Model Order Reduction of Higher Order Discrete Time Systems using Modified Pole Clustering Technique

N.Sai Dinesh [1], Dr.M.Siva Kumar [2], D.Srinivasa Rao [3]

1. PG Student, Department of Electrical and Electronics Engineering, Gudlavalleru Engineering College, Gudlavalleru , AP, India.

2. Professor & Head of the Department, Department of Electrical and Electronics Engineering, Gudlavalleru Engineering College, Gudlavalleru, AP, India.

3.Associate Professor, Department of Electrical and Electronics Engineering, Gudlavalleru Engineering College, Gudlavalleru,AP,India.

Abstract

In this paper, a new method is presented to derive a reduced order model for a discrete time systems. This method is based on modified pole clustering technique and pade approximations which are conceptually simple and computer oriented. The denominator polynomial of the reduced order model is obtained by using modified pole clustering technique and numerator coefficients are obtained by Pade approximations. This method generates stable reduced models if the original higher order system is stable. The proposed method is illustrated with the help of typical numerical examples considered from the literature.

Keywords: **Model Order Reduction, Modified Pole Clustering, Pade approximation, Cluster centre, Inverse Distance Measure.**

I. INTRODUCTION

The modeling of a higher order system is one of the most important subjects in engineering and sciences. A model is often too complicated to be used in real life problems. It is an un-debated conclusion that, the development of mathematical model of physical system made it feasible to analyze and design. So the procedures based on the physical considerations or mathematical models are used to achieve simpler models than the original one. Whenever a physical system is represented by a mathematical model it may a transfer function of very high order. Available methods for analysis and design may become cumbersome when applied to a system of higher order. At this juncture, application of large scale order reduction methods is inevitable to reduce computational effort and process time. Efforts towards obtaining low order models from high order systems are related to the aims of deriving stable reduced order models from the stable original ones and ensuring that reduced-order model matches some quantities of the original one. Many methods are available in the international literature that addresses the main objective of the modeling of large-scale systems. Several methods are available in the literature for the order reduction of linear continuous systems in time domain as well as frequency domain [1]-[8].The methods belonging to time domain are Lageurre polynomials [9] and Krylov method [10] the methods belonging to the frequency domain are Routh Approximation Method suggested by Hutton and Fried Land [11], continued fraction expansion method [12] given by Shamash, Moment Matching Method and Pade Approximation methods. The reduced order model obtained in the frequency domain gives better matching of the impulse response with its high order system. Many of these methods can be easily extended to discrete time systems by applying simple transformations [13,14].

In this paper, the authors proposed a method for the order reduction of high order discrete time systems using modified pole clustering technique. The original higher order discrete system is transformed to continuous time system by applying linear transformation and the reduced order model is derived for the continuous time system, by using modified pole clustering technique and pade approximation. And finally corresponding inverse transformation yields reduced order model in discrete time system.

II. PROBLEM FORMULATION

Let the transfer function of higher order original Discrete Time System of order 'n' be

$$G(Z) = \frac{N(Z)}{D(Z)} = \frac{e_0 + e_1 z + e_2 z^2 + \dots\dots\dots + e_{n-1} z^{n-1}}{f_0 + f_1 z + f_2 z^2 + \dots\dots\dots\dots + f_n z^n}$$

Convert the higher order discrete time system into 'P' domain using linear transformation Z=P+1.

$$G(P) = G(Z)_{|Z=P+1}$$

Therefore the transfer function of higher order original system of order 'n' in P-domain is

$$G(P) = \frac{N(p)}{D(p)} = \frac{e_0+e_1p+e_2p^2+\cdots+e_{n-1}p^{n-1}}{f_0+f_1p+f_2p^2+\cdots+f_np^n} \quad (1)$$

Where e_i; $0 \le i \le n-1$ and f_i ; $0 \le i \le n$ are scalar constants.

