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Modeling of Cumulative Damage Degradation and Random Shocks for a Constant- Interval Preventive Maintenance System

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Abstract:- **This research work presents the modeling of total expected maintenance cost of a system subjected to cumulative deterioration and random shocks under constant interval preventive maintenance (PM). An accelerated life testing methodology was used to study the functional deterioration of a centrifugal pump subjected to cumulative damage and random shocks situations. A constant interval policy was used to evaluate the optimum time interval that will reduce the total expected cost of the unit. Results obtained from the study shows that, random shocks raises the number of failures and therefore, reduces the reliability of the system. The total expected cost ranges from two thousand naira only (₦2000.00) to sixteen thousand naira only (₦16,000.00), depending on the shock and shock's duration. The model and method adopted are very useful and thus recommended as a tool for studying similar mechanical systems.**

Key words: Modeling, constant- interval, degradation, cumulative damage, random shocks.

I INTRODUCTION

Modeling is an abstract mathematical representation of the behavior of a system. It postulates how systems will behave in a particular situation. Deterioration can be described as a kind of reduction in value. A deteriorating state depicts state of a system in which the system performs only limited function, which are lower than the specified functions [1].Degraded system will produce excessive noise and vibration [2]. Failure is the end of performance of the functions of the system. Failure can be reduced by preventive maintenance. Preventive maintenance is the weaving together of technical and engineering actions to keep an item or system or bring it back to a functional state, whereby it can perform the needed functions. Failure of an asset terminates reliability, thereby leading to costs or unreliability [3]. In summary, maintenance is all the actions which have the objective of returning a system to another state- a functional and acceptable state [4]. Machines failures are normally caused by poor maintenance practice and inability to predict future problems during usage [5].

[6] described machine deterioration as the irrecoverable accumulation of damage in the system over its useful life that leads to its breakdown in a later time. There exist many stages of deterioration which make the efficiency of the machine to decrease, this type of breakdown is known as soft failure. Hard failure occurs without warning.Note that in spite of planned preventive maintenance measure, failures are bound to occur but they can be reduced to a minimum extent [7].

A degradation model usually gives two relevant information, such as deterioration processes of the indicators and the linkage between the indicators and the failure events [8].

Deterioration models can be divided into two groups: normal degradation and accelerated degradation models. Normal degradation models are employed to determine the degradation applied at normal operating conditions. Accelerated degradation models make use of data gotten under stressed conditions. Normal degradation come with stress and without stress factors [9].

An example without stress factor is the general degradation path models and Markov models. An example of stressed factor is the accumulative damage [10].

Continuous- time model is a stochastic process that allows the index values to take a continuous set of values, such as Weiner processes [11].

. [12] studied the optimal reliability maintenance policy for a system subject to cumulative shock model [13] investigated an optimal maintenance practice for system subject to random shocks and showed by a $\partial - shock\ model\cdot$

In this research, constant interval preventive replacement policy was modeled and data generated under accelerated system failure condition and applied on the mode for verification.

II METHODLOGY

The items that were used in the research study include: A $0.015m^3$ holding tank, fixed with a stirrer 0.745kW centrifugal pump, coupled to a single phase electric motor, a ball valve with 0.0254m hoses. A pressure guage was provided to measure the inlet and outlet pressures of the pump. Figure 1 shows the top view of the experimental setup used for the research work.

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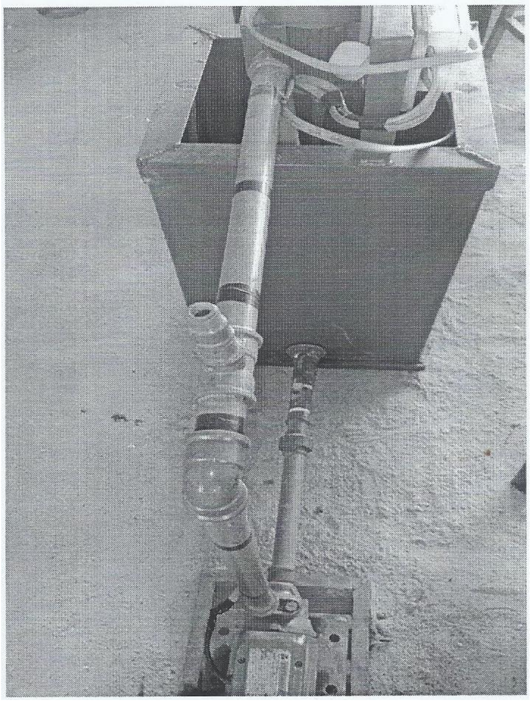

