Model Analysis of Composite Fuselage like Structures with Cutouts

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ABSTRACT

The present study presents a new methodology of composite fuselage carried out with cut outs and without cut outs. Cut outs are necessary for fuselage to fit window glass, Doors, wings, etc. The stress intensity at cut outs depends up on orientation. Initially the work is carried out without cut out and different orientations of cut outs. In the later case it is observed that the stresses are different with different orientations of cut out depending upon loading condition. All the variations are plotted on the graph to compare easily. The stresses are more if the cut out area is in the perpendicular direction of load applied.

Keywords: Composites, Fuselage, Cut outs.

INTRODUCTION

Cervantes, J. A. and Palazotto, A. N are presented that thin cylindrical shells are most sensitive to buckling. As the compressive load is applied on the shells, it leads to failure at certain load, which is critical load. As the cylindrical shells have wide range of applications in varied fields, its structural design for safety is very much essential. Tremendous work has been carried out earlier in the field of buckling of thin cylindrical shells with cutouts.

The basics of the composite materials, properties, laminate procedure, Interlaminar, Intralaminar and types of fibres these topics are presented by the Madhujit Mukhopadhyay .

Mark W. Hilburger presented results from a numerical study of the response of thin-walled compression-loaded quasi-isotropic laminated composite cylindrical shells with unreinforced and reinforced square cutouts.

Hashin Z presented a paper on progressive failure methodology to simulate the initiation and material degradation of a laminated panel due to intralaminar and interlaminar failures. Initiation of intralaminar failure can be by a matrix-cracking mode, a fibre-matrix shear mode, and a fibre failure mode.

Almroth, B. O. studies have been conducted on the response of compression-loaded curved shells with reinforced cutouts,

and the few results that do exist are limited to isotropic shells

The traditional method for the preliminary design of a reinforced cutout in a thin-walled shell structure is based on a linear analysis of a flat plate with a square cutout .Kuhn, P.,

Developed progressive failure methodology by simulating the geometrically nonlinear response and failure of a flat panel tested in shear and a curved panel tested in axial compression .Rouse M.

The intralaminar failure modes considered in these references are matrix-cracking, fibre-matrix shear failure, and fibre failure. Hashin

An analysis-based approach .for developing shell-buckling design criteria for laminated composite cylindrical shells that accurately accounts for the effects of initial geometric imperfections is presented by Mark W.Hilburger, Michael P. Nemeth, and James H. Starnes.

The high-fidelity analysis procedure has been successfully applied to the analysis of other similar compression-loaded shells with cutouts, and the predicted results have been verified with selected experiments are presented by Starnes, J. H., Jr., Hilburger, M. W., and Nemeth, M. P.,

INTRODUCTION TO FUSELAGE CUT-OUTS

Cut outs are necessary for fuselage to fit window glass, Doors, wings, etc. The stress intensity at cut outs depends up on orientation. Generally the stress variations will be compared in above cases. Initially the work is carried out with without cut out and different orientations of cut out. In this case it is observed that the stresses are different with different orientations of cut out depending upon loading condition.

The aircraft structure is continually faced with requirements for openings at webs and panels to provide access or to let other members such as control rods, hydraulic lines, electrical wire bundles, etc., pass through. Other cut-outs such as windows, doors, servicing panels, hatches, bomb-bays, inspection access holes, etc. cause a recurring head-ache for the structural engineer. As soon as one makes a hole in a load-bearing skin, a stronger surrounding structure must be introduced to provide alternate paths to carry the loads. Perhaps the most noticeable feature of cutouts is the rounding of the corners; sharp corners cause excessively high stress concentrations.

The most efficient structure is when the load path is most direct. Cutouts in structures invariably increase the structural weight because the structure adjacent to the cutout must be increased to carry the load which would have been carried in the cutout panel, plus the forces due to the redistribution of this load. The procedure used in the analysis for the effect of cutouts may be considered to the following two methods, depending primarily upon the geometrical relationship of the cutouts of the structure.

*When cutouts are relatively small to medium in size the effect is localized. This means that only structure in the immediacy vicinity of the cutout is appreciably involved in the redistribution; i.e. Donut doubler or with round flanged holes.

*Sections having relatively large size cutouts must consider the effects of the cutout in computing the section properties. This means that the entire section will be affected by the cutout instead of the effect being localized, such as framing cutouts in webs of large rectangular openings. If the holes are so large, some shear is carried by frame action of the adjacent members. This should be taken into account if the caps are to be subjected to high axial loads as the ability to carry axial loads will be reduced.