Therefore, it is required to derive a 'k^{th}' order reduced model in P domain. It is given by

$$R_k(p) = \frac{N_k(p)}{D_k(p)} = \frac{a_0+a_1p+a_2p^2+\cdots+a_{k-1}p^{k-1}}{b_0+b_1p+b_2p^2+\cdots+b_kp^k} (2)$$

where a_i; $0 \le i \le k-1$ and b_i ; $0 \le i \le k$ are scalar constants.

By applying inverse transformation, the reduced order model in Z-domain is obtained.

$$R_k(Z) = R_k(p)_{|P=Z-1}$$

III.REDUCTION PROCEDURE

The proposed method for getting the k^{th} order reduced model, consists of the following two steps:

Step1: Determination of the denominator polynomial for the k^{th} order reduced model, using modified pole clustering technique.

Step2: Determination of the numerator of k^{th} order reduced model using Pade approximation.

The following rules are used for clustering the poles of the original system to get the denominator polynomial for reduced order models.

a. Separate clusters should be made for real poles and complex poles.
b. Poles on the jw-axis have to be retained in the reduced order model.
c. Clusters of poles in the left half s-plane should not contain any pole of the right half s-plane and vice-versa.

By using a simple method, "Inverse Distance Measure", the cluster center can be formed as follows:

Let there be a r real poles in one cluster are $p_1, p_2, p_3 \dots\dots\dots p_r$,then Inverse Distance Measure (IDM) identifies cluster center as

$$p_u = \left[\left(\sum_{i=1}^{r} \left(\frac{1}{p_i} \right) \right) \div r \right]^{-1} \quad \dots (3)$$

where $|p_1| < |p_2| < |p_3| \ldots \ldots |p_r|$,then modified cluster center can be obtained by using the algorithm.

Step 1: Let r real poles in a cluster be $|p_1| < |p_2| < |p_3| \ldots \ldots \ldots \ldots . . |p_r|$.

Step 2: Set j=1.

Step 3: Find pole cluster centre $c_{j} = \left[\sum_{i=1}^{r}\left(\frac{-1}{|p_i|}\right)\div r\right]^{-1}$

Step 4: Set j=j+1

Step 5: Find a modified cluster centre from

$$c_j = \left[\left(\frac{-1}{|p_1|} + \frac{-1}{|c_{j-1}|}\right) \div 2\right]^{-1}$$

Step 6: Is r=j? if No, and then go to step 4, otherwise go to step 7

Step 7: Modified cluster centre of the k^{th} cluster as $p_{uk} = c_j$

Let m pair of complex conjugate poles in the cluster be

$$[(\alpha_1 \pm j\beta_1), (\alpha_2 \pm j\beta_2), \ldots . . (\alpha_m \pm j\beta_m)]$$

then the complex center is in the form of $A_u \pm jB_u$.

Where $A_u = \left[\left(\sum_{i=1}^{m}\left(\frac{1}{\alpha_i}\right)\right) \div m\right]^{-1}$

and $\beta_u = \left[\left(\sum_{i=1}^{m}\left(\frac{1}{\beta_i}\right)\right) \div m\right]^{-1}$ (4)

One of the following cases may occur, for synthesizing the k^{th} order denominator polynomial.

Case 1: If all the modified cluster centers are real, then reduced denominator polynomial of order 'k', can be taken as

$$D_k(p) = (p - p_{u1})(p - p_{u2}). \ldots (p - p_{uk})$$

where $p_{u1}, p_{u2}, \ldots . p_{uk}$ are $1^{st}, 2^{nd} \ldots \ldots k^{th}$ modified cluster center respectively. (5)

Case 2:If all the modified cluster centers are complex conjugate, then the reduced denominator polynomial of order 'k' can be taken as

$$D_k(p) = (p - \dot{p}_{u1})(p - \acute{p}_{u1})(p - \dot{p}_{u2})(p - \acute{p}_{u2}) \ldots . . (p - \dot{p}_{uk/2})(p - \acute{p}_{uk/2}) \quad (6)$$

where $\dot{p}_{u1}$ and $\acute{p}_{u1}$ are complex conjugate cluster centers or $\dot{p}_{u1} = A_u + jB_u$ and $\acute{p}_{u1} = A_u - jB_u$

