

# Modeling and Design Optimization of a Solar Heater for Efficient Operation

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**Abstract** — Electricity in remote parts of the world has always been an issue for many underdeveloped countries and unreached parts of the world. The need for hot water is a luxury that many will never get to experience due to the remoteness of many places where people live. A solar water heater could be the answer to this problem with their low to no energy cost and the ability to transport and assemble easily. But the problem that arises with many of these products is the efficiency that they can run at. For example, during winter months and during the night, they are going to cool off faster and the hot water will be gone due to less energy being absorbed into the system.

In this study, a theoretical side of this product was explored, and the system performance was optimized. The optimized parameters are based on theory of heat transfer mechanism on conduction, convection and radiation process on a roof mounted solar water heater. These parameters were modelled and optimized by using the tool of stochastic mechanics and heat transfer mechanism and properties. It was determined that materials with high thermal conductivities are desired since they revealed more efficient and optimal heat transfer. Furthermore, it is also shown that the geometry of the design impacted the system efficiency during operation. The vertical axis gave better operation efficiency, and the horizontal axis gave better emissivity of the design.

**Keywords**— *Modelling, Design, Optimization, Efficiency and Solar Heater*

## ABBREVIATIONS

$Q$  = Heat Transfer  
 $K$  = Thermal Conductivity  
 $T_1$  = Initial Temperature  
 $T_2$  = Final Temperature  
 $L$  = Length; or half thickness of a plane wall  
 $h$  = Convection heat transfer coefficient,  $W/m^2 \cdot ^\circ C$   
 $A_s$  = Surface Area,  $m^2$   
 $T_s$  = Surface Temperature,  $^\circ C$  or K  
 $T$  = Free stream or surrounding medium temperature,  $^\circ C$   
 $\epsilon$  = Emissivity  
 $\sigma$  = Stefan – Boltzmann Constant  
 $R$  = Resistance  
 $t$  = Thickness, m

model in the current study. These data were used to find out the best times of the day and months of the year for sunlight to shine on the product and the best absorption rates was revealed.

$Bi$  = Boit number

$L_c$  = Corrected fin length, m

$V$  = Velocity, m/s

## I. INTRODUCTION

Solar water heaters have been around since the 1890s, when the first system was patented and became commercialized ([1],[5]). The ability to improve this concept has been tried and tried again, especially in recent history and not just in the use of solar energy for heating, but for power as well [1]. Due to the awareness of climate change being evident in the world today, solar energy is being sought out like never before, for example, as in solar lights, solar cookers, solar batteries, solar cameras, etc. All these products share one thing in common, which is converting solar energy into another form of energy.

For a solar water heater, the emphasis is on converting solar energy to thermal energy. For this reason, materials that conduct and insulate are of great importance, specifically, ones that have high conduction and convection coefficients [2]. Another important factor is the ability to harness solar energy, especially when it comes to the emissivity of the components [3]. This design is dependent on the sun, and when and how much radiation is available [4]. With these factors taken into consideration, the costs and benefits of designs can be compared. The heat transfer is the main point of analysis in experimentally determining these factors. By determining efficient conductive and inductive materials and perfecting solar energy absorption with emissivity, the design of a solar water heater can be cost effective and maximized in benefits. The aim therefore, is to try and produce a more efficient solar water heater that can be used in all parts of the world and can be affordable. The proposed design is efficient and therefore the operating parameters for mechanism of heat and heat transfer properties are modelled and optimized for efficient operation. The tool of stochastic material property was identified at the optimal heat transfer mechanism.

Empirical data was obtained to validate the theoretical derived

## II. METHODOLOGY

### A. Theoretical background

Solar water heating systems have had different designs since their inception. The first ones started out with black boxes holding water, which then transitioned to the use of flat plate solar collectors [4]. Today's solar water heaters consists of four main parts - a solar collector, water tanks, connecting pipes, and a control system [1]. The addition of a pump is also a common design, thus, these types of solar water heaters are known as active [1]. While not necessary, this addition does allow for greater efficiency, compared with more cost [6]. Leaving the pump out of the system makes it a passive solar water heater. This is where the water flows naturally through the system [4]. Figures 1 and 2 compare the two systems.

