Model based Adaptive PID Controller with Parallel Feedforward Compensator for Steam Turbine Speed Control

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Abstract—In this paper, a new design method of model based adaptive PID controller with parallel feedforward compensator is proposed and improvement of steam turbine control is investigated. The method can be applied to plants, which satisfy the Almost Strict Positive Realness (ASPR) conditions. In order to satisfy ASPR conditions, Parallel Feedforward Compensator (PFC) is introduced. Also, practical applications of the method are discussed and the proposed method is applied to steam turbine speed control system through numerical simulations.

Keywords—PID, PFC, Steam Turbine Speed Control, Adaptive PID Control, ASPR.

1. INTRODUCTION

A power plant comprises of boiler, gas turbine and steam turbines as major components. Frequency stabilizing is very important for power system security. Because of that, there have been many signs of progress and interests in power systems control in recent years. The new control algorithms and techniques are applied to the power system to obtain the best performance.

Thus, adaptive control seems one good solution to this problem and has a growing interest in the industrial equipment.

Most common automatic tuning method is proposed by J.G. Ziegler and N. B. Nichols in 1942 [1]. The method determines $K_p$, $K_v$ and $K_y$ gains of PID by using rules, which are developed by Ziegler and Nichols. This method is still used in industrial equipment and commercial sectors. However, it has limited applications on systems, since the rules are based on open-loop responses and cannot perform on closed loop environment.

Since Ziegler-Nichols tuning method has restrictions on closed-loop processes and therefore other tuning methods have been developed. K. Aström and T. Hägglund proposed “Phase Margin Method” [2]. However, the drawback of this method is that it may not perform well on systems with big time delays causing oscillations in closed loop systems.

On the other hand, Popov proposed hyperstability in [3] and introduced the notion of strict positive realness for which Linear Time Invariant (LTI) system is hyper stable asymptotically if the transfer function of the system is strictly positive real”. M. Hakimi and H. Khaloozadeh in [4] had studied this concept and approached the problem in the frequency domain by using Taylor Series Expansion and Maximum Modulus Principle and showed strict positive realness in the frequency domain.

A robust optimal LQG controller was proposed for non-minimum phase plant by H. Zargarzadeh and M. M. Arefi in [14]. Robustness of the proposed controller was investigated and for experimental purposes, the controller was later reduced to a PI controller. As compared to loop-shaping H∞ control, proposed LQG controller showed improvement in energy consumption and time responses.

S. Ozcelik and H. Kaufman presented Direct Model Reference Adaptive Control (DMRAC) algorithm for Single Input Single Output (SISO) plants in [5], where Kharitonov’s theorem was used for feed forward compensator design to satisfy Almost Strictly Positive Real (ASPR) conditions. This algorithm was later extended to Multi-Input Multi-Output (MIMO) systems in [6,12].
Z. Iwai, I. Mizumoto and Y. Nakashima developed a parallel feedforward compensator for (ASPR) systems and developed tuning rules for MIMO systems in [7, 10], where an approximate model of the system was used in order keep the stability of the MIMO plant since ASPR conditions were not satisfied in real applications. The drawback of the algorithm is that it has practical and theoretical restrictions and may not be applied to broad range systems due to its control structure.

Hence, a new model based adaptive PID controller with parallel feedforward compensator is proposed and applied to steam turbine speed control system. Comparisons between proposed algorithm and conventional PID controller are explained and numerical simulations are pursued.

II. MODELING OF STEAM TURBINE SPEED CONTROL SYSTEM

A. Modeling of Steam Turbine

Steam enters from main stop valve and flows to governor valve. Governor controls the High Pressure section pressure and then it moves to reheat. Reheated steam passes to Intermediate Pressure section through the intermediate control valve and piping. Finally, steam leaves turbine from exhaust section through the crossover.

The simplified block diagram of steam turbine is shown in Figure 1 below.