Figure 1: Side View of Experimental Setup

An accelerated life testing procedure was applied in this study. The holding tank mounted with a stirrer, initially had $0.015m^3$ of water pumped and circulated back to the tank. The inlet and outlet pressures of the water from the pump were measured. Some fixed amount of sand was put into the tank and the stirrer switched on while pumping activity was performed and the inlet pressure, outlet pressure, vibration of the pump measured .This was repeated with different amount of sand at a constant interval and also repeated at random intervals.

Modeling of PM Policy

Optimal constant-interval preventive replacement (PR) policy was modeled . Under the policy, PM was performed on the system after it had operated a total of t_p hours, irrespective of the number of occurring failures within the interval. In a situation where failure happened prior to t_p, minimal repair was carried out. PM renewed the system to become as good as new. Minimal repair did not change the failure rate of the system, as emphasized in [2]. The choice of the horizon or planning period in the study was important, a position maintained also according to [4] which, they had presented earlier and solved a planning horizon problem.

The model for PM is presented in Equation (1):

$$\mathrm{EC}(t_p) = \frac{C_p + C_f H(t_p)}{t_p} \qquad (1)$$

where,

$\mathrm{EC}(t_p)$ is total expected cost due to PM and minimal repair cost per expected length of interval,

C_p is the preventive maintenance cost,

C_f is the failure cost, and

$\mathrm{H}(t_p)$ is the failure cost function.

The failure cost function is given in Equation (2):

$$\mathrm{H}(t_p) = \int_0^{t_p} r(t)dt\,, \qquad (2)$$

Which is the expected number of failures in the interval $[0, t_p]$.

The failure rate function is given by Equation (3):

$$\mathrm{r(t)} = \frac{f(t)}{1 - f(t)}\,, \qquad (3)$$

where,

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f(t) it the probability density function and is given in Equation (4):

$$f(t)=\frac{1}{N} \quad 0<t\leq N \tag{4}$$

where N is uniformly distributed in the interval [$0<t\leq N$]. Substituting N into Equation (4) gives, Equation (5) given below.

$$r(t) = \frac{\frac{1}{N}}{1-\frac{1}{N}t} = \frac{1}{N-t} \tag{5}$$

Equation 5 is substituted into Equation 2, resulting into Equation (6) given below

$$H(t_p) = \int_0^{c_p} \frac{1}{N-t}dt = -\ln[(N-t_p)] \tag{6}$$

When Equation 6 is substituted into Equation (1), Equation (7) is obtained:

$$EC(t_p) = \frac{C_p + C_f \ln\frac{N}{N-t_p}}{t_p} \tag{7}$$

The magnitude of the shock can be calculated from Equation (8):

$$s = \frac{f_2 - f_1}{f_1} \tag{8}$$

where,

f_1 is the measured frequency when there is no sand in the system and f_2 is the measured frequency when sand is put into the system. $0 \leq s \leq 1$.

A Golden section method was employed to find t_p which minimizes the expected maintenance cost for-Equation (7) under this policy.

The value of C_p varies from ₦1,000.00 to ₦1,500.00only, and C_f varies from₦,1000.00 to ₦10,000.00only. The projection was for seven series: from one to six under random studies and one for a constant –interval method. The purchase price of the pimp was ₦4,0000.00 only.

III RESULTS AND DISCUSSION

RESULTS

The results obtained at the end of the research are presented under sub-section 4.1.