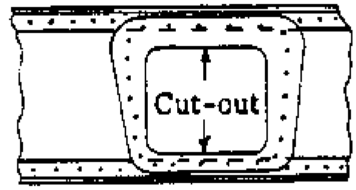

Fig: Rectangle with rounded corners cutout

The objective of the analysis is to determine the stress state with fuselage cut outs and without cut outs with different orientations.

- For windows the fuselage must include cut-outs.
- Cut outs are necessary for fuselage to fit window glass, Doors, wings, etc.
- The ideal shape for a cut-out in a fuselage is an ellipse, and many aircraft have windows this shape.
- Elliptical shapes are not very practical for cabin doors, and the more usual shape is a rectangular with rounded corners.

CUTOUTS IN WEBS OR SKIN PANELS

The aircraft structure is continually faced with requirements for opening up webs and panels to provide access or to let other members such as control rods, hydraulic lines, electrical wire bundles etc.. pass through. The designer should be familiar with the various methods of structural cutouts. There are several ways of providing cutouts.

There are two ways to determine the loads in area framed around the cutout. The first is to assume shear flow equal and opposite to that present with no cutout and determine corresponding balancing loads in the framed area. Adding this load system to the original one will give you the final loads. The other is to use standard procedures assuming the shear to be carried I reasonable proportions on each side of the cutout.

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METHODOLGY FOR THE ANALYSIS OF FUSELAGE CUTOUTS

PROBLEM

The finite element modeling and 3-dimentional analysis is done to study the Inter laminar shear stress, Buckling factor and Deformation By using Eigen value buckling analysis with and without reinforcement of shell by use of Graphite /Epoxy composite material subjected to compressive load.

SHELL181 Finite Strain Shell

SHELL181 is suitable for analyzing thin to moderately-thick shell structures. It is a four-noded element with six degrees of freedom at each node: translations in the x, y, and z directions, and rotations about the x, y, and z axes. The degenerate triangular option should only be used as filler elements in mesh generation.

SHELL181 is well-suited for linear, large rotation, and/or large strain nonlinear applications. Change in shell thickness is accounted for in nonlinear analyses. In the element domain, both full and reduced integration schemes are supported. SHELL181 accounts for follower effects of distributed pressures.

SHELL181 can be used instead of SHELL43for many problems that have convergence difficulty with SHELL43. See Section 14.181 of the *ANSYS Theory Reference* for more details about this element.

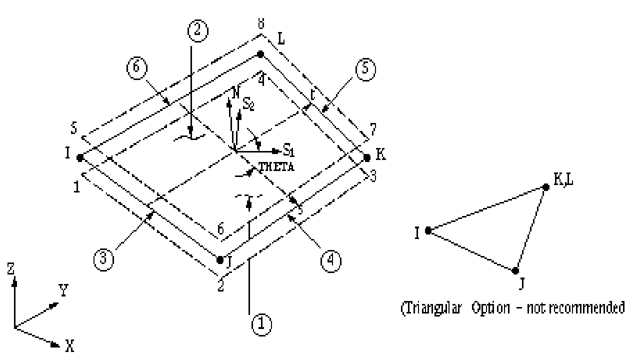


Fig: SHELL181 FINITE STRAIN SHELL

INPUT DATA

The geometry, node locations, and the coordinate system for this element are shown in Figure7.1 the element is defined by four nodes: I, J, K, and L. Element formulation is based on logarithmic strain and true stress measures. Element kinematics allow for finite membrane strains (stretching). However, the curvature changes within an increment are assumed to be small.

The thickness of the shell may be defined at each of its nodes. The thickness is assumed to vary smoothly over the area of the element. If the element has a constant thickness, only TK (I) needs to be input. If the thickness is not constant, all four thicknesses must be input. The element supports degeneration into a triangular form; however, use of the triangular form is not recommended, except as mesh filler elements.

The default orientation for this element has the S_1 (shell surface coordinate) axis aligned with the first parametric direction of the element at the centre of the element. In the most general case, the axis can be defined as:

SHELL181 uses a penalty method to relate the independent rotational degrees of freedom about the normal (to the shell surface) with the in-plane components of displacements. The ANSYS program chooses appropriate penalty stiffness by default. However, you can change the default value if necessary by using the tenth real constant.