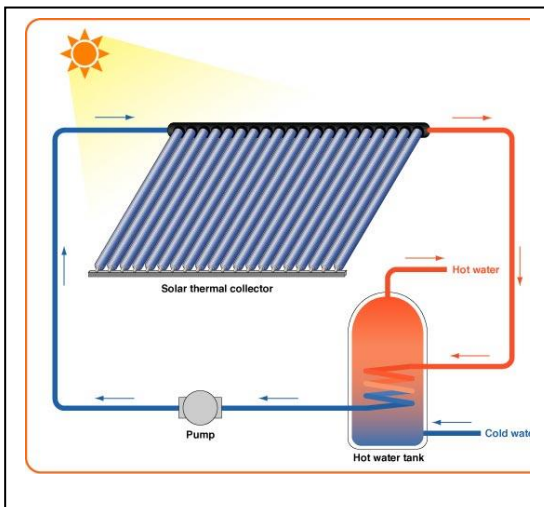


Fig. 1. Active Solar Water Heating System ([7])

The passive systems can be broken down further into thermosyphon or integral collector storage types. These types are identified by how the solar collector relates to the water tanks. The thermosyphon type uses buoyancy force to allow the heated water to rise into the water tank, while the integral collector storage type uses the solar collector and the water tank in one component ([1],[6]). Some systems even use a heat exchanger fluid to bridge the transition between the solar collector and water tanks. These are generally used where freezing temperatures are common ([4],[5]).

Water tanks, connecting pipes, and a control system are generalized components for solar water heating systems. However, the solar collector is widely different depending on the design. There are commonly two designs of the collector.

From fundamental notion on heat transfer process in transient heat conduction, lumped system analysis gives the relationship between parameters and geometry. The criteria for lumped system analysis was used to analyze heat transfer and natural phenomenon on geometry length of the design system as shown below as the dynamics changes with time during operation. Equation (8-14) give the fundamental design relationship between lumped analysis and time, Boit number, Nusselt number, Rayleigh number and the mechanism that impact the system.

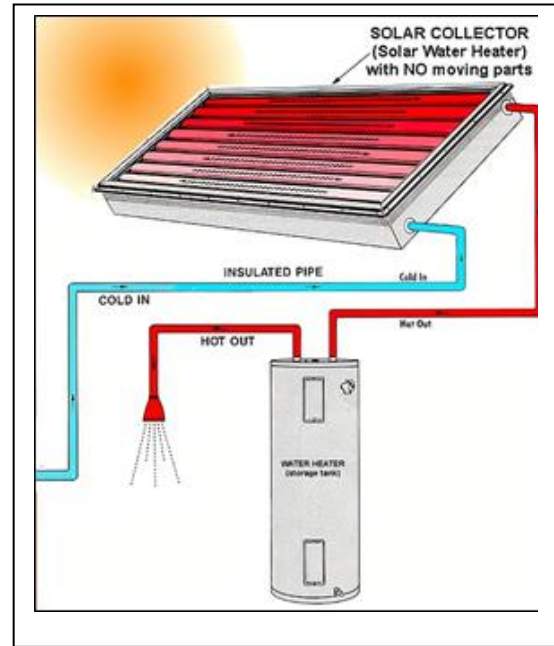


Fig. 2. Passive Solar Water Heating System [8]

One being a concentrated collector, and the other being a nonconcentrated collector [5]. Concentrated collectors use light reflection and mirrors to emphasize solar radiation collection, while nonconcentrated collectors use a wide area of conductive components to capture the solar radiation. Components such as flat plates or tubing ([4],[5]). Depending on the design, the efficiency of the system can be determined using several heat transfer concepts for conduction, convection and radiation given by as,

$$Q_{conduction} = kA \frac{(T_1 - T_2)}{L} \quad (1)$$

$$Q_{convection} = hA_s(T_s - T_{\infty}) \quad (2)$$

$$Q_{radiation} = \epsilon \sigma A_s(T_s^4 - T_{surr}^4) \quad (3)$$

The derived resistance for conduction, convection, radiation are

$$R_{conduction} = \frac{L}{kA} \quad (4)$$

$$R_{convection} = \frac{1}{hA_s} \quad (5)$$

$$R_{radiation} = \frac{1}{h_{rad}A_s} \quad (6)$$

$$R_{total} = R_1 + R_2 + R_3 + \dots R_n \quad (7)$$

## III. DESIGN OF THE SOLAR WATER HEATER

The generalized solar water heater design consists of a passive system with a nonconcentrated solar collector. It is unique, however, in that we have two water tanks, with tubing connecting the two. The first water tank as well as the tubing