Table 1 presents the data obtained from the experimental setup

Table 1: Normal working condition (2.97 hours)

Time (hours)	Sum time (hours)	Inlet Pressure $x10^5$ N/m^2	Outlet Pressure $x10^5$ N/m^2	Amount of sand (kg)	Sum of sand (kg)	Flow rate m^3/hr	Vibration value Hz	No. of failures
0.33	0.33	0.50	0.50	0.00	0.00	120	51	0
0.33	0.66	0.50	0.50	0.00	0.00	120	51	
0.33	0.99	0.50	0.50	0.00	0.00	120	51	
0.33	1.32	0.50	0.50	0.00	0.00	120	51	
0.33	1.65	0.45	0.45	0.00	0.00	120	51	
0.33	1.98	0.45	0.45	0.00	0.00	120	51	
0.33	2.31	0.45	0.45	0.00	0.00	120	51	
0.33	2.64	0.45	0.45	0.00	0.00	120	51	
0.33	2.97	0.45	0.45	0.00	0.00	120	51	

In Table 2 results with cumulative damage and random shock at1.08 hours is shown.

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Table 2 : Results with cumulative damage degradation and random shock at1.08 hours

Time (hours)	Sum time (hours)	Inlet Pressure $x10^5$ N/m^2	Outlet Pressure $x10^5$ N/m^2	Amount of sand (kg)	Sum of sand	Shock Magnitude (kg)	Flow rate m^3/hr	Vibration Hz	No. of failures
0.33	0.33	0.35	0.35	0.0010	0.0010	0.18	120	60.1	2
0.25	0.58	0.25	0.25	0.0020	0.0030	0.18	120	60.0	
0.17	0.75	0.25	0.20	0.0030	0.0060	0.18	112	60.0	
0.17	0.92	0.15	0.12	0.0040	0.0100	0.24	112	63.2	
.08	1.00	0.10	0.05	0.0050	0.015	0.24	112	63.3	
0.08	1.08	0.10	0.05	0.0060	0.0210	0.24	112	63.3	

Accordingly, various results obtained for cumulative damage degradation and random shocks in are as presented in Table 4, Table 5,Table 6,Table7,Table 8, Table 9 and Table10 respectively at 1.00hour, 1.41 hours, 1.08 hours, 1.08 hours, 0.84 hours, 1.98 hours,1.98 hours and1.98 hours, respectively.

Table 3 : Data with cumulative damage degradation and random shock at1.00 hours

Time (hours)	Sum time (hours)	Inlet Pressure $x10^5$ N/m^2	Outlet Pressure $x10^5$ N/m^2	Amount of sand (kg)	Sum of sand (kg)	Shock Magnitude	Flow rate m^3/hr	Vibration Hz	No. of failure
0.25	0.25	0.25	0.25	0.003	0.003	0.27	108	65.0	2
0.25	0.50	0.20	0.45	0.005	0.008	0.27	108	65.0	
0.17	0.67	0.15	0.60	0.007	0.015	0.28	100	65.4	
0.17	0.84	0.10	0.70	0.009	0.024	0.29	100	65.9	
0.08	0.92	0.00	0.70	0.01	0.035	0.29	100	65.9	
0.08	1.00	0.00	0.70	0.01	0.038	0.31	96	67.0	

Table 4 : Results obtained with cumulative damage degradation and random shock at1.41 hours

Time (hours)	Sum time (hours)	Inlet Pressure $x10^5$ N/m^2	Outlet Pressure $x10^5$ N/m^2	Amount of sand (kg)	Sum of Sand (kg)	Shock Magnitude	Flow rate m^3/hr	Vibration Hz	No. of failures
0.33	0.33	0.25	0.22	0.0020	0.002	0.31	96	67.0	3
0.33	0.66	0.20	0.18	0.004	0.006	0.32	96	67.2	
0.25	0.91	0.10	0.08	0.006	0.012	0.32	96	67.5	
0.25	1.16	0.05	0.03	0.008	0.020	0.33	96	67.9	
0.17	1.33	0.05	0.00	0.010	0.030	0.37	88	70.0	
0.08	1.41	0.00	0.00	0.012	0.042	0.37	88	70.0	

Table 5: Results obtained with cumulative damage degradation and random shock at1.08 hours

