# Modeling of Cutting Force Considered Helix Angle and Piecewise Continuous Cuttingaper style

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*Abstract* - In this paper, the geometries of helix angle and the piecewise continuous regions in the cutting is considered, and the mechanical model considering the helix angle and the piecewise continuous regions of the cutting is established. The milling experiment is carried out to verify the validity and reliability of the model. It is found that the helix angle has a great influence on the milling force, and the helix angle can obviously affect the peak shape of the cutting force.

Keywords: Helical angle; Cutting force; Piecewise continuous cutting

# I. INTRODUCTION

With the rapid development of modern industry, high efficiency, high precision, high flexibility and greening has become an important development direction of contemporary advanced manufacturing industry. High-speed cutting is an important branch of advanced manufacturing technology and have been widely applied to the mold processing, aerospace, electronics, precision instruments and other fields. In the highspeed cutting environment, the workpiece and the milling cutter is difficult to smooth processing.

Tsai [1] based on the tool rotation eccentricity and tool deformation during the machining process, and assuming that the unit cutting force coefficient is constant, the milling force model is established. Song[2] based on the precise thin-walled parts milling force model analysis the phenomenon of cutter relieving between workpiece and tool. The tool parameters and process parameters are studied to establish a more accurate milling force model. Altintas [3] establishes the milling force model of the side blade by establishing the cutting layer area and the blade contact length change function. In this paper, the influence of the helix angle is taken into account in the modeling of the cutting force. The experimental results show that the helix angle has a specific effect on the cutting force.

# II. CUTTING FORCE MODELING

In the milling process, the workpiece, tool and machine tool together to form a cutting process dynamics system[4], The cutting force of the system considering the helix angle is modeled, and the mechanical properties of the milling are analyzed in detail. The cutting prediction factors are more comprehensive and provide the theoretical basis for further optimization of the cutting parameters.

When considering the helix angle, the instantaneous milling height of the teeth will change continuously as the cutting process progresses. As shown in Fig. 1, the feed direction is the positive direction of x, and the micro-cutting force of a point on the tooth is taken as the object of study. The micro-cutting force is the vector sum of tangential  $dF_a(t,z)$  and radial ...differential cutting forces i.e.

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$$\lim_{d \to j} \lim_{a \to j} \lim_{a \to j} \lim_{a \to j} \frac{d F_{ij}}{d F_{ij}} d F_{ij}.$$
(1)

As shown in Fig. 2, the rotation angle of the jth tooth is defined as  $\phi_i$ , and the expression of the jth tooth at t is the



Figure 2 Force Analysis of Helical Milling Cutter

$$\phi_{j}(t,z) = \omega t + \frac{2\pi}{N} (j-1) - z \frac{\tan \beta}{R}.$$
(2)

where,  $\beta$  is the helix angle of the milling cutter,  $\omega$  is the angular velocity of spindle rotation (rad / s), N is the number of teeth. The tangential ( $dF_a(t,z)$ ) and radial ( $dF_a(t,z)$ )

differential cutting forces are decomposed into the  $\boldsymbol{x}$  and  $\boldsymbol{y}$  directions, i.e.

$$F_{x_{j}}(t) = (-F_{t_{j}} \cos \phi_{j}(t) - F_{t_{j}} \sin \phi_{j}(t))$$
  

$$F_{y_{i}}(t) = (F_{t_{i}} \sin \phi_{j}(t) - F_{t_{i}} \cos \phi_{i}(t)).$$
(3)

Assuming that the cutting force is proportional to the cutting area, a static cutting force model is established. If considering the interaction between the rake face and the chip, the flank face and the machined surface, tangential  $dF_{ij}(t,z)$  and radial  $dF_{ij}(t,z)$  differential cutting forces can be expressed as

$$dF_{ij}(t,z) = k_i h_j(t,z) dz + k_{ie} dz$$

$$dF_{ii}(t,z) = k_i h_i(t,z) dz + k_{ie} dz.$$
(4)

where  $k_t$  and  $k_t$  are the tangential and normal cutting force coefficients, respectively.  $k_{te}$  and  $k_{re}$  are the tangential and normal edge force coefficients, respectively. $h_j(t)$  is the instantaneous cutting thickness, here only consider the static cutting thickness, i.e.,

$$h_i(t, z) = f_t \sin \phi_i(t, z).$$
<sup>(5)</sup>

where  $f_t$  is the feed per tooth (mm / tooth),  $f_t = V / (N \cdot n)$ , V is the feed rate of the milling cutter, and n is the spindle speed.

Considering the milling process is intermittent process[5]. Each tooth has no cutting state in each rotation cycle of the tooth, and at least one knife is in the cutting state to exist the cutting force. Fig. 3 is a schematic model of cutting process. The diagram of (a) and (b) is vertical milling cutter cutting process used to describe the two types of cutting process. They are all composed of (1), (2), (3) three processes. Where the slash indicates the cutter teeth and the box represents the workpiece.