Time of lumped analysis system

$$\frac{T_t - T_\infty}{T_i - T_\infty} = e^{-bt} \rightarrow b = \frac{hA_s}{\rho c_p V} \quad (8)$$

Boit number

$$Bi = \frac{hL_c}{k} \rightarrow L_c = \frac{V}{A_s} \quad (9)$$

Nusselt number for  $0^\circ < \Theta < 60^\circ$

$$Nu = \left[ 0.6 + \frac{0.397 R_a^{\frac{1}{8}}}{\left[ 1 + \left( \frac{0.559}{Pr} \right)^{\frac{9}{16}} \right]^{\frac{1}{4}}} \right]^2 \quad (10)$$

Rayleigh number

$$R_a = \frac{g \cos \theta \beta (T_1 - T_2) P_r}{\nu^2} \quad (11)$$

Volume expansion coefficient

$$\beta = \frac{1}{T_f} \quad (12)$$

Convection heat transfer coefficient

$$h = \frac{k}{L} Nu \quad (13)$$

Heat Transfer through a body

$$Q = \frac{T_1 - T_2}{R_{Total}} \quad (14)$$

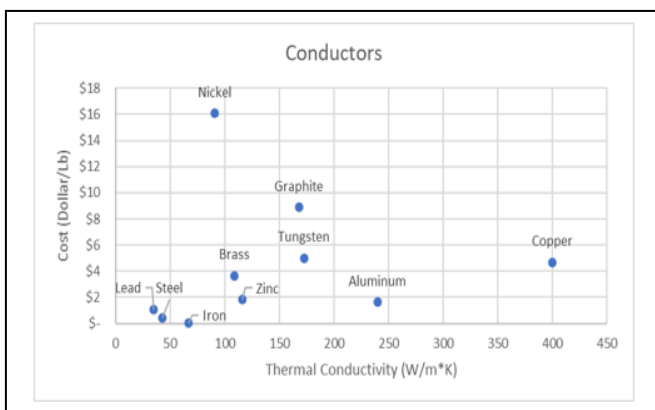


Fig. 4. Comparison of conductive materials

acts as the solar collector, while the second water tank is insulated. The connecting water pipes are located at the inlet and outlet, with the outlet pipe being insulated. The controlling system will be added separately, as this monitors water temperatures and pressure.

The focus of the design is on the solar collector, as this is where the actual water heating occurs. The process requires the application of blackbody surface for the bottom of the solar collector under the tubes to aid in capturing solar radiation. The reason for the tubing is to create more surface area for the water, allowing heat to transfer faster. The tubing and initial water tank are where the heating of the water takes place. The second water tank is then insulated to maintain the temperature of the heated water for as long as possible. The entire system is encased in glass and would theoretically be placed on a roof, or a place of high sunlight exposure.

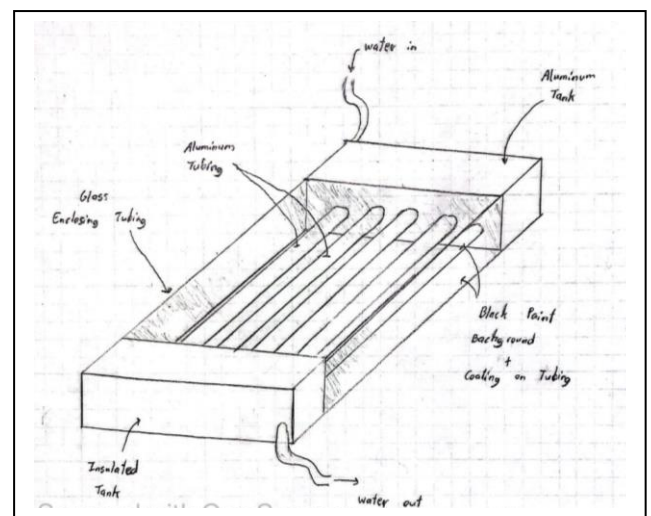


Fig. 3. Sketch of Design

#### IV. RESULTS AND DISCUSSION

An important part of the design process was to determine which materials to use. Because of the importance of capturing solar energy and transforming it into heat, materials of conductance were taken into consideration. Materials of high thermal conductivities are desired; however, cost is also a factor. The comparison of common conductive materials between costs and conductivities is shown below.