Time (hours) $x10^{-1}$	Sum of time (hours) $x10^{-1}$	Inlet Pressure $x10^6$ N/m^2	Outlet Pressure $x10^6$ N/m^2	Amount of sand (kg) $x10^{-3}$	Sum of sand (kg) $x10^{-3}$	Shock Magnitude $x10^{-1}$	Flow rate m^3/hr $x10^1$	Vibration Hz $x10^1$
2.5	2.5	2.5	2.0	1	1	3.3	8.8	7.00
5.0	7.5	2.0	2.0	3	4	3.9	8.8	7..0
7.5	15.0	1.5	1.2	5	9	4.0	8.8	7.14
9.2	24.2	1.0	0.5	7	16	4.0	8.4	7.14
10.9	35.1	0.5	0.0	9	25	4.1	8.0	7.20
11.7	46.8	99.9	0.0	11	36	4.1	8.0	7.20

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Table 6: Results Obtained with cumulative damage degradation and random shock at1.08 hours

Time (hours)	Sum time (hours)	Inlet Pressure $x10^5$ N/m^2	Outlet Pressure $x10^5$ N/m^2	Amount of sand (kg)	Sum of sand (kg)	Shock Magnitude	Flow rate m^3/hr	Vibration Value Hz	Number of failures
0.25	0.25	0.20	0.20	0.002	0.002	0.47	72	75.1	
0.17	0.42	0.20	0.15	0.003	0.005	0.48	72	75.3	
0.17	0.59	0.15	0.15	0.004	0.009	0.48	60	75.3	
0.17	0.76	0.10	0.05	0.005	0.014	0.48	60	75.7	
0.08	0.84	0.05	0.00	0.006	0.020	0.49	60	76	
0.08	0.92	0.00	0.00	0.007	0.027	0.49	60	76	

Table 7: Results obtained with cumulative damage degradation and random shock at 0.84 hours

Time (hours)	Sum of time (hours)	Inlet Pressure $x10^5$ N/m^2	Outlet Pressure $x10^5$ N/m^2	Amount of sand (kg)	Sum of sand (kg)	Shock Magnitude	Flow rate m^3/hr	Vibration Value Hz	Number of failures
0.17	0.17	0.15	0.15	0.4	0.4	0.51	60	77	2
0.17	0.34	0.15	0.15	0.8	1.2	0.57	60	80	
0.17	0.51	0.10	0.10	1.2	2.4	0.61	60	82	
0.17	0.68	0.00	0.00	1.6	4.0	0.63	0	83	
0.08	0.76	0.00	0.00	2.0	6.0	0.67	0	85	
0.08	0.84	0.00	0.00	2.4	8.4	θ	0	-	

(-): shows that the pump stopped working or seized. θ is a sign of a very high shock value

Table 8: Results obtained with cumulative damage degradation and random shock at 1.98 hours

Time (hours) $x10^{-1}$	Sum time (hours) $x10^{-1}$	Inlet Pressure $x10^6$ N/m^2	Outlet Pressure $x10^6$ N/m^2	Amount of sand (kg) $x10^{-3}$	Sum of sand (kg) $x10^{-3}$	Shock Magnitude $x10^{-1}$	Flow rate m^3/hr $x10^1$	Vibration Value Hz $x10^1$
3.3	33	3.5	3.0	1	1	1.8	12	6.01
3.3	6.6	3.0	2.7	2	3	1.9	12	6.09
3..3	9.9	3.0	2.5	3	6	2.4	11.2	6.30
33	1.32	2.5	2.0	4	10	2.5	11.2	6.39
3.3	16.5	2.5	1.5	5	15	2.6	10.4	64.3
3.3	1.98	1.5	1.0	6	21	2.7	10.4	6.49

Table 9: Results obtained with cumulative damage degradation and random shock at1.98 hours