![Fig.1. Block Diagram of Steam Turbine [8]](Image 1)

\[ F_{HP}, F_{IP} \text{ and } F_{LP} \text{ being power fractions, determine power, which comes from High Pressure, Intermediate Pressure and Low Pressure sides. Sum of the fractions is equal to 1} \ (F_{HP} + F_{IP} + F_{LP} = 1). \ \Delta E \text{ is valve power. } P_g \text{ is mechanical power, which is equal to torque.} \ T_{ch}, T_{rh} \text{ and } T_c \text{ are the time constants of steam turbine chest, re-heater, and crossover piping, respectively. It is assumed that } T_{ch} \text{ is negligible as compared to } T_{ch}. \text{ Thus, simplified transfer function of the turbine system, which is the ratio of mechanical power to gate position is [9]}

\[
\frac{\Delta P_g}{\Delta E} = \frac{F_{HP}}{T_{ch} s + 1} + \frac{1 - F_{HP}}{(T_{ch} s + 1)(T_{rh} s + 1)}
\]

B. Speed Governing Mechanism and Modeling

The Speed Governor, a type of transducer senses error in speed and converts shaft speed to position output. Control valves conduct input to the turbine and are activated by the governor. Speed control mechanism has other electrical and mechanical components such as linkages, servomotors, levers, and amplifiers. Generic speed governing system is shown in Figure 2 below. There are two valve nonlinearities; Rate limit represents the saturation of the servomotor position, and integrator limits the position of the valve. Limits are shown by per unit (p.u). A new model based adaptive PID controller with parallel feedforward compensator algorithm is applied to steam turbine speed control system.

![Fig.2. Generic Speed Governing System Model Presentation [5]](Image 2)

III. MODEL BASED ADAPTIVE PID CONTROLLER WITH PARALLEL FEEDFORWARD COMPENSATOR

In this section, the Model Based Adaptive PID Control algorithm is proposed. The algorithm proposed by Iwai [10] has practical, theoretical restrictions and can’t be applied to broad range systems due to its control structure. Hence, the new Model Based Adaptive PID Controller with Parallel Feedforward Compensator is proposed. If the steady-state algorithm of analysis proposed by Iwai is studied, there are conditions, which need to be satisfied firstly for zero steady state response. These are,

\[
- G_{FFC} = 0
\]

\[
- G (0) \neq 0 \text{ and } k_i \neq 0
\]

where \( G_{FFC} \) is the transfer function of feedforward compensator and \( k_i \) is the integral gain of adaptive PID. \( G_{FFC} \) (s) is obtained from \( G_{FFC} (s) = G_{ASPR} (s) - G^* (s) \), where \( G_{ASPR} (s) \) is an ASPR transfer function and \( G^* (s) \) is the nominal transfer function. Since \( G_{FFC} (0) \) should be zero and is calculated using \( G^* (s) \), all physical systems cannot satisfy the above conditions. Hence, steady state error will remain for the controlled plant. To this effect, the new adaptive PID controller is developed to improve the algorithm and to eliminate the error in the system so that adaptive PID can be applied to sophisticated systems.
A. Problem Setup

Let’s consider SISO n\textsuperscript{th} order plant:

\[ x_p(t) = A_p x_p(t) + B_p u_p(t) \]
\[ y_p(t) = C_p x_p(t) \]  \hspace{1cm} (3.3)

where \( x_p \) is the state vector, \( u_p \) is the control vector, \( y_p \) is the output vector and \( A_p \) and \( B_p \) are matrix dimensions. The plant can be expressed by transfer function of

\[ G_p(s) = C_p (s I - A_p)^{-1} B_p \] \hspace{1cm} (3.4)

Theorem 1 [11]

Equation (3.4) is ASPR if it satisfies the conditions below,

- \( N_p(s) \) is Routh-Hurwitz polynomial.
- Relative degree of the transfer function is 0 or 1.
- Leading coefficient is positive

Since most of the plants do not satisfy the ASPR conditions, Parallel Feedforward Compensator is introduced.

B. Parallel Feedforward Compensator Design

Nominal plant parameters are known and uncertainties are represented as additive perturbations. Plant uncertainties will be presented by transfer functions. The aim is to develop parallel feedforward compensator so that augmented plant satisfies ASPR conditions and desired responses are achieved despite changes in parameters.

Now consider a non-ASPR SISO plant in the form of \( G_p \),

\[ G_p = \frac{C_m s^m + C_{m-1} s^{m-1} + \ldots + C_0}{B_n s^n + B_{n-1} s^{n-1} + \ldots + B_0} \] \hspace{1cm} (3.5)

Where coefficients \( B_{n-j} \) and \( C_{m-j} \) may take values within the bounds below,

\[ C_{m-j} \leq \bar{C}_{m-j}, j = 0,1,\ldots,m \] \hspace{1cm} (3.6)
\[ B_{n-j} \leq \bar{B}_{n-j}, j = 0,1,\ldots,n \] \hspace{1cm} (3.7)

If the nominal plant parameters are known and \( G_{nom}(s) \) is constructed, uncertainty can be shown as frequency dependent additive perturbation. Hence, actual plant becomes \( G_p(s) = G_{nom}(s) + \Delta(s) \), where \( \Delta(s) \) is additive perturbation.