SHELL181 supports uniform reduced integration, full integration, and full integration with incompatible modes. SHELL181 can be associated with linear elastic, elastoplastic, or hyperelastic material properties. Only isotropic and orthotropic linear elastic properties can be input for elasticity. The von Misses isotropic hardening plasticity models can be invoked with BISO (bilinear isotropic hardening) and MISO (multilinear isotropic hardening) options. Kinematic hardening plasticity and creep material models are not available for this element. Invoking plasticity assumes that the elastic properties are isotropic (i.e., if orthotropic elasticity is used with plasticity, ANSYS assumes the isotropic elastic modulus=EX and Poisson's ratio=NUXY).Poisson's ratio is used to specify the compressibility of the material. If less than 0, Poisson's ratio is set to 0; if greater than or equal to 0.5, Poisson's ratio is set to 0.5 (fully incompressible).

Both isotropic and orthotropic thermal expansion coefficients can be input using the MP, ALPX command.

SHELL181 INPUT SUMMARY

Element Name	SHELL 181
Nodes	I,J,K,L

Degrees of Freedom
UX,UY,UZ.ROTX,ROTY,ROTZ

REAL CONSTANTS

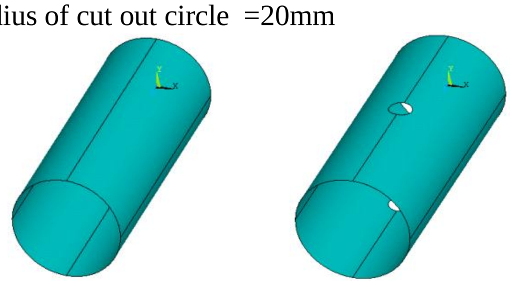
No.	**Name**	**Description**
1	TK(I)	Thickness at node I
2	TK(J)	Thickness at node J
3	TK(K)	Thickness at node K
4	TK(L)	Thickness at node L

DIMENSIONS OF FUSELAGE STRUCTURE

FUSELAGE WITHOUT CUTOUT
Length of the shell =500mm
Radius of the shell =100mm
Thickness of shell =4mm

FUSELAGE WITH CIRCULAR CUTOUT

Representative cutouts with small dimensions(less than the typical aircraft cutouts for windows) are assumed since the objective of the project is to only compare the structural performance of the various shapes of the cutouts.
Length of the shell =500mm
Radius of the shell =100mm
Thickness of shell =4mm
Radius of cut out circle =20mm



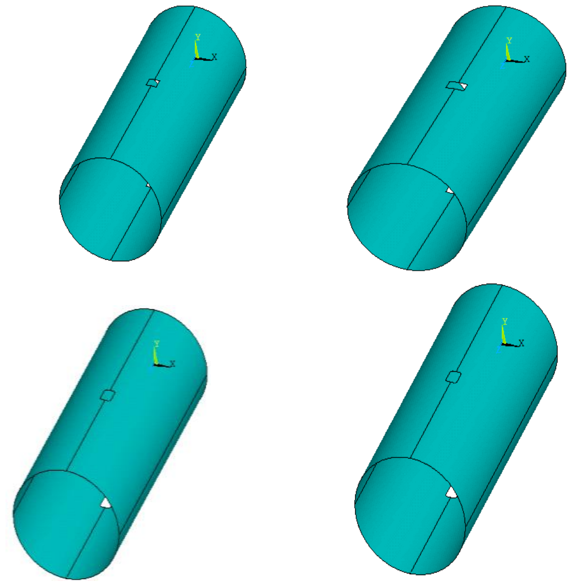


Fig. Cylinders without cutout, with Circular, Square, Rectangle cutouts and Rectangle with rounded corners cutouts

FUSELAGE WITH SQUARE CUTOUT

Length of the shell =500mm
Radius of the shell =100mm
Thickness of shell =4mm
Length of the rectangle =25mm
Breath of the rectangle =25mm

FUSELAGE WITH RECTANGLE CUTOUT

Length of the shell =500mm
Radius of the shell =100mm
Thickness of shell =4mm
Length of the rectangle =20mm
Breath of the rectangle =30mm

RECTANGLE CYLINDER WITH ROUNDED CORNERS CUTOUT

Length of the shell =500mm
Radius of the shell =100mm
Thickness of shell =4mm
Length of the rectangle =20mm
Breath of the rectangle =30mm
Radius of the corner =5mm