Hence, fiber glass was chosen as the insulating material. This material was used for the second holding tank and the outlet connecting pipes. The blackbody background of the design also needed to be carefully decided, as this component helps increase the solar radiation captured by the system. For radiation, materials with higher emissivity are better at capturing it. The figure below compares the costs and emissivity of common blackbody materials.

It is observed from Fig.4 that Nickel is having the highest cost even at low thermal conductivity and copper is having an average cost even at high thermal conductivity. The cost of the material per pound is on the vertical axis while the thermal conductivity is along the horizontal. The ideal choice would be the highest thermal conductive material at the lowest cost. Therefore, aluminum was chosen to be used for the tubing and the initial water tank. Our design also requires efficient insulation. A goal for the design is to keep the heated water hot for as long as possible, and insulation will aid in this objective. Common insulative materials are compared below on thermal conductivity and price.

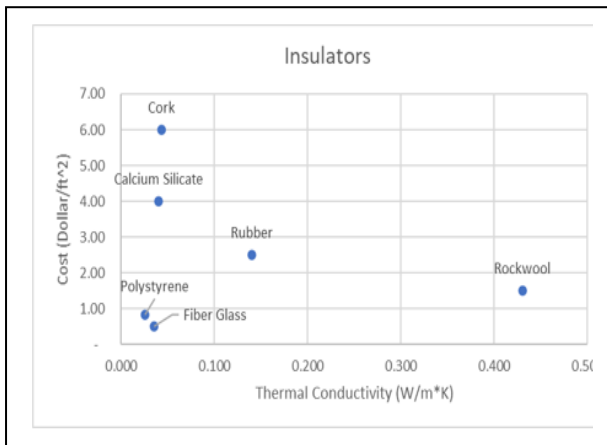


Fig. 5. Comparison of Insulative Materials

It is observed from Fig.5 that cork is having more cost even at low thermal conductivity when compared with rockwool that is having low cost with higher thermal conductivity. The vertical axis of Fig 5 is the cost per foot squared, while the horizontal axis is the thermal conductivity. When it comes to insulation, the lower the conductivity, the better. Also, lower cost is ideal.

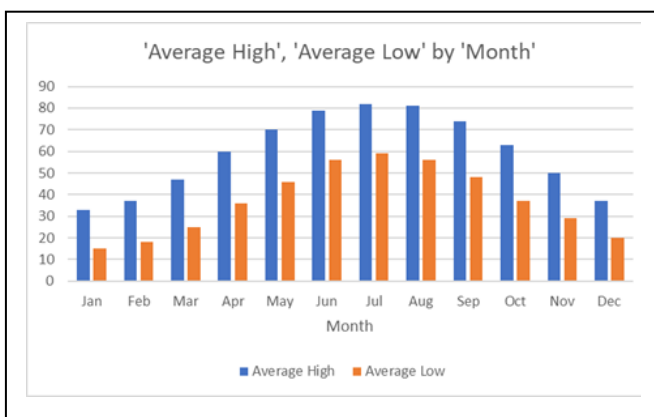


Fig. 7. Graph of monthly temperatures

The vertical axis of Fig. 7 is the temperature in degree Fahrenheit, and the horizontal axis is the month ranging from January to December. This data shows the high and low temperatures for each of the months of the year for Mount Vernon, Ohio. Other things to be considered are the number of days that it may be cloudy and raining and that the

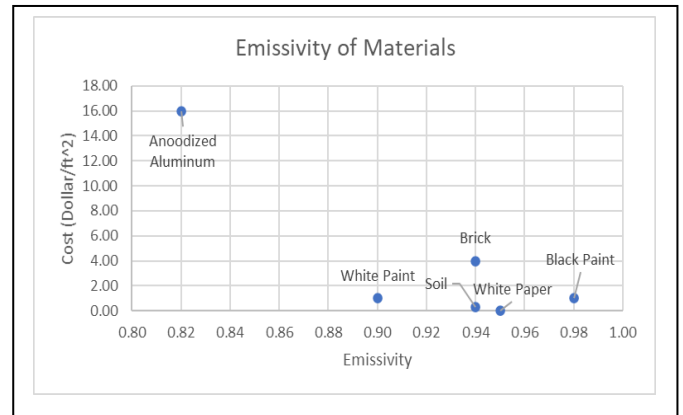


Fig. 6. Comparison of blackbody materials

The vertical axis of Fig 6 is cost per foot squared, and the horizontal axis is the emissivity. The desire is to have a materials with a higher emissivity and lower cost. With that in mind, the black paint was chosen as the blackbody material. This material was applied under the aluminum tubing across the entire interior floor.