Then defining,

\[ \Delta(s) = G_p - G_{nom} \] \hspace{1cm} (3.8)

It is clear that the uncertainty is a function of plant parameters varying within the ranges. Hence, a parallel feedforward compensator that satisfies the worst case uncertainty should be designed. An optimization procedure below will be used to get the worst case uncertainty at each frequency.

Define a vector whose elements are plant parameters in equation (3.6),

\[ V = [C_m C_{m-1} \ldots C_0 B_n B_{n-1} \ldots B_0] \]

Maximize \( \| V(j\omega) \| \) at each \( \omega \)

Subject to,

\[ C_{m-j} \leq \bar{C}_{m-j}, j = 0,1,\ldots,m \]
\[ B_{n-j} \leq \bar{B}_{n-j}, j = 0,1,\ldots,n \] \hspace{1cm} (3.9)

where \( \Delta(j\omega) \) is perturbation. Given the worst-case uncertainty for each frequency, it is supposed that there is a rational function which is \( W(s) \in RH_\infty \) such that

\[ \| W(j\omega) \| \leq \max \| \Delta(j\omega) \|, \forall \omega \] \hspace{1cm} (3.10)

The following assumptions are made for the plant:

-- Nominal plant (\( G_{nom} \)) is known, minimum phase and stable.
-- Actual plant is stable.
-- \( \Delta(s) \) satisfies (3.10).

If the following augmented plant with parallel feedforward compensator is considered as,

\[ G_{a}(s) = G_p(s) + G_{PFC}(s) \] \hspace{1cm} (3.11)

The following theorem gives the design conditions of parallel feedforward compensator. Also, augmented plant satisfies ASPR conditions in the presence of plant uncertainty [12].

Theorem 1:

If \( G_{PFC}(s) \) is designed as conditions below, the augmented plant \( G_{a}(s) = G_p(s) + G_{PFC}(s) \) will be ASPR.

-- \( G_{PFC}(s) \) is stable and relative degree one.
-- The augmented nominal plant \( G_{nom}(s) + G_{PFC}(s) \) is ASPR.

-- \( \bar{\Delta}(s) \in RH_\infty \) and \( \| \bar{\Delta}(s) \| \leq \epsilon \), \( \epsilon < 1 \)

Where

\[ \bar{\Delta}(s) = (G_{nom}(s) + G_{PFC}(s))^{-1} W(s) \] \hspace{1cm} (3.12)

\( \bar{\Delta}(s) \) is the uncertainty of the augmented plant.

(Please refer to the proof of theorem)
Consider the closed loop system comprised of the nominal plant $G_{nom}(s)$ and additive perturbation which is $\Delta(s)$ and controller $C(s)$ as shown in Figure 4, the actual plant is

$$G_p(s) = G_{nom}(s) + \Delta(s) \quad (3.13)$$

The transfer function from $w$ to $z$ can be shown as

$$w = (1 + C(s)G_{nom}(s))^{-1} C(s) z \quad (3.14)$$

By using small gain theorem, the closed loop system is stable if and only if

$$\left\| \begin{bmatrix} \Delta \\ G_{PFC} \end{bmatrix} \right\|_\infty < 1 \quad (3.15)$$

or it can be written as,

$$\left\| (1 + G_{nom}(s)C(s))^{-1} C(s) \Delta(s) \right\|_\infty < 1 \quad (3.16)$$

The robust stability condition by using parallel feedforward compensator is

$$\left\| \begin{bmatrix} \Delta \\ G_{PFC} \end{bmatrix} \right\|_\infty < 1 \quad (3.17)$$

By replacing $\Delta$ with $W$, one can have

$$\left\| \begin{bmatrix} W \\ G_{nom} + G_{PFC} \end{bmatrix} \right\|_\infty < 1 \quad (3.18)$$

C. Adaptive PID Controller Design

Suppose that augmented system is ASPR. The system can be stabilized by using equation below,

$$V(t) = -k(t) g(t) \quad (3.19)$$

where,

$$k(t) = [k_p(t), k_d(t), k_i(t)] \quad (3.20)$$

$$g(t) = [y_a, \dot{y}_a, w] \quad (3.21)$$

$$\dot{w} = y_a \quad (3.22)$$

The adaptive gain vector $k(t)$ is tuned by adaptive law below,

$$\dot{k}(t) = \Gamma g(t) y_a(t) \quad (3.23)$$

In this case $\lim_{t \to \infty} \epsilon(t) = 0$. Zero steady state error condition is satisfied.