Case 3: If (k-2) cluster centers are real and one pair of cluster center is complex conjugate, then the denominator polynomial of the k^{th} order reduced model can be obtained as

$$D_k(p) = (p - p_{u1})(p - p_{u2}) \ldots . . (p - p_{u(k-2)})(p - \dot{p}_{u1})(p - \acute{p}_{u1}) \quad (7)$$

Step 2: Determination of the numerator of k^{th} order reduced model using Pade approximations.

The original n^{th}order system can be expanded in power series about p=0 as

$$G(p) = \frac{N(p)}{D(p)} = \frac{e_0 + e_1 p + e_2 p^2 + \cdots + e_{n-1} p^{n-1}}{f_0 + f_1 p + f_2 p^2 + \cdots + f_n p^n}$$

$$= c_0 + c_1 p + c_2 p^2 + \cdots \ldots \ldots \quad (8)$$

The coefficients of power series expansion are calculated as follows:

$$c_0 = e_0$$

$$c_i = \frac{1}{f_0}\left[e_i - \sum_{j=1}^{i} f_j c_{i-j}\right], i > 0$$

$$e_i = 0, i > n - 1 \quad (9)$$

The reduced k^{th} order model is written as

$$R_k(p) = \frac{N_k(p)}{D_k(p)}$$

$$= \frac{a_0 + a_1 p + a_2 p^2 + \cdots + a_{k-1} p^{k-1}}{b_0 + b_1 p + b_2 p^2 + \cdots + b_k p^k} \quad (10)$$

Here, D_k(s) can be determined through equations

(5-7).

For $R_k(s)$ of equation (10) to be Pade approximants of G(s) of equation (8), we have

$$a_0 = b_0 c_0$$

$$a_1 = b_0 c_1 + b_1 c_0$$

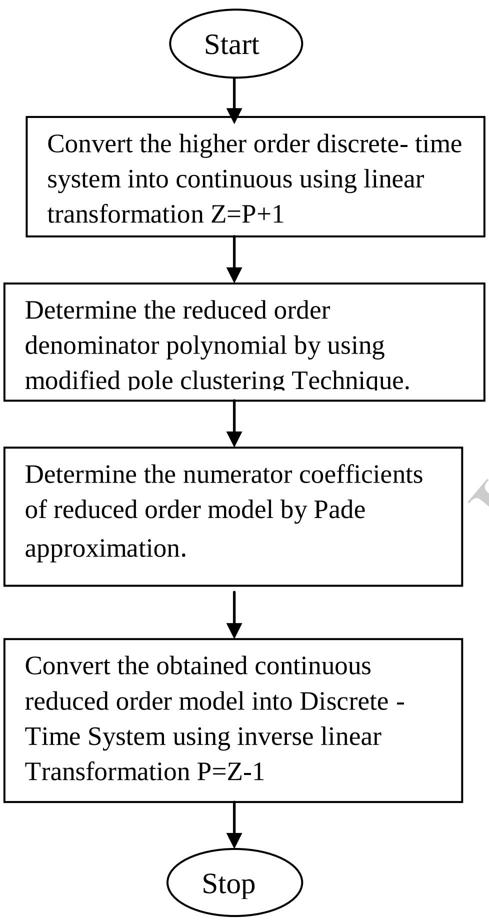
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (11)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_{k-1} = b_0 c_{k-1} + b_1 c_{k-2} + \cdots + b_{k-1} c_1 + b_k c_0$$

the coefficients a_j ; j=0,1,2,3…..k-1 can be found by solving above k linear equations.

