of a MEMS to a step input. Since the steady state value was independent of time or pulse width, we found out that the position of the moving plate could be determined with a high degree of closeness to the numerical (MATLAB) value. In the case of a ramp input, the final position of the plate depends on the pulse width. If $V_s(t) = t$, then (4) becomes

$$m\frac{d^2y}{dt^2} + d_c\frac{dy}{dt} + ky = \frac{A\varepsilon_0 t^2}{2(d-y)^2} \quad (14)$$

With $y(0) = 0, y'(0) = 0$, and $B = 1$, we can obtain the ramp response by integrating (13) twice to obtain

$$y(t) = \frac{N_1\left(\exp\left(-(c_2+a)t\right)+1\right)}{\left(c_2+a\right)^2} + \frac{N_2\left(\exp\left((c_2-a)t\right)-1\right)}{\left(c_2-a\right)^2} + \frac{c_3 t^2}{2} + \frac{N_1(c_2-a) - N_2(c_2+a)}{\left(c_2{}^2 - a^2\right)}t \quad (15)$$

Figs9, 10, and 11 show a comparison between the analytical and numerical solutions for (14) with pulse widths of 1, 2, and 3 s respectively. The percentage error between the two analyses wereapproximately 0%, 6%, and 20% respectively.

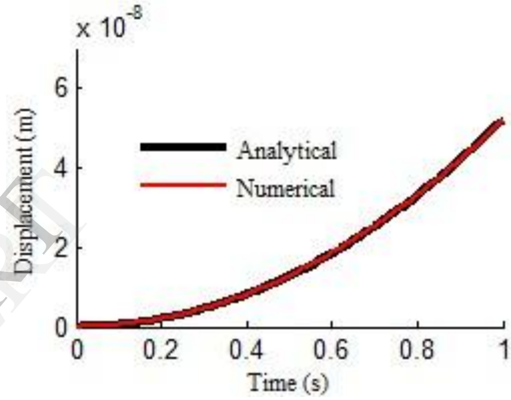

Fig.9. Numerical and Analytical solutions with $V_s = t$.
Error = 0 %

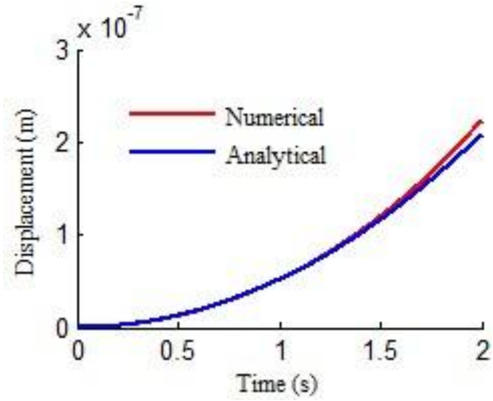

Fig.10. Numerical and Analytical solutions with $V_s = t$.
Error = 6 %

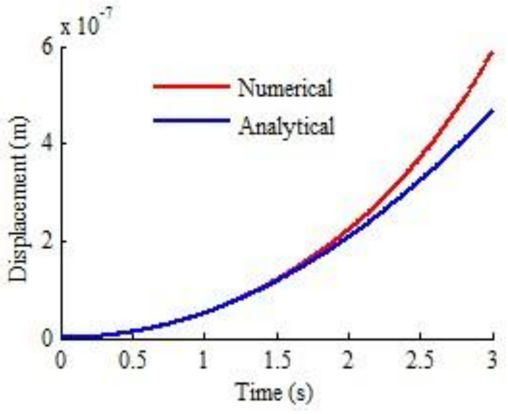

Fig.11. Numerical and Analytical solutions with $V_s = t$.
Error = 20 %

The analysis of a ramp input showed that the approximation done is impractical for $t > 2.5$. As shown in Fig.12, at $t = 2.5$ s the error is 6% and increases rapidly to 20 % at $t = 3$ s.

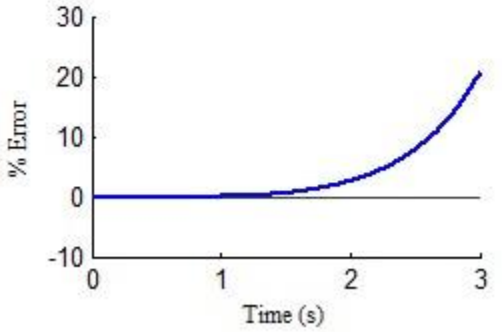

Fig.12. Percent Error between Numerical and Analytical solutions with $V_s = t$.