Error Analysis

Suppose the parallel feedforward compensator has the form of Figure 4, then conditions below have to be satisfied,

- $k_i > 0$
- $G_p(0) \neq 0$
- $R(s)$ is step input

Then, $\lim_{t \to \infty} \epsilon(t) = 0$

Define,

$$y_m = G_m(s) R(s), y_f = u_c G_{PFC}(s), y_p = u_c G_p(s) \quad (3.24)$$

$$y_{ma} = y_m + y_f, y_{pa} = y_p + y_f, G_{PID} = C(s)/s \quad (3.25)$$

![Fig.4. New Model Based Adaptive PID with Feedforward Control System](image)

From Figure 4, one can have the following signals,

$$E_a(s) = y_{ma} - y_{pa} = y_m + y_f - y_p - y_f = y_m - y_p \quad (3.27)$$

By taking (3.24) and (3.25) into (3.27), one can define,

$$E_a(s) = G_m(s) R(s) - u_c G_p(s) \quad (3.28)$$

and

$$E_a(s) = s G_m(s) R(s) - C(s) \quad (3.29)$$

$$E_a(s) = \frac{s G_m(s) R(s)}{s + G_p(s) C(s)} \quad (3.30)$$

Using final value theorem, one can have,

$$\lim_{s \to 0} s E_a(s) = \lim_{s \to 0} s \frac{s G_m(s) R(s)}{s + G_p(s) C(s)} \quad (3.31)$$

$$\lim_{s \to 0} \frac{0 G_m(0)}{0 - G_p(0) C(0)} = 0 \quad (3.32)$$
IV. SIMULATION RESULTS

In previous chapters, steam turbine system was discussed and design of new adaptive PID control algorithm was introduced. In this section, an adaptive PID control system with parallel feedforward compensator is implemented. The objective is to control the speed of steam turbine under changing operating conditions. Design procedure utilizes transfer functions which are obtained from equations from Figure 1. Steam turbine system has a governor, turbine and load part. Under changing operating conditions of the turbine system, the transfer function of the plant are \( G_{\text{plant}}(s) = G_{\text{governor}} \cdot G_{\text{turbine}} \cdot G_{\text{load}} \). Simulations are performed using the following nominal parameters and their respective ranges as shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_1 )</td>
<td>1.6</td>
<td>1.333 to 2</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>4.497</td>
<td>3.124 to 7.028</td>
</tr>
<tr>
<td>( b_3 )</td>
<td>1.777</td>
<td>1.029 to 3.472</td>
</tr>
<tr>
<td>( c_1 )</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>6.753</td>
<td>7.417 to 6.311</td>
</tr>
<tr>
<td>( c_3 )</td>
<td>32.54</td>
<td>26.68 to 41.79</td>
</tr>
<tr>
<td>( c_4 )</td>
<td>83.73</td>
<td>63.44 to 119</td>
</tr>
<tr>
<td>( c_5 )</td>
<td>106.3</td>
<td>73.19 to 168.2</td>
</tr>
<tr>
<td>( c_6 )</td>
<td>36.88</td>
<td>21.35 to 72.04</td>
</tr>
</tbody>
</table>

Nominal plant transfer function \( G_{\text{nom}}(s) \) is

\[
G_{\text{nom}}(s) = \left[ \frac{1.6s^2 + 4.49s + 1.77}{0.4s^5 + 6.753s^4 + 32.34s^3 + 83.73s^2 + 106.3s + 36.88} \right]
\]

The actual plant \( G_p(s) \) can be written as

\[
G_p(s) = \frac{b_1s^2 + b_2s + b_3}{c_1s^5 + c_2s^4 + c_3s^3 + c_4s^2 + c_5s + c_6}
\]

(4.2)

The uncertainty is an additive perturbation in transfer function and can be obtained from (3.9). The worst-case uncertainty was calculated by optimization procedure by equation (3.9) for 200 frequencies. Then \( \omega \) that satisfies equation (3.10) is given below

\[
W(s) = \left[ \frac{0.7}{1.2s^2 + 3s + 5} \right]
\]

(4.3)

The magnitude response showing worst case uncertainty and bounding function \( W(s) \) is shown in Figure 5.