FLOW CHART FOR ORDER REDUCTION



IV. NUMERICAL EXAMPLES

EXAMPLE 1

Consider a 4th order discrete-time system given by Younseok Choo [15]:

$$G(Z) = \frac{N(Z)}{D(Z)}$$

$$= \frac{2Z^4 + 1.8Z^3 + 0.8Z^2 + 0.1Z - 0.1}{Z^4 - 1.2Z^3 + 0.3Z^2 + 0.1Z + 0.02}$$

Substituting Z=P+1,

$$G(P) = \frac{N(P)}{D(P)}$$

$$= \frac{2p^4 + 9.8p^3 + 18.2p^2 + 15.1p + 4.6}{p^4 + 2.8p^3 + 2.7p^2 + 1.1p + 0.22}$$

The poles are : $-1.1199 \pm j0.1351$,

$$-0.2801 \pm j0.3073$$

By using the above modified pole clustering algorithm, modified cluster center can be formed from the poles as

$$p_{u1} = -0.6400 \pm j0.1570$$

The denominator polynomial $D_k(p)$ of reduced order model is obtained by using the equation (6) as

$$D_k(p) = (p - p'_{u1})(p - p'_{u2})$$

$$D_k(p) = p^2 + 1.28p + 0.43425$$

Therefore, the 2^{nd} order reduced model is

$$R_k(p) = \frac{a_0 + a_1 p}{0.2074435 + 0.53334p + p^2}$$

The numerator coefficients are obtained by pade approximations. By using equations (9) and (11),

$$a_0 = 9.0797$$

$$a_1 = 11.1702$$

Therefore, finally 2^{nd}-order reduced model is obtained as

$$R_2(p) = \frac{9.0797 + 11.1702p}{0.43425 + 1.28p + p^2}$$

By applying inverse transformation P = Z-1, we obtain the reduced order model of discrete time system as

$$R_2(Z) = \frac{-2.0905 + 11.1702Z}{0.15425 - 0.72Z + Z^2}$$

For comparison purposes, a second order approximant by Younseok Choo [15]and [16] is found to be

$$G_2{}^1(Z) = \frac{4.87768Z - 2.55604}{Z^2 - 1.50888Z + 0.63166}$$

$$G_2{}^{11}(Z) = \frac{4.2Z - 0.9023114}{Z^2 - 1.442346Z + 0.616743}$$

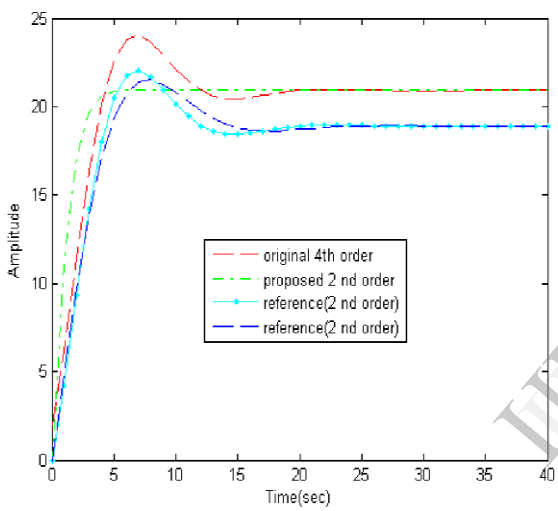


Figure 1: Comparison of Step response of high order system and reduced order models for example1.

It has been observed from Fig1 that the 2nd order reduced model obtained by the proposed method gives good step response that the models obtained from the methods given in [15] and [16].

EXAMPLE 2

Consider a 6th order discrete-time system,

$$G(Z) = \frac{N(Z)}{D(Z)}$$

$$\frac{2.04(Z-0.75)(Z-0.9423592)(Z-0.717487)(Z-0.5195656)(Z-0.5)}{(Z-0.3)(Z-0.5)(Z-0.75)(Z-0.85)(Z-0.9)(Z-0.95)}$$

Substituting Z=P+1,

$$G(P) = \frac{N(P)}{D(P)}$$

$$= \frac{2.04p^5 + 3.203998p^4 + 1.8770978p^3 + 0.5001582p^2 + 0.0577944p + 0.0019949}{p^6 + 1.75p^5 + 1.1125p^4 + 0.323125p^3 + 0.45212p^2 + 0.002893p + 0.0000656}$$

The poles are: -0.7,-0.5,-0.25,-0.15,-0.1,

-0.05.

