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Modeling and Identification of Linear Systems from Input-Output Data

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Abstract

System Identification is the determination of the system model of a dynamic system based on measured input-output data. In this paper concentration is made on different aspects of system identification, different models, parameter estimation methods and model validation. Here it is assumed that all data is available at once i.e. the variability of the system is studied rather than doing real on-line calculations. Non-parametric method i.e. spectral analysis and parametric method i.e. Least squares and Instrumental Variable methods are used as process identification methods. Some of the recursive (on-line) algorithms are also studied. Simple demonstrations are performed to support these aspects

1. Introduction

The problem of modeling and system identification has attracted considerable attraction during the past twenty years mostly because of a large number of applications in diverse fields like chemical processes, biomedical systems, socioeconomic systems, transportation, ecology, electric power systems, hydrology, aeronautics [9].

System Identification is the field of mathematical modeling of systems from experimental data. Some experiments are performed on the system; a model is then fitted to the recorded data by assigning suitable numerical values to its parameter [6]. The system identification is necessary to establish a model based on which the controller can be designed, and it is useful for tuning and simulation before applying the controller to the real system [9].

2. Problem Statements

1. Given input output data (u(t), y(t)), generated by a system G, y= $G[u]$, find a system G_1 that approximates G and provides an estimate of the size of the approximation error. Compare the output of the simulated model output with the actual output.

2. Compute different parametric models and compute its spectrum with different methods and compare these spectrums with each other.

3. Compute the different models using different approaches and plot the estimated parameters and the true parameters as a function of time for comparison.

3. System Identification

System Identification is an experimental approach. Some experiments are performed on the system; a model is then fitted to the recorded data by assigning suitable numerical values to its parameters. The issues associated with input-output system identification can be classified in four main categories:
a) System Approximation b) System Parameterization c) Parameter Estimation d) Implementation

System approximation deals with the sense in which the assumed class of system model approximates the actual system. The system identification has three basic ingredients 1) Set of models 2) Data 3) Selection Criterion. Once these have been decided upon, we have at least implicitly, defined a model: the one in the set that best describes the data according to the criterion. But is it good enough? It is the objective of model validation. How do we check the quality of a model? The simplest and most pragmatic approach is to simulate the obtained model with the validation data, evaluate their performance and pick the one with the best fit to measured data. The basic method for model validation is to examine the residuals from the identification process. It is useful to determine such factors as step responses, impulse responses, poles & zeros, model errors and prediction errors. Some of the tools that are useful for discarding models as well as for developing confidence in them are the purpose of modeling, feasibility of physical parameters, applying model reduction techniques, parameter confidence interval and simulation.

System parametrization deals with the manner adjustable parameters enter the description of the class of the system models under consideration. For example such parameters can be the poles & zeros of a transfer function or thee coefficients of the numerator & denominator polynomials etc. Parametric models are required for the so-called 'parametric system identification', however non-parametric methods can be employed as well. The most difficult choice for the user is to find a suitable model structure to fit the data. In particular multivariable systems with several outputs can be difficult because the couplings between several inputs and outputs lead to more complex models. The structures involved are richer and more parameters are required to obtain a good fit. Generally it is preferable to work with state space models in the multivariable case, since the model structure complexity is easier to deal with. It is just a matter of choosing the model order. Choosing the order of the system too high is not recommended since it causes several problems of both numerical and theoretical nature. If you have difficulties in obtaining good models for a multi-output system, model one output at a time to find out which are the difficulties ones to handle. Models to be used for simulations could very well be built up from single output models, one output at a time. Some of the aspects that are considered while selecting a model structure includes the type of model set, size of model set, model parametrization, quality of model, price of model, a priori consideration, comparison of model structures and validation.

The issue of parameter estimation arises after having decided on the structure of a parametric model and the number of adjustable parameters. These estimates are substituted in our parametric model to define the identified system.