After getting bounding function, parallel feedforward compensator is to be derived. By using Theorem 1 which satisfies equation (3.12), the parallel feedforward compensator can be found. Depending on the plant structure, the relative degree of the compensator can be chosen either 0 or 1. Relative degree in this simulation is chosen 1 since turbine plant structure satisfied ASPR conditions.

\[
G_{\text{PFC}}(s) = \left[ \frac{h_1s + h_2}{s^2 + 2s + 1} \right]
\]

(4.4)

The denominator part of parallel feedforward structure is chosen by designers regarding of the plant type. Since 2nd degree polynomial is chosen, numerator part should be first or second degree polynomial. If parallel feedforward structure is formed with low degree polynomial, system response will be faster compared to higher order polynomial structure. Parallel feedforward compensator from optimization method using Lagrange Multiplier is obtained as,

\[
G_{\text{PFC}}(s) = \left[ \frac{0.1703s + 0.09219}{s^2 + 2s + 1} \right]
\]

(4.5)

The augmented plant transfer function becomes as \( G_{a}(s) = G_p(s) + G_{\text{PFC}}(s) \), where

\[
G_{a}(s) = \left[ \frac{0.0681s^5 + 1.187s^4 + 7.763s^3 + 24.85s^2 + 38.19s + 24.13}{0.4s^5 + 7.553s^4 + 155.6s^3 + 306.3s^2 + 333.2s^1 + 180.1s + 36.88} \right]
\]

(4.6)

\( \omega \) being weights for adaptive gains must be positive and can be chosen arbitrarily depending on the system. For this simulation, \( \omega_1 = 0.04, \omega_2 = 0.8, \omega_3 = 0.04 \) are chosen. Simulations are done for different cases to observe the changing conditions of the steam turbine. Step input is applied for both PID and adaptive PID controllers and results are shown.

Changes in turbine time constants are dependent on the formula
\[ T_v = V \frac{\partial \rho}{\partial P} P_o \]  

(4.7)

\( P_o = \) Rated pressure  
\( Q_o = \) Rated flow out of the vessel

Time constants depend on the steam pressure, flow and steam density. In real systems, steam coming through and going through operation may not be stable. Changes in pressure and flow will affect the turbine time constants and the system. Changes in plant parameters from the nominal values are assumed to be 25-30% for simulations. Simulation results for conventional PID controller were obtained for \( K_p = 1, K_i = 0.4, K_d = 0.1 \). Simulations are carried out for 8 different cases as shown in Table II below.

**TABLE II  
DIFFERENT PLANT CASES FOR SIMULATIONS**

<table>
<thead>
<tr>
<th>Cases</th>
<th>Turbine Chest</th>
<th>Turbine Re-heater</th>
<th>Turbine Crossover</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
<td>7.5</td>
<td>0.4</td>
</tr>
<tr>
<td>2</td>
<td>0.3</td>
<td>9</td>
<td>0.48</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>6</td>
<td>0.32</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>9</td>
<td>0.48</td>
</tr>
<tr>
<td>5</td>
<td>0.3</td>
<td>6</td>
<td>0.32</td>
</tr>
<tr>
<td>6</td>
<td>0.25</td>
<td>6</td>
<td>0.48</td>
</tr>
<tr>
<td>7</td>
<td>0.3</td>
<td>6</td>
<td>0.48</td>
</tr>
<tr>
<td>8</td>
<td>0.3</td>
<td>6</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Simulations for the uncontrolled system responses for a unit step input for all the 8 cases are shown in Figure 6. It is important to note that uncontrolled system responses cannot reach the steady-state value of unity.

As seen in Figure 8, 2% percent drop in droop causes increasing settling time in PID controller for all the 8 cases. However, Adaptive PID Controller can compensate this change and performs well in all responses.

![Fig.7. Turbine Governor Representation with Droop](image)

![Fig.8. Time vs. Unit-Step Response under 2 % Droop Change](image)

However, if droop drop increases further to 4%, PID cannot control and three system responses are unacceptable and considered unstable from a practical point of view as seen in Figure 9. However, the new model based adaptive PID controller can successfully control the system and maintains desired performance for all cases.

![Fig.9. Time vs. Unit-Step Response under 4 % Droop Change](image)

Figures 10 and 11 show error history for adaptive PID and regular PID, respectively. While errors in all the 8 cases in adaptive PID approach zero, PID controller errors are taking...
too long with three cases showing unacceptable decay.

V. CONCLUSION

In this report, model based adaptive PID controller is introduced. This new algorithm is applied to steam turbine speed control system under droop change. From the results obtained during simulations, the new model based adaptive PID control algorithm can cope with changing operational conditions and is a better alternative to conventional PID controller.

REFERENCES