Time (hours)	Sum time (hours)	Inlet Pressure $x10^5$ N/m^2	Outlet Pressure $x10^5$ N/m^2	Amount of sand (kg)	Sum of sand (kg)	Shock Magnitude	Flow rate m^3/hr	Vibration Value Hz	Number of failures
0.33	0.33	0.30	0.25	0.001	0.001	0.24	112	63.0	2
0.33	0.66	0.30	0.20	0.002	0.003	0.25	112	63.6	
0.33	0.99	0.25	0.17	0.003	0.006	0.26	104	64.2	
0.33	1.32	0.25	0.15	0.004	0.010	0.27	104	65.0	
0.33	1.65	0.20	0.10	0.005	0.015	0.28	96	65.5	
0.33	1.98	0.10	0.00	0.006	0.921	0.37	96	70	

Table 10: Results obtained with cumulative damage degradation and random shock at1.98 hours

Time (hours) $x10^{-1}$	Sum time (hours) $x10^{-1}$	Pressure inlet $x10^6$ N/m^2	Outlet Pressure $x10^6$ N/m^2	Amount of sand (kg) $x10^{-3}$	Sum of sand (kg) $x10^{-3}$	Shock Magnitude $x10^{-1}$	Flow rate m^3/hr $x10^1$	Vibration Value Hz $x10^1$

3.3	3.3	2.5	2.1	1	1	3.1	9.6	6.70
3.3	6.6	2.0	1.3	2	3	3.3	9.6	6.80
3.3	9.9	2.0	1.0	3	6	3.5	8.8	6.88
.33	13.2	.15	.08	4	10	3.6	8.0	6.96
3.3	16.5	1.0	0.5	5	15	3.9	8.0	7..08
3.3	19.8	0.5	0.0	6	21	4.0	7.2	7.1.6

The cumulative data set for the centrifugal Pump is presented in Table 11.Table 12 presents the summary of random readings and Table 13 presents the summary of steady readings taken during the experiment.

Table 11: Cumulative Result set

S/N	TBF $x10^1$	Number of failures	Sum Time (TBF)	Sum Failure	Down time (hours)	Number of Replacement	Maintenance Time Spent (hours)	Remarks
1.0	0.297	2.0	2.97	2.0	0.00	0	0	Without sand
2.0	0.108	2.0	4.05	4.0	3.00	0	1	With sand
3.0	0.100	2.0	5.05	6.0	1.15	0	1	With sand
4.0	0.141	3.0	5.46	9.0	6.00	1	1	With sand
5.0	0.117	2.0	6.63	11.0	0.30	0	0.5	With sand
6.0	0.092	2.0	7.55	13.0	1.00	0	1	With sand
7.0	0.084	2.0	8.39	15.0	0.00	0	0	With sand

Table 12: Tabulation of Random Readings

S/N	Time before Failure TBF (hours)	Number of Failures (n)	Failure rate (λ) per hour $x10^{-1}$
1.0	1.08	2.0	5.4
2.0	1.00	2.0	5.0
3.0	1.41	3.0	3.8
4,0	1.17	2.0	5.9
5,0	0.92	2.0	4.5
6,0	0.84	2.0	4.2

Table 13: Summary (Steady Readings)

S/N	Time TBF (hrs)	Number of Failures (N)	Failure rate (λ) Per hour
1	2.97	0	0
2	1.98	2	0.99
3	1.98	2	0.99
4	1.98	2	0.99

IV DISCUSSION

From Table 1, the time of 0.33 hours was used to measure the required parameters in the accelerated experimental setup. The inlet and outlet pressures had the same value and remained the same throughout the study period. No sand was added to the water and the volume flow rate of $120m^3/hr$ was constant at operating vibration frequency of $51Hz$.