By using the above modified pole clustering algorithm, modified cluster centers can be formed from the real poles as

$$p_{u1} = -0.057144$$

$$p_{u2} = -0.15939$$

The denominator polynomial $D_k(p)$ of reduced order model is obtained by using the equation (5) as

$$D_k(p) = (p - p_{u1})(p - p_{u2})$$

$$= (p + 0.057144)(p + 0.15939)$$

$$D_k(p) = p^2 + 0.216534p + 0.009108$$

Therefore, the 2^{nd} order reduced model is

$$R_k(p) = \frac{a_0 + a_1 p}{0.009108 + 0.216534p + p^2}$$

The numerator coefficients are obtained by pade approximations. By using equations (9) and (11),

$$a_0 = 0.2769748$$

$$a_1 = 2.39431$$

Therefore, finally 2^{nd}-order reduced model is obtained as

$$R_2(p) = \frac{0.2769748 + 2.39431p}{0.009108 + 0.216534p + p^2}$$

By applying inverse transformation P = Z-1,

we obtain the reduced order model of discrete time system as

$$R_2(Z) = \frac{-2.1173352 + 2.39431Z}{0.792574 - 1.783466Z + Z^2}$$

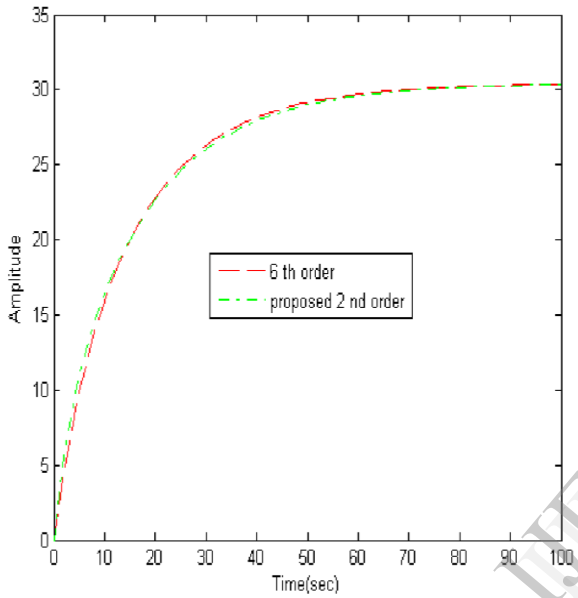


Figure 2: Comparison of Step response of high order system and reduced order models for example2.

V.CONCLUSION

A new method is presented to determine the reduced order model of a higher order discrete time system. The denominator polynomial of the reduced order model is obtained by using modified pole clustering while the numerator coefficients are obtained by pade approximation. The effectiveness of proposed method is illustrated with the help of examples chosen from the literature and the responses of the original and reduced system are compared graphically as shown in fig.1

VI. REFERENCES

[1] A.K.Sinha, J. Pal, Simulation based reduced order modeling using clustering technique, *Computer and Electrical Engg. ,* 16(3), 1990 159-169.

[2] S.K.Nagar, and S.K.Singh, An algorithmic approach for system decomposition and balanced realized model reduction, *Journal of Franklin Inst.*, Vol.341, pp. 615-630,2004.

[3] S.Mukherjee, Satakshi and R.C.Mittal,"Model order reduction using response-matching technique", *Journal of Franklin Inst.*,Vol. 342, pp. 503-519,2005.

[4] G.Parmar, S.Mukherjee and R.Prasad, Relative mapping errors of linear time invariant systems caused by Particle swarm optimized reduced model,Int. *J. Computer, Information and systems science and Engineering*, Vol. 1, No. 1, pp. 83-89,2007.