TABLE 1. AVERAGE MONTHLY TEMPERATURES FOR MOUNT VERNON, OHIO (°F) [2]

Month	Average High	Average Low
Jan	33	15
Feb	37	18
Mar	47	25
Apr	60	36
May	70	46
Jun	79	56
Jul	82	59
Aug	81	56
Sep	74	48
Oct	63	37
Nov	50	29
Dec	37	20

Table 1 shows the average high and low temperatures for all the twelve months of the year in Mount Vernon, Ohio. Over the course of the year, the average temperature typically varies from 15°F to 82°F. The data in the table can be depicted in the form of a graph, as shown in Fig. 7.

Surface area of top of half of pipe and pipe volume

There are 6 pipes:

Diameter – 0.0508m

Length – 2.2m

Thickness of pipes – 0.00129m

$$A_s = \frac{\pi DL}{2}$$

$$A_s = 0.175m^2$$

$$V = \pi \left(\frac{D}{2}\right)^2 L$$

$$V = 0.004m^3$$

Fiber glass insulation

Thickness – 0.0254

apparatus is going to cool off after the sun sets. With this data we can see the best months this product will work at the highest efficiency. Another thing that is to be kept in mind is that the average temperature of water we want coming out of the shower head or anywhere else. For example, the average temperature of water coming out of a shower needs to be at least between 99°F - 105°F. So, the apparatuses will have to at least reach similar temperatures on the inside to heat to the desired temperature.

Dimensions:

#### Surface area and volume of apparatus

Width – 2m

Length – 2.2m

Height – 0.2m

Thickness of box sides – 0.003175m

$$A_s = W \times L$$

$$A_s = 4.4\text{m}^2$$

$$V = W \times L \times h$$

$$V = 0.88\text{m}^3$$

Surface area of top of water tank and tank volume - There are two tanks

Width - 2m

Length – 0.4m

Height – 0.2m

Thickness of tank walls – 0.003175m

$$A_s = W \times L$$

$$A_s = 0.8\text{m}^2$$

$$V = W \times L \times h$$

$$V = 0.16\text{m}^3$$

#### Surface area of Glass Panel

Width – 2m

Length – 2.2m

Thickness of glass – 0.00635m

$$A_s = W \times L$$

$$A_s = 4.4\text{m}^2$$

#### Total volume of water in the Apparatus

$$V_{\text{Pipes}} = \left(\pi \left(\frac{D}{2}\right)^2 L\right)6$$

$$V_{\text{Tanks}} = (W \times L \times h)2$$

$$V_{\text{Total Water}} = V_{\text{Pipes}} + V_{\text{Tanks}}$$

$$V_{\text{Total Water}} = 0.34675\text{m}^3$$

In deciding on these dimensions, the following was considered. An average ten minute shower requires about 21 gallons or 0.07949365m<sup>3</sup> of water, which is about 2.1 gallons per min. This obviously depends on the shower head and the hot water temperature setting. This volume should provide about 4 showers before running out of hot water. Several other modifications can be made to allow more water for a bigger family as well as to allow water for other things in the household such as washing dishes and laundry. However, if the volume is increased too much, then it will take longer for the water to get up to temperature and in some cases, will never get heated up.

## V. CONCLUSION

The aim of this paper is to try and produce a more efficient and affordable solar water heater that can be used in all parts of the world. The tool of stochastic mechanics and heat transfer theory was used in modeling and designing the current system. The optimal geometry and material property was identified at the

optimal heat transfer mechanism. Empirical data was obtained to validate the theoretical derived model in the current study.

Various materials were compared with respect to cost as well as their thermal conductivity, insulation and emissivity properties. It was determined that aluminium with its high thermal conductivity would be used for the tubing and the initial water tank, fiber glass material with its lower thermal conductivity would be used for the second holding tank and the outlet connecting pipes, while black paint with its high emissivity would be used as the backbody material and thus applied under the aluminium tubing in the design.

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