International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 11, November-2016

Modeling of the Dynamic Drying System

Gokhan Koyunlu
Faculty of Engineering
Nile University of Nigeria
Abuja/Nigeria

***Abstract*— Differential–algebraic equations (DAEs) naturally come up in many engineering problems , as well as numerical and analytical difficulties. In this paper , Paper drying unit was modelled by using DAEs in terms of fundamental equations of thermodynamic . Meanwhile, the index of DAE seems to be main numerically difficult. Hence , in order to solve it , the original DAE was transformed into an equivalent DAE with lower index.**

Keywords*— *Differential Algebraic Equations, Paper Drying unit, thermodynamics

I. INTRODUCTION

In this paper , a model for the dynamic drying system ,which is currently used in the pulp industry, has been mathematically developed. Here, The model of Dynamic Drying System is used for only paper drying system cylinders . Dynamic Drying System has four inputs which are Steam, Paper in comes from press size, and Supply air comes from heat recovery system, leakage comes from air system. The Dynamic Drying System has four outputs. One is Exhaust, the other is condense . Lastly paper out is the last point for paper producing with ignoring coating, which means that we are not interested pre-drying unit. By this modeling , we are able to control total mass of flow and steam .

1.1 DIFFERENTIAL ALGEBRAIC EQUATIONS (DAEs)

Adams, Runge and Kutta has developed the numerical solution of ordinary differential equations. The theory for Differential Algebraic Equations (DAEs) has not been studied as the same as ODEs at the past . It was firstly studied by Gear and Petzold [1]. Guzel,N and Bayram, M [2`] has been introductorily depicted DAE as follows

$$F(t, y, y') = 0 \qquad (1.1)$$

Here $\frac{\partial F}{\partial y'}$ is singular. The rank and structure of this Jacobian matrix may be dependent on the solution of y(t), and we will always assume that it is independent of t in order to make essay

1.1 Semi Explicit DAEs

$$y' = f(y, z, t) \qquad (1.2)$$
$$0 = g(y, z, t)$$

The index is 1 if $\frac{\partial g}{\partial t}$ is nonsingular, That's why this is a special case of (1.1) . We can divide into differential variables x(t) and algebraic variables z(t) if DAE has semi-explicit index-1. So z can be gotten as a function of y(t) by inverse function theorem.

1.2 Index Reduction

Another way for dealing with the problem of instability is to build up a new equation system by carrying out *index reduction* on the original DAE system. In order to understand index reduction, we can advise to look at the definition of the differentiation index, see [3]. This index gives the number of times *m* that depicts how many times we will differentiate following the equation system.

$$\frac{d^i}{dx^i} y = \frac{d^i}{dx^i} f(y, z)$$

$$0 = \frac{d^i}{dx^i} g(y)$$

with *x* being the independent variable. When we are able to rearrange equations, then it will be called the primary ODE. The principle of index reduction is to differentiate the equation system. It will give us a new set of equations, so that a new equation system with index one lower can be set up using algebraic manipulations. This reduction will be continued in following steps for lowering the index of the system. Finally this reduction enable us the use of methods for lower index problems.

1.7 Modelling of Paper Drying Unit

Ghosh [3] has been giving fundamentals of paper drying , Actually there are two main drying units , one is pre drying , the other is drying . Hereby, we assume that pre-drying unit is ignored

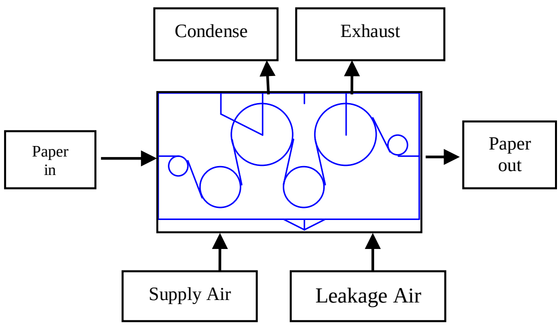

Figure 1.1 The Open Scheme of Drying Model System.

Input parameters

The mass flow main stream	$\dot{m}_s$
The mass flow paper in	$\dot{m}_{pin}$
The mass flow of the supply air	$\dot{m}_{sup}$
The mass flow of the leakage air	$\dot{m}_{leak}$

Output parameters

The mass flow Exhaust	$\dot{m}_{exh}$
The mass flow Condense	$\dot{m}_{cond}$
The mass flow Paper out	$\dot{m}_{pout}$

2.0 Mass and Energy Balance For The Dynamic Drying System

Mass Balance for the Dynamic Drying System

$$\frac{dM}{dt} = \dot{m}_s + \dot{m}_{pin} + \dot{m}_{sup} + \dot{m}_{leak} - \dot{m}_{exh} - \dot{m}_{cond} - \dot{m}_{pout} \quad (2.1)$$