As shown in Tables 2 to Tables 7, randomly selected time interval was used with a fixed amount of sharp sand put into the tank and pumping was done and some parameters were measured. In Table 2, when 0.0010kg of sharp sand was put into the system during 0.33 hours, the volumetric flow rate recorded was $120\ x10^5\ N/m^2$ and the vibration of the pump rose from 51Hz to 60.1 Hz. This gave a shock of 0.18. At the inlet and outlet pressures of 0.15 $x10^5\ N/m^2$ and 0.12 $x10^5\ N/m^2$ the shock was 0.24 with vibration magnitude of 63.2Hz. At the inlet and outlet pressures of 0.10 $x10^5\ N/m^2$ and 0.05 $x10^5\ N/m^2$ the shock was still 0.24 and the vibration magnitude of the pump rose to 63.3Hz. At this instance the system failure two. Similar behaviours were obtained in Table 3 by randomly adding of sharp sand at the span of 1.00 hour. The peak vibration magnitude frequency of 67Hz and the lowest vibration magnitude frequency of 65Hz were recorded. In Table 4 the highest shock magnitude was 0.37 at a vibration magnitude of 70Hz and the lowest vibration magnitude of 67Hz at a span of 1.41hours with three failures.

In Table 5 and Table 6, respectively the lowest volumetric flow rate were $80m^3/hr$, $60m^3/hr$ at a vibration magnitudes of 70Hz and 76 Hz and $88m^3/hr$, $72m^3/hr$ at a vibration magnitudes of 72Hz and 75.1Hz at spans of 1.08 hours and at a shock magnitudes of 0.41 and 0.49, respectively.

Other results are given for pressures, quantity of sand, shock magnitude, vibration frequency and number of failures for random methods in Tables, 7, 8, 9 and 10 for 1.98 hours respectively.
Table 11 shows cumulative data set for the centrifugal pump without sand and with sand.
The synopsis of random readings for each of pump activity is given in Table 12.
Similarly, Table 13 shows the values under steady readings. From Table 13, it can be observed that the reliability obtained was high compared to the ones obtained from Tables 2 to 10; indicating that random shocks are most devastating than cumulative degradation damage.
The failure rate under each condition was computed and used to obtain the reliability theof the pump from one to twelve months during the systematic search for t_p -the optimum interval. From Figure1 presented below, the total expected cost varies from two thousand naira (₦2,000.00) only to sixteen thousand naira (₦16,000.00) only.
Figure 2 shows the expected cost over time interval.

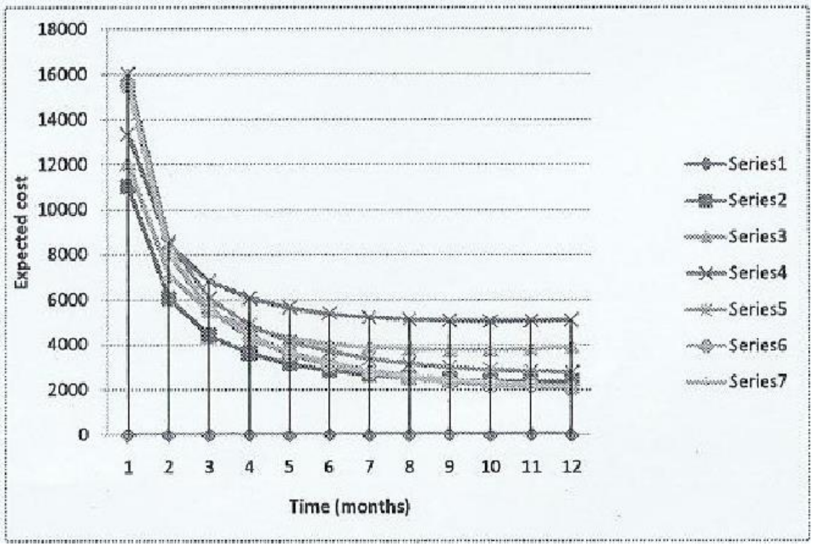

Figure 2 : Expected Cost Versus Time

IV CONCLUSION

The cumulative degradation damage shock for a system operating under a normal operating condition is less devastating than that with random shocks conditions. The amount of sharp sand put into the system affects the, pressures, volumetric flow rate and the vibration magnitude. This is depicted in the magnitude of the shock obtained. A system operating under shock conditions will have many number of failures compared to a system without shock conditions and this will affect the reliability of the system and hence the total expected cost of the system will rise. A high reliability will reduce the total expected cost of the system and increase the availability of the overall system. The model and methodology presented are useful in determining the cumulative damage degradation and random shocks for a given system.

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