MATERIAL PROPERTIES

ALUMINUM ALLOY
Young’s modulus, E_l = $70GP_a$

Major Poisson's ratio, υ = 0.30
Density =2.7gm/cc

GLASS FABRIC EPOXY
Young's modulus in X-direction Ex =22.925GP_a
Young's modulus in Y-direction Ey =22.925GP_a
Young's modulus in Z-direction Ez =12.4GP_a
Poisson's ratio, Nuxy =0.12
Poisson's ratio Nuyz =0.2
Poisson's ratio, Nuzx =0.2
In-plane shear modulus Gxy =4.7GP_a
In-plane shear modulus Gyz =4.2 GP_a
In-plane shear modulus Gzx =4.2GP_a
Density =1.8gm/cc

CARBON EPOXY
Young's modulus in X-direction Ex =120GP_a
Young's modulus in Y-direction Ey =10GP_a
Young's modulus in Z-direction Ez =10GP_a
Poisson's ratio, Nuxy =0.16
Poisson's ratio Nuyz =0.2
Poisson's ratio, Nuzx =0.16
In-plane shear modulus Gxy =5.2GP_a
In-plane shear modulus Gyz =3.8GP_a
In-plane shear modulus Gzx =6GP_a
Density =1.6gm/cc

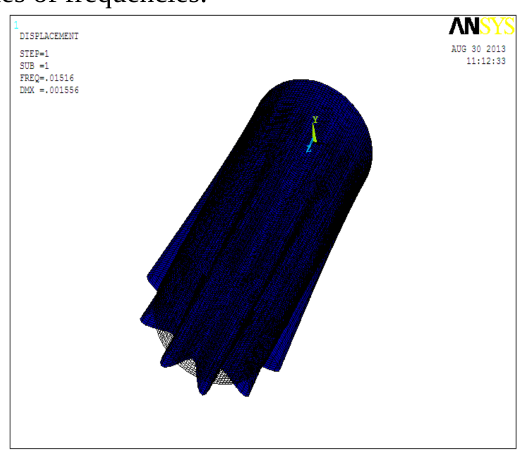
RESULTS AND DISCUSSIONS

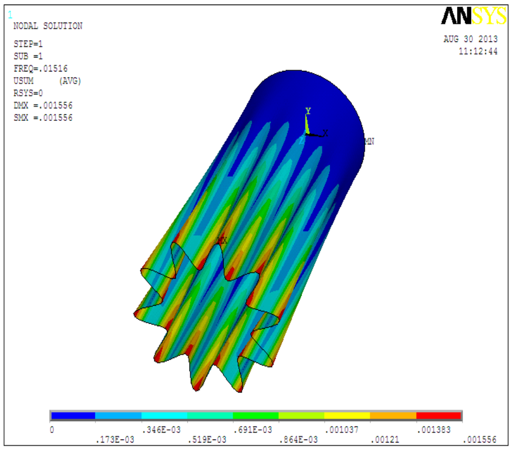
Results illustrating the behavior of fuselage with aluminum metal for without cut-out are presented. Then, results illustrating the behavior of a fuselage with composite materials for without cut-out are presented. In second case, the behavior of fuselage with aluminum metal for circle shaped cut-out is presented. Then, results illustrating the behavior of a fuselage with composite materials for circle shaped cut-out are presented. In third case, the behavior of fuselage with aluminum metal for square shaped cut-out is presented. Then, results illustrating the behavior of a fuselage with composite materials for square shaped cut-out are presented.
In fourth case, the behavior of fuselage with aluminum metal for rectangle shaped cut-out is presented. Then, results illustrating the behavior of a fuselage with composite materials for rectangle shaped cut out are presented. In fifth case, the behavior of fuselage with aluminum metal for vertical shaped rectangle cut-out is presented. Then, results illustrating the behavior of a fuselage with composite materials for vertical shaped rectangle cut-out are presented. In sixth case, the behavior of fuselage with aluminum metal for rectangle with rounded corners is presented. Then, results illustrating the behavior of a fuselage with composite materials for rectangle with rounded corners are presented.

ANALYSIS OF FUSELAGE WITHOUT CUT-OUT
In analysis of Fuselage we are considering both model analysis and static analysis for aluminum alloy and for composite materials. At first we are going to discuss Model analysis of Fuselage with without cut-out for Aluminum alloy.

MODEL ANALYSIS OF FUSELAGE WITHOUT CUT-OUT FOR ALUMINUM ALLOY.

In case of Model analysis it is consider ten modes of frequencies.