Energy Balance for the Dynamic Drying System

$$\frac{dU}{dt} = \dot{m}_s h_s + \dot{m}_{pin} h_s + \dot{m}_{sup} h_{sup} + \dot{m}_{leak} h_{leak} - \dot{m}_{exh} h_{exh} - \dot{m}_{cond} h_{cond} - \dot{m}_{pout} h_{pout} \quad (2.2)$$

Now, Eqs. (2.1) and (2.2) are not sufficient for our drying part modeling. Hence, we attempted to write this modeling in terms of differential algebraic forms as follows the following algebraic equations

Total mass of flow and steam in system

$$M = M_f + M_s \quad (2.3)$$

Total Energy in system

The following equations ,which are given in [4] and [5] , are used to formulate on the basis internal energy , U(t) , and specific energy u(t) ;

$$U(t) = M(t).u(t) \quad (2.4)$$
$$u(t) = h(t) - p(t).v(t) \quad (2.5)$$

where h(t) is enthalpy , p(t) is pressure , v(t) is specific volume

$$U = U_f + U_s = M_f u_f + M_s u_s$$
$$M_f\left(h_f - pv_f\right) + M_s(h_s - pv_s) \quad (2.6)$$

Total volume of Flow and Steam

$$V = V_f + V_s = M_f v_f + M_s v_s \quad (2.7)$$

The Differential Algebraic Equations defined by above Eqs. have the following dynamic variables:

$$y^T = [M, U] \quad (2.8)$$

And the following algebraic variables:

$$z^T = [M_f, M_s, \dot{m}_s] \quad (2.9)$$

By applying the following DAEs

$$y'(t) = f(y, z, t)$$
$$0 = g(y, z, t)$$

Where y(t) contains the differential variables and z(t) the algebraic variables. We can write the dynamic drying system as follows.

$$y'(t) = \begin{bmatrix} \dot{m}_{pin} + \dot{m}_{sup} + \dot{m}_{leak} - \dot{m}_{exh} - \dot{m}_{cond} - \dot{m}_{pout} \\ \dot{m}_{pin} h_{pin} + \dot{m}_{sup} h_{sup} + \dot{m}_{leak} h_{leak} - \dot{m}_{exh} h_{exh} - \dot{m}_{cond} h_{cond} - \dot{m}_{pout} h_{pout} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & h_s \end{bmatrix} z = f(y, z, t) \quad (2.10)$$

and

$$0 = \begin{bmatrix} 0 \\ 0 \\ -V \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} .y + \begin{bmatrix} 1 & 1 & 0 \\ h_f - pv_f & h_s - pv_s & 0 \\ v_f & v_s & 0 \end{bmatrix} .z = g(y, z, t) \quad (2.11)$$

i.e.

$$\frac{\partial g}{\partial z} = \begin{bmatrix} 1 & 1 & 0 \\ h_f - pv_f & h_s - pv_s & 0 \\ v_f & v_s & 0 \end{bmatrix} \quad (2.12)$$

The last column of (2.12) is 0 . Therefore $\frac{\partial g}{\partial z}$ is singular. The DAE system's index is greater than 1, we are expected the integration can cause difficulties.

2.3 Index Reduction of DAE for the Dynamic Drying System

Now , our aim is that we would like to reduce the index of DAE such that we can write the DAE in terms of Ordinary Differential Equation.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 11, November-2016

$$\tilde{y}(t) = f(\tilde{y}, t)$$
$$0 = g(\tilde{y}, \tilde{z}, t) \quad (2.13)$$

With the following algebraic variables:

$$\tilde{y}^T = [M, M_s, M_f, U] \quad (2.14)$$

$$\tilde{z}^T = [\dot{m}_s] \quad (2.15)$$

Firstly we will differentiate the mass balance for the dynamic drying system Eq. (2.3)

$$\frac{dM}{dt} = \frac{dM_f}{dt} + \frac{dM_s}{dt} \quad (2.16)$$

And we will differentiate the total volume of the dynamic drying system Eq. (2.7)

$$0 = \frac{dV}{dt} = v_s.\frac{dM_s}{dt} + M_s.\frac{dv_s}{dt} + v_f.\frac{dM_f}{dt} + M_f\frac{dv_f}{dt} \quad (2.17)$$

And by applying the chain rule for differentiation

$$0 = v_s.\frac{dM_s}{dt} + M_s.\frac{dv_s}{dp}.\frac{dp}{dt} + v_w.\frac{dM_f}{dt} + M_f.\frac{dv_f}{dp}.\frac{dp}{dt} \quad (2.18)$$