[5] V.Singh, D.Chandra and H.Kar, Improved Routh Pade approximants: A Computer aided approach, IEEE Trans. *Autom.* Control, 49(2), pp .292-296,2004.

[6] C.P.Therapos, Balanced minimal realization of SISO systems, Electronics letters, Vol.19, No.11, pp. 242-2-426, 1983.

[7] Y.Shamash, Stable reduced order models using Pade type approximations, IEEE Trans. Vol. AC-19, pp.615-616, 1974.

[8] Sastry G.V.K.R Raja Rao and Mallikarjuna Rao P., Large scale interval system modeling using Routh approximants, Electrical letters, 36(8), pp.768-769,2000.

[9] Model order reduction in the Time Domain using Laguerre Polynomials and Krylov Methods, Y.Chen, V.Balakrishnan, C-K. Koh and K.Roy,Proceedings of the 2002 Design, Automation and Test in Europe Conference and Exhibition DATE.02 2002 IEEE.

[10] Practical issues of Model Order Reduction with Krylov-subspace method, Pieter Heres, Wil Schilders.

[11] Hutton, M.F and Friedland,B,"Routh Approximation for reducing order of linear Time invariant system",IEEE Trans, On Automatic Control, Vol.20, 1975, PP.329-337.

[12] Shieh, L.S and Goldman, M.J, Continued Fraction Expansion and Inversion of cauer Third Form, IEEE *Trans.On* Circuits and systems, Vol.CAS-21, No.3 May 1974, pp.341-345.

[13] Y.P.Shih, and W.T.Wu.,"Simplification of z-tranfer functions by continued fraction" Int.J.Contr.,Vo.17, pp.1089-1094,1973.

[14] Chuang C, "Homographic transformation for the simplification of discrete-time functions by Pade approximation",Int.J. Contr.,Vo.22, pp.721-729,1975.

[15] Y. Choo, "Suboptimal Bilinear Routh Approximant for Discrete Systems,"ASME Trans.,Dyn.Sys.Meas. Contr., 128, 2006 ,pp.
742-745.

[16] VEGA Based Routh-Pade Approximants For Discrete Time Systems: A Computer Aided Approac, IACSIT International Journal of Engineering and Technology Vol.1, No.5, December 2009, ISSN:1793-8236.

VII.BIOGRAPHIES

Dr Mangipudi Siva Kumar was born in Amalapuram, E. G. Dist, Andhra Pradesh, India, in 1971. He received bachelor's degree in Electrical & Electronics Engineering from JNTU College of Engineering, Kakinada and M.E and Ph.D degree in control systems from Andhra University College of Engineering, Visakhapatnam, in 2002 and 2010 respectively. His research interests include model order reduction, interval system analysis, design of PI/PID controllers for Interval systems, sliding mode control, Power system protection and control. Presently he is working as Professor & H.O.D of Electrical Engineering department, Gudlavalleru Engineering College, Gudlavalleru, A.P, India. He received best paper awards in several national conferences held in India.

D.Srinivasa Rao was born in Nara kodur, Guntur Dist, Andhra Pradesh, India, in 1969. He received bachelor's degree in Electrical & Electronics Engineering from JNTU College of Engineering, Kakinada and M.Tech in Electrical Machines & Industrial Drives from NIT Warangal in 2003.Now doing Ph.D in the area of control systems in JNTU Kakinada.His research interests include model order reduction, interval system analysis, design of PI/PID controllers for Interval systems, Multi-level Inverters, Presently he is working as Associate Professor in Electrical Engineering department, Gudlavalleru Engineering College, Gudlavalleru, A.P, India. He presented various papers in national and International Conferences held in India.

N.SaiDinesh a PG Student completed B.Tech in Sri Sunflower College of Engineering and Technology, and pursuing M.Tech.control systems in Gudlavalleru Engineering College, Gudlavalleru.