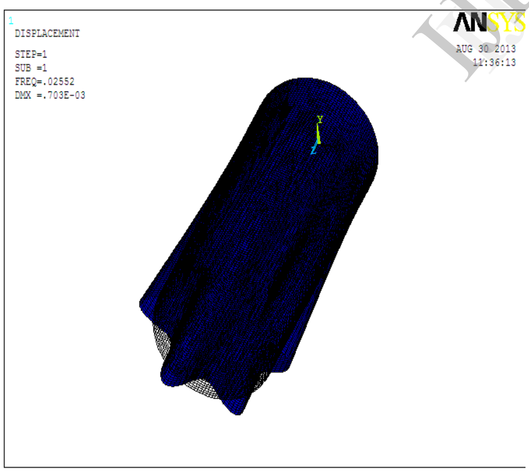


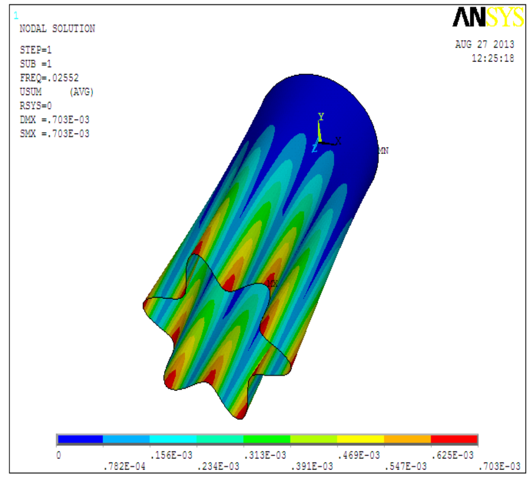


MODEL ANALYSIS OF COMPOSITE MATERIALS FUSELAGE WITHOUT CUT-OUT

GLASS FABRIC EPOXY

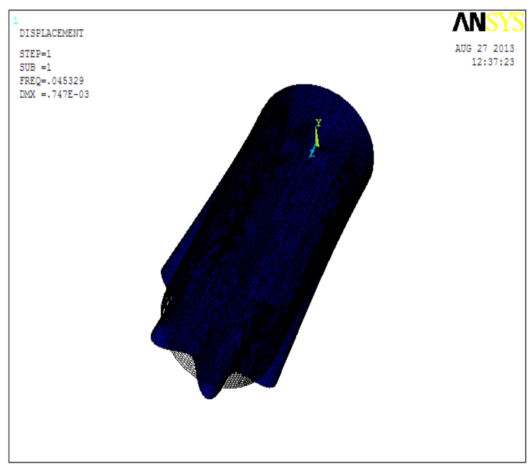
In case of Composite materials we consider two types of composites at first we consider Glass Fabric Epoxy and for this composite we consider different orientations such as 0, 60,120,180,240,300,360. It is consider ten modes of frequencies. Out of ten here now we are considering deformation and displacement vector sum for mode 1 and mode 10.





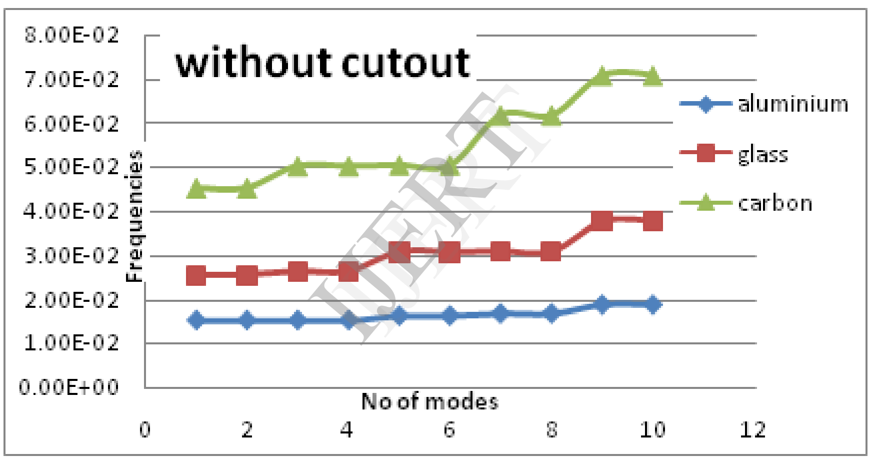
CARBON EPOXY

In case of Composite materials we consider two types of composites here in second case we consider Carbon Epoxy and for this composite we consider different orientations such as 0, 60,120,180,240,300,360. It is consider ten modes of frequencies. Out of ten here now we are considering deformation and displacement vector sum for mode 1 and mode 10.