The parameter estimation principle for sampled model is illustrated in figure (1). A discrete time model with adjustable parameters is implemented using computer. The error between the system output at instant 't', y (t), and the output predicted by the model $\hat{y}(t)$ (known as the prediction error) is used by a parameter adaptation algorithm (PAA), which at each sampling instant, will modify the model parameters in order to minimize this error.

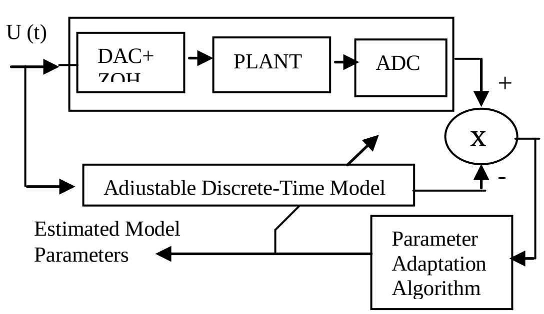

Figure 1. Parameter estimation principle

We may now establish a system identification procedure for SISO systems.

- Select a test input sequence
- Collect experimental data.
- Form the matrix Φ and vector B by processing the I/O data.
- Compute the LS solution.
- Compute the transfer function of the identified system.
- Compute residual error and obtain an estimate of the uncertainty bound.

Assumed that the experiment design and data processing are performed in an appropriate manner, consistent with the limitations of the algorithm.

Finally under implementation we consider various practical issues such as selection of test inputs, fine tuning of the estimation algorithm, computational trade offs etc. The objective of experimental design is to make the collected data set as informative as possible for the models being built from the data. Some of the important points that must be taken into consideration are:

- The input signal must expose all the relevant properties of the system i.e. the input must contain at least as many different frequencies as the order of thee linear model being built.
- A typical good sampling frequency is 10 times the bandwidth of the system.
- In time domain, these suggestions are useful:
- Use binary (two-level) inputs if linear models are being built. This gives maximal variance for amplitude-constrained inputs
- Check the changes between the levels so that the input occasionally stays on one level long enough for a step response from the system to settle, more or less.

4. Implementation and Results

1) Consider the model of the following kind.

$$y(t) + a_1\, y(t-1) + \ldots + an_a\, y(t-n_a) = u(t-n_k) + \ldots + bn_b\, u(t-n_b-n_k+1) \qquad (1)$$

We start by forming some simulated data. Estimate the parameters of the model. Compare the output of the simulated model output with the actual output.

The input is generated as a Random Binary Sequence. The input-output is shown in figure (2). A second order model of ARX type is considered. The order of the system is tested and thee parameters of the model are estimated. The output of the simulated model is compared with the actual output for validation as shown in figure (3) and the residuals are shown in figure (4). The chosen structure and the selected structure is given in the results.

2) The input output of the simulated model for the second problem is shown in figure (5). The comparison of spectrums of the model by different methods is shown in the figure (6) and (8). A 5^{th} order AR model is computed by instrumental variable method and modified

covariance method and their spectrums are compared as shown in figure (9). The spectrums of the AR model and ARMAX model are compared as shown in figure (7).

3) Output error model of the input output data is computed. The estimated parameters and the true parameters are shown in table (1). The four parameters are plotted as a function of time using forgetting factor approach as shown in figure (10). Figure (11) shows the plot of parameters from ARX model using Kalman filter approach.

Table 1. Results of the estimated parameters

Sr. No.	Parameters	Assumed Parameters (G)	Estimated Parameters (G_1)
01	A1	-1.5	-1.4368
02	A2	0.7	0.6370
03	B1	1.0	0.9563
04	B2	0.5	0.6281

Chosen Structure: sN_n = 5 5 1
i.e. Orders and delays: $n_a = 5$; $n_b = 5$; $n_k = 1$.
Number of parameters to be estimated = 4.
Selected Structure: N_{ns} = 2 2 1
i.e. Selected Orders and delays: $n_a = 2$; $n_b = 2$; $n_k = 1$.