The picture of (c) and (d) are special case when the cutting force reaches the maximum, which is used to represent the integral interval of contact angle. Define the judgment function to determine whether the cutting force is cut, that is, if and only if the tooth in the (1), (2), (3) three process, the cutting force is not 0. The expression is

$$\mathbf{G}(\boldsymbol{\phi}_{j}(t)) = \begin{cases} 1, \ \boldsymbol{\phi}_{st} < \boldsymbol{\phi}_{j}(t) < \boldsymbol{\phi}_{ex} + \boldsymbol{\phi}_{lag} \\ 0, \text{ others} \end{cases}$$
(6)

where  $\phi_{st}$  and  $\phi_{ex}$  are the cut in angle and the cut-out angle

espectively, 
$$\phi_{lag} = z \frac{\tan \beta}{R}$$

The cutting force is the integral of the micro-cutting force on the entire axial milling height along the cutting edge

$$F(t) = \begin{pmatrix} F_{x}(t) \\ F_{y}(t) \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{N} \int_{0}^{a_{p}} dF_{xj}(t,z) \\ \sum_{j=1}^{N} \int_{0}^{a_{p}} dF_{yj}(t,z) \end{pmatrix}.$$
 (7)

where  $F_x(t)$  is the cutting force in the x direction.  $F_y(t)$  is the cutting force in the y direction.  $dF_{xj}(t,z)$  and  $dF_{yj}(t,z)$  are the micro-cutting forces of the j-th teeth of the tooth teeth at the micro-element height of dz, respectively. Taking into account the processing of thin-walled parts, so ignore the tool and the workpiece in the x direction of cutting force, The cutting force can be expressed as

$$F(t) = \sum_{j=1}^{N} \int_{0}^{a_{p}} \mathrm{d}F_{j}(t, z).$$
(8)

Eq. 2 is derivatived for the variable z, getting  $dz = -\frac{R}{\tan \beta} d\phi$ , combining the Eq. 3, 4, 5, 6 with Eq. 8, one can gain

$$F(t) = -\sum_{j=1}^{N} \frac{R}{\tan \beta_{j}} \int_{\phi_{j}(t,0)}^{\phi_{j}(t,ap)} G(\phi) \,\mathbf{K}(t) d\phi.$$
(9)

Among them,  $\mathbf{K}(\mathbf{t}) = (\mathbf{s} \cdot K_t - \mathbf{c} \cdot K_r) \mathbf{f}_t \cdot \mathbf{s} + (K_{te} \cdot \mathbf{s} - K_{re} \cdot \mathbf{c})$ 

It can be seen from the literature. F(t) in the Eq. 9 is a piecewise function, and its integral interval cut off angle changes with time. As shown in Fig. 3 (a), (1), (2), (3) three processes just constitute a complete milling cycle, which is part of the cutter teeth cut. (2) is completely cut into the teeth. (3) is part of the cutter teeth cut. Compared with Fig. 3(a) and Fig. 3 (b), When  $a_p > (\phi_{ex} - \phi_{st})R \arctan \beta$ , it is Fig. 3 (a) type of cutting process, otherwise it is Fig. 3(b) class cutting .The integral interval can be expressed as

$$\phi_{sj}(t) = \begin{cases} \phi_{st}, \phi_{st} < \phi_{j}(t) < \phi_{st} + \phi_{\text{lag}} \\ \phi_{j}(t) - \phi_{\text{lag}}, \phi_{st} + \phi_{\text{lag}} < \phi_{j}(t) < \phi_{ex} + \phi_{\text{lag}} \\ 0, \text{ others} \end{cases}$$
(10)

$$\phi_{ej}(t) = \begin{cases} \phi_{j}(t), & \phi_{st} < \phi_{j}(t) < \phi_{ex} \\ \phi_{ex}, & \phi_{ex} < \phi_{j}(t) < \phi_{ex} + \phi_{lag} \\ 0, & \text{others} \end{cases}$$
(11)

#### III. EXPERIMENTAL RESULTS

As shown to Fig. 4,In order to verify the reliability of the model, cutting force verification experiments are carried out on the DMC 75 V linear five-axis high-speed machining center. Force measuring system Kistler9257B fixed on the table to measure the X, Y and Z three directions of the cutting force. The tool uses a carbide end mill.



Figure 4 Cutting force test setup

The tool parameters and cutting parameters are the number of teeth N = 3, the tool radius R = 6 mm, the helix angle  $\beta = 30$ °, the axial cutting depth ap=10mm, the depth of cut rate  $a_e$ =0.5, feed speed V = 240mm / min,spindle speed n = 2000 r / min, Tangential and normal cutting force coefficients are:kt= $8.354 \times 108$ N/m<sup>2</sup>, $k_r = 2.445 \times 10^8$  N/m<sup>2</sup>;Tangential and radial edge force coefficient are: $k_{te}=2.88 \times 10^4$  N/m<sup>2</sup>,  $k_{re}=2.64 \times 10^4$  N/m<sup>2</sup>.Under the condition of up-milling, the above-mentioned milling cutter and cutting force parameters are used to carry out the experiment to show the change of cutting force as shown in Fig. 5.

For ease of comparison, the simulation results based on the predictive theory of this paper are also added to the graph. It can be seen from the figure that the model results predict the results of the cutting force and the test results, whether the cutting force curve cycle, waveform or amplitude are very good consistency.



#### **IV. CONCLUSION**

Considering the geometries of helix angle and the piecewise continuous regions of the cutting, an analytical model for predicting the cutting force for milling process with

helix angle cutter is presented. The analysis results show that the angle of helix has a significant effect on the cutting force, the helix angle on the cutting force of the size and the impact of the cutting cycle are obvious. Therefore, when the milling process is modeled in engineering, the helix angle can not be ignored.

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