V. SINUSOIDAL INPUT RESPONSE AND ANALYSIS

In this section the sinusoidal response is analyzed and an approximate closed form solution for the displacement y is obtained. Let $V_s(t) = V_m \sin(wt)$ then (4) becomes

$$m\frac{d^2y}{dt^2} + d_c\frac{dy}{dt} + k\,y = \frac{A\varepsilon_0 V_m{}^2 \sin^2(wt)}{2(d-y)^2} \quad (16)$$

Expanding the right hand side of (16) as a geometric series with $V_m = 1$,

$$\frac{A\varepsilon_0 \sin^2(wt)}{2(d-y)^2} = \frac{A\varepsilon_0 \sin^2(wt)}{2d^2\left(1-\frac{y}{d}\right)^2} =$$

$$\frac{A\varepsilon_0 \sin^2(wt)}{2d^2}\left(\sum_{n=0}^{\infty}\left(\frac{y}{d}\right)^n\right)^2$$

$$= \frac{A\varepsilon_0 \sin^2(wt)}{2d^2}\left(\sum_{n=0}^{\infty}(n+1)\left(\frac{y}{d}\right)^n\right)$$

$$= \frac{A\varepsilon_0 \sin^2(wt)}{2d^2}\left(\sum_{n=0}^{\infty}(n+1)\left(\frac{y}{d}\right)^n\right)$$

$$\sum_{n=1}^{\infty}(n+1)\left(\frac{y}{d}\right)^n = \left(\frac{1}{\left(1-\frac{y}{d}\right)^2} - 1\right)$$

Then for $y_{\max} = \frac{9A\varepsilon_0}{8d^2k} = 1.1475\text{x}10^{-7}$ we get,

$$\left(\frac{1}{\left(1-\frac{y}{d}\right)^2} - 1\right) = \left(\frac{1}{\left(1-\frac{1.1475\text{x}10^{-7}}{5\text{x}10^{-6}}\right)^2} - 1\right) \approx 0.023$$

Since $0.0234 << 1$, then (15) reduces to

$$m\frac{d^2y}{dt^2} + d_c\frac{dy}{dt} + k\,y \approx \frac{A\varepsilon_0 V_m{}^2\left(1-\cos(2wt)\right)}{4d^2}$$
(17)

The solution to (17) is

$$y(t) = M\exp\left((-a+c_1)t\right) + N\exp\left((-a-c_1)t\right) + A_1\cos(2wt) + B_1\sin(2wt) + C_1 \quad (18)$$

where

$$c_1 = \sqrt{\frac{{d_c}^2}{4m^2} - \frac{k}{m}},\ a = \frac{d_c}{2m}$$

$$C_1 = \left(A\varepsilon_0 {V_m}^2\right) / \left(4d^2 k\right)$$

$$B_1 = \left(-2kC_1 d_c w\right) / \left(\left(k - 4mw^2\right)^2 + 4w^2 {d_c}^2\right)$$

$$A_1 = \left(B_1\left(k - 4w^2 m\right)\right) / \left(2wd_c\right)$$

$$M = -\left(c_1 + a\right)\left(C_1 + A_1 + 2wB_1\right) / \left(2c_1\right)$$

$$N = -C_1 - A_1 - M$$

Figs 13 – 18 show the plots of the displacement y with a sinusoidal inputs with different frequencies and amplitudes.

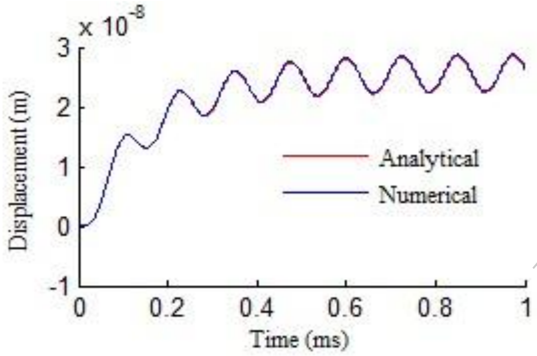

Fig.13. Numerical and Analytical solutions with $V_s = \sin(8000\pi t)$. Error = 1%.

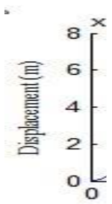

Fig.14. Numerical and Analytical solutions with $V_s = \sin(8\pi t)$. Error = 2 %.