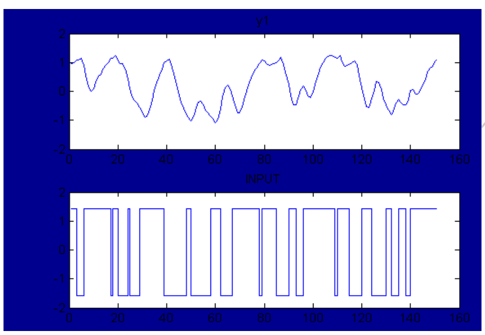

Figure 2. Input output of the system

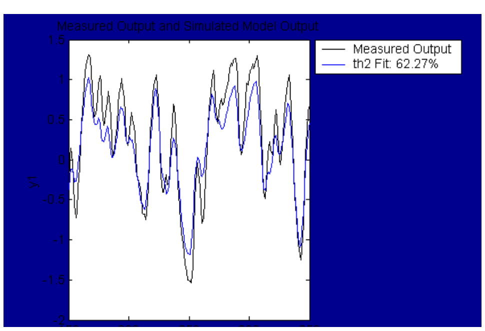

Figure 3. Comparison of the output of simulated model with actual output.

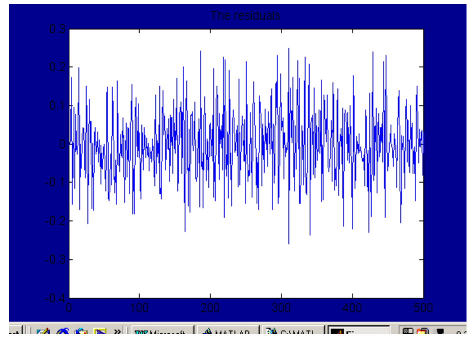

Figure 4. Residuals

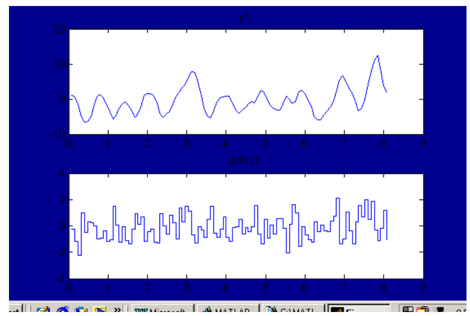

Figure 5. The input output of the simulated model

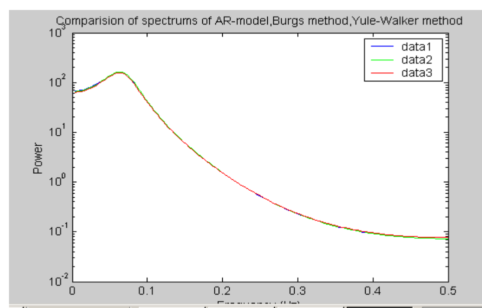

Figure 6. The comparison of spectrums of the model by Burgs and Yule Waker methods

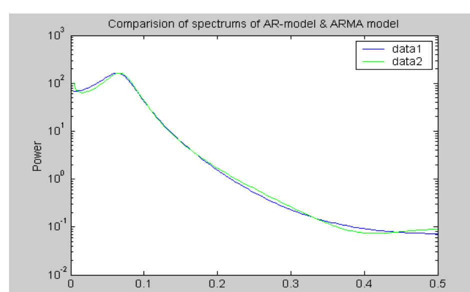

Figure 7. The spectrums of the AR and ARMAX model

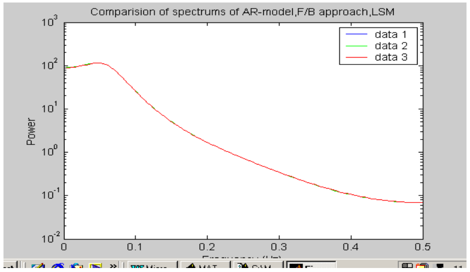

Figure 8. The comparison of spectrums of the AR model by LSM and feedback approach

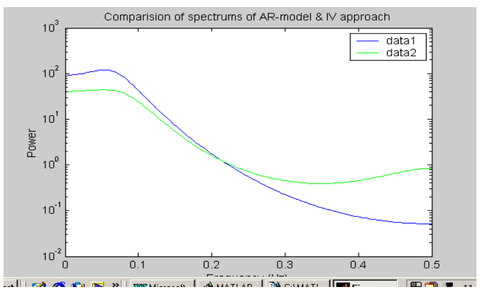

Figure 9. Comparison of spectrums of AR model by instrumental variable and modified covariance method.