Differentiation and reduction of the total energy content in the Dynamic Drying System Eq.(2.6)

$$\frac{dU}{dt} = (h_w - p.v_w)\frac{dM_w}{dt} + M_f.\frac{dp}{dt}.\left[\frac{dh_f}{dp} - p.\frac{dv_f}{dp} - v_f\right] + (h_s - pv_s)\frac{dM_s}{dt} + M_s.\frac{dp}{dt}\left[\frac{dh_s}{dp} - p.\frac{dv_s}{dp} - v_s\right] \quad (2.19)$$

Rewriting Eqs. [2.1] and [2.2] yields

$$\dot{m}_s = -(\dot{m}_{pin} + \dot{m}_{sup} + \dot{m}_{leak}) + \dot{m}_{exh} + \dot{m}_{cond1} + \dot{m}_{cond2} + \dot{m}_{pout} + \frac{dM}{dt} \quad (2.20)$$

$$\dot{m}_s = \frac{-1}{h_s}\left[(\dot{m}_{pin}h_{pin} + \dot{m}_{sup}h_{sup} + \dot{m}_{leak}h_{leak}) - (\dot{m}_{exh}h_{exh} + \dot{m}_{cond1}h_{cond1} + \dot{m}_{pout}h_{pout} + \frac{dU}{dt})\right] \quad (2.21)$$

By using these equations , we can eliminate $\dot{m}_s$, and we can write as following

$$\dot{m}_{pin}(1 - \frac{h_{pin}}{h_s}) + \dot{m}_{sup}(1 - \frac{h_{sup}}{h_s}) + \dot{m}_{leak}(1 - \frac{h_{leak}}{h_s}) - m_{exh}(1 - \frac{h_{exh}}{h_s}) - \dot{m}_{cond1}(1 - \frac{h_{cond1}}{h_s}) - \dot{m}_{pout}(1 - \frac{h_{pout}}{h_s}) = \frac{dM}{dt} - \frac{1}{h_s}\frac{dU}{dt} \quad (2.22)$$

Eqs. (2.19)-(2.20) and (2.22) form ODE which can be written in the form

$$A(\tilde{y}, t).\tilde{y}' = B(\tilde{y}, t).\tilde{y} + C \quad (2.23)$$

$$\tilde{y} = \tilde{y}_0 \quad for \quad t = t_0 \quad (2.24)$$

$$\tilde{y}'^T = \left[\frac{dM}{dt}, \frac{dM_s}{dt}, \frac{dM_f}{dt}, \frac{dU}{dt}\right] \quad (2.25)$$

$$\tilde{y}^T = [M, M_s, M_f, U]$$

and $\tilde{z}^T = [\dot{m}]$ (2.26)

where

$$A = \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & v_s & v_f & 0 \\ 0 & -(h_s - p.v_s) & -(h_f - p.v_f) & 1 \\ 1 & 0 & 0 & \frac{-1}{h_s} \end{bmatrix} \quad (2.27)$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{dp}{dt}.\frac{dv_s}{dp} & \frac{-dp}{dt}\frac{dv_f}{dp} & 0 \\ 0 & \frac{dp}{dt}(\frac{dh_f}{dp} - p.\frac{dv_f}{dp} - v_f) & \frac{dp}{dt}\left(\frac{dh_s}{dp} - p.\frac{dv_s}{dp} - v_s\right) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.28)$$

$$C^T = \left[0, 0, 0, \dot{m}_{pin}(1 - \frac{h_{pin}}{h_s}) + \dot{m}_{sup}(1 - \frac{h_{sup}}{h_s}) + \dot{m}_{leak}(1 - \frac{h_{leak}}{h_s}) - m_{exh}(1 - \frac{h_{exh}}{h_s}) - \dot{m}_{cond1}(1 - \frac{h_{cond1}}{h_s}) - \dot{m}_{pout}(1 - \frac{h_{pout}}{h_s})\right] \quad (2.29)$$

CONCLUSION

We used Differential Algebraic Equations for Simple Paper Drying Modeling in Pulp Industry . By the way , we are able to control the mass of steam , flow and energy in the system

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 11, November-2016

REFERENCES

[1] Asher, M. and Petzold, R., *Computer Methods for Ordinary Differential Equations and Differential Algebraic Equations*. USA, 1998

[2] Guzel, N. and Bayram, M., "Numerical Solution of Differential Algebraic Equations with Index 2", *Applied Mathematics and Computation*, Vol 174. pp1279-1289, 2006

[3] Ghosh, A.K., "Fundamentals of Paper Drying", *Evaporation Condensation and Heat Transfer*, pp. 536-581, In Tech, Crotia

[4] Guggenheim, E.A., *Thermodynamics*, North Holland Publisching Company, Amsterdam, 1959

[5] Venkanna, B. and Swati, B., *Basic Thermodynamics* , PHI Learning Pvt. Ltd, 2010