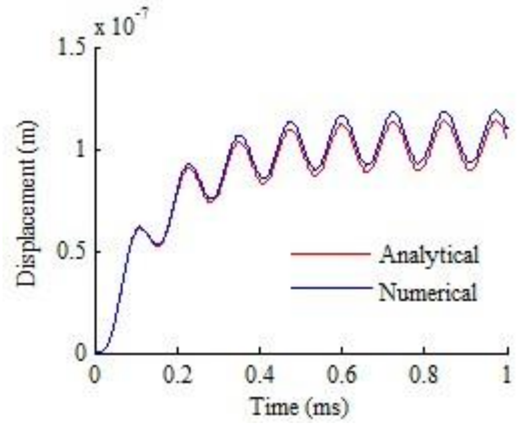

Fig.15. Numerical and Analytical solutions with $V_s = 2\sin(8000\pi t)$. Error = 4 %.

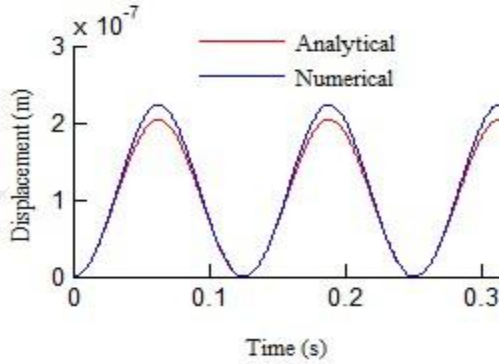

Fig.16. Numerical and Analytical solutions with $V_s = 2\sin(8\pi t)$. Error = 8.7%.

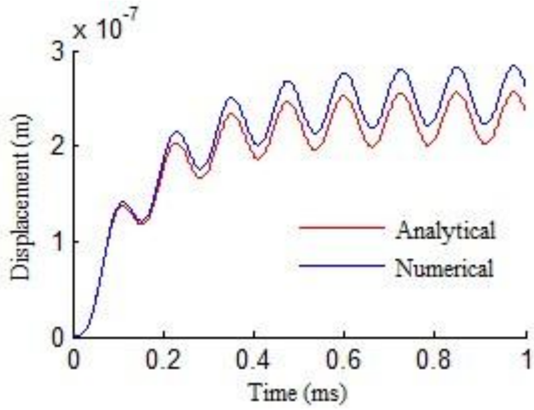

Fig.17. Numerical and Analytical solutions with $V_s = 3\sin(8000\pi t)$. Error = 9 %.

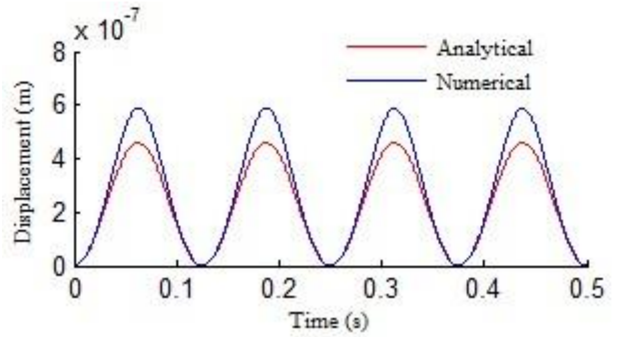

Fig.17. Numerical and Analytical solutions with $V_s = 3\sin(8\pi t)$. Error = 22 %.

For the low frequency inputs,the differences between the numerical and analytical solutions were higher than the high frequency inputs with the same peak-to-peak values. For high frequency inputs the displacement $y$is smaller due to the more frequent switching of the electrostatic force. Results show that for an input of almost 80% of the pull-in-voltage, the difference between the numerical and analytical values was less than 9% except for the case where the input was a sinusoidal with a frequency 4 Hz where the maximum difference was 22 %.

For a low frequency input such as 4 Hz, the frequency of the electrostatic driving force is 8 Hz which is much smaller than

$$f_{MEMS} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 4.3453 \text{ kHz}$$

the natural frequency of the MEMS. That is the plate will travel a bigger distance than the case for higher frequencies. With larger values for y at low frequencies, the approximation of the geometric series in y with the first two terms will yield larger errors.

V. CONCLUSION

An approximate closed form solution to the non-linear differential equation governing the motion of the upper plate of a MEMS device was derived for different input signals. Comparison between the approximate closed form and numerical solutions were made. We found that the differences between the solutions, numerical and analytical, with pull-in-voltage of 3.81 V, were 0%, 0.6% and 4.6% for step inputs with magnitudes 1, 2 and 3 V respectively. For a ramp input, the differences were 0%, 6%, and 20% with pulse duration of 1, 2, and 3 seconds respectively. For the sinusoidal inputs of magnitudes 1, 2, and 3 V, and a frequency of 4 KHz, the differences were 1%, 4% and 9% respectively, and 2%, 8.7%, and 22% for a frequency of 4Hz.

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