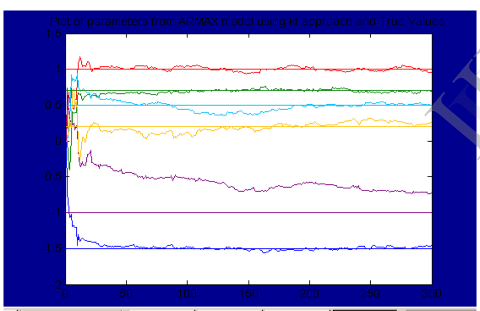

Figure 10. The plot of parameters from ARMAX model using forgetting factor approach.

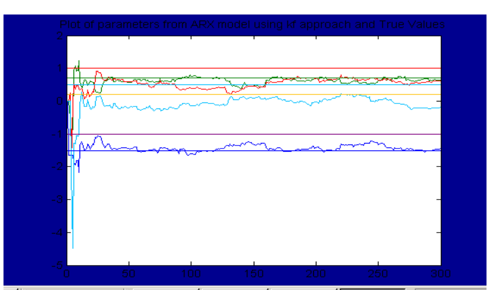

Figure 11. The plot of parameters from ARX model using Kalman filter approach.

5. Conclusion

System Identification with neural network found to be very much useful easy and accurate, specifically for non-linear systems. We find that it is useful to resort to the physical entities, Bode plots and simulation plots, rather than getting stuck in the decimals of parameter estimates, estimated accuracies and loss functions. For industrial scale processes and complex processes, more difficulties will be encountered.

6. Scope for future work

There is much scope for future work in doing system identification by using the advanced intelligent techniques such as Genetic algorithms, neural networks, and Fuzzy logic. Wavelet transforms can be used in the case of Time-varying systems. Algorithms can be developed for the system identification with input-output noisy data and missing measurements.

7. References

[1] G. A. BEKEY and G.N. SARIDIS *Identification and system parameter estimation.* Proceedings of the Sixth IFAC symposium, Washington DC, USA, 7-11 June 1982.

[2] I. D. LANDAU *System Identification and control design using P.I.M. + Software.* Prentice-Hall, 1990.

[3] Jeng-Ming Chen and Bor-Sen Chen *"System Parameter Estimation with Input/Output Noisy Data and Missing Measurements,"* IEEE Trans. Signal Processing, Vol. 48, No. 6, pp. 1548-1558, June 2000.

[4] K. TSAKALIS. *EEE 480/482.*

[5] Laszlo Gerencser and David A. Maluf *"Identification of Time Varying Stochastic System."*

[6] L. LJUNG, *System Identification: Theory for the user.* Prentice-Hall, Inc., Englewood Cliffs, N.J., 1987.

[7] N.K.SINHA and B.KUSZTA, *Modeling and Identification of dynamic systems.* Van Nostrand Reinhold Company, 1983.

[8] S. SALIVAHANAN, A VALLAVARAJ and C.GNANAPRIYA, *Digital Signal Processing,*Tata McGraw-Hill Publishing Company Ltd, New Delhi, 2000.

[9] T. SODERSTROM and P. STOICA *System Identification.* Prentice-Hall International (UK) Ltd, 1989.

[10] M. GOPAL, *Digital Control and State Variable Methods, 2nd ed.,* Tata McGraw-Hill Publishing Company Ltd, New Delhi, 2003.