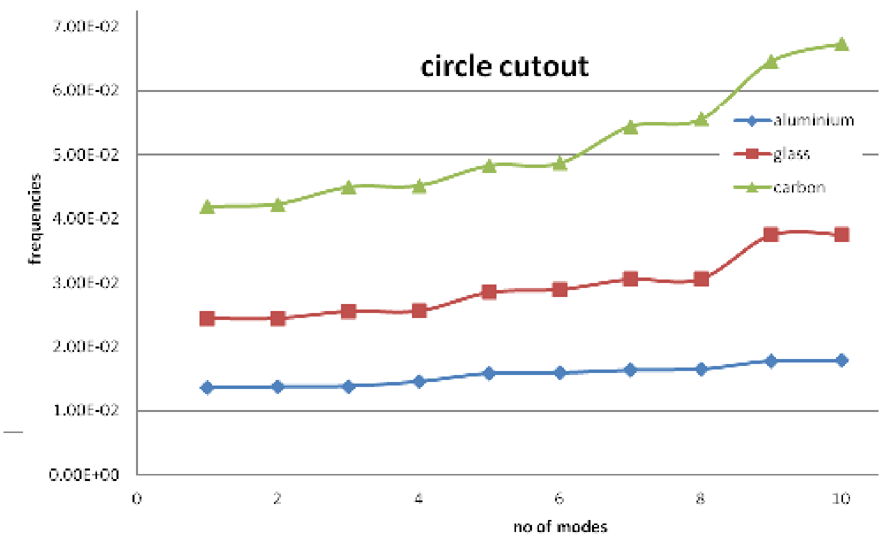
No of modes	Aluminum	Glass Fabric	Carbon Epoxy
1	1.52E-02	2.55E-02	4.53E-02
2	1.52E-02	2.55E-02	4.53E-02
3	1.52E-02	2.62E-02	5.03E-02
4	1.52E-02	2.62E-02	5.03E-02
5	1.62E-02	3.06E-02	5.05E-02
6	1.62E-02	3.06E-02	5.05E-02
7	1.67E-02	3.08E-02	6.18E-02
8	1.67E-02	3.08E-02	6.18E-02
9	1.86E-02	3.76E-02	7.07E-02
10	1.86E-02	3.76E-02	7.07E-02

Table: shows the modes of cylinder without cut-out frequencies of aluminum alloy, glass fabric, carbon epoxy without cot-outs

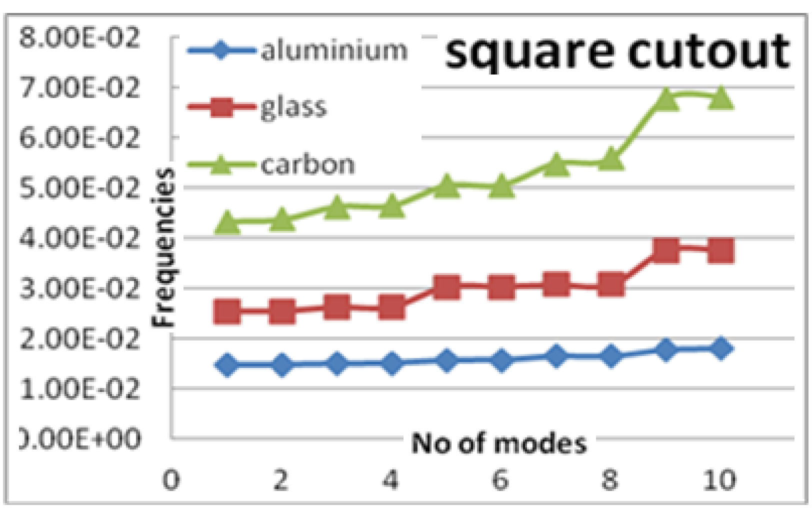


No of modes	Aluminium	Glass Fabric	Carbon Epoxy
1	1.36E-02	2.44E-02	4.19E-02
2	1.38E-02	2.44E-02	4.23E-02
3	1.38E-02	2.56E-02	4.50E-02
4	1.46E-02	2.56E-02	4.52E-02
5	1.59E-02	2.85E-02	4.83E-02
6	1.60E-02	2.89E-02	4.87E-02
7	1.64E-02	3.06E-02	5.44E-02
8	1.65E-02	3.06E-02	5.55E-02
9	1.78E-02	3.75E-02	6.46E-02
10	1.79E-02	3.76E-02	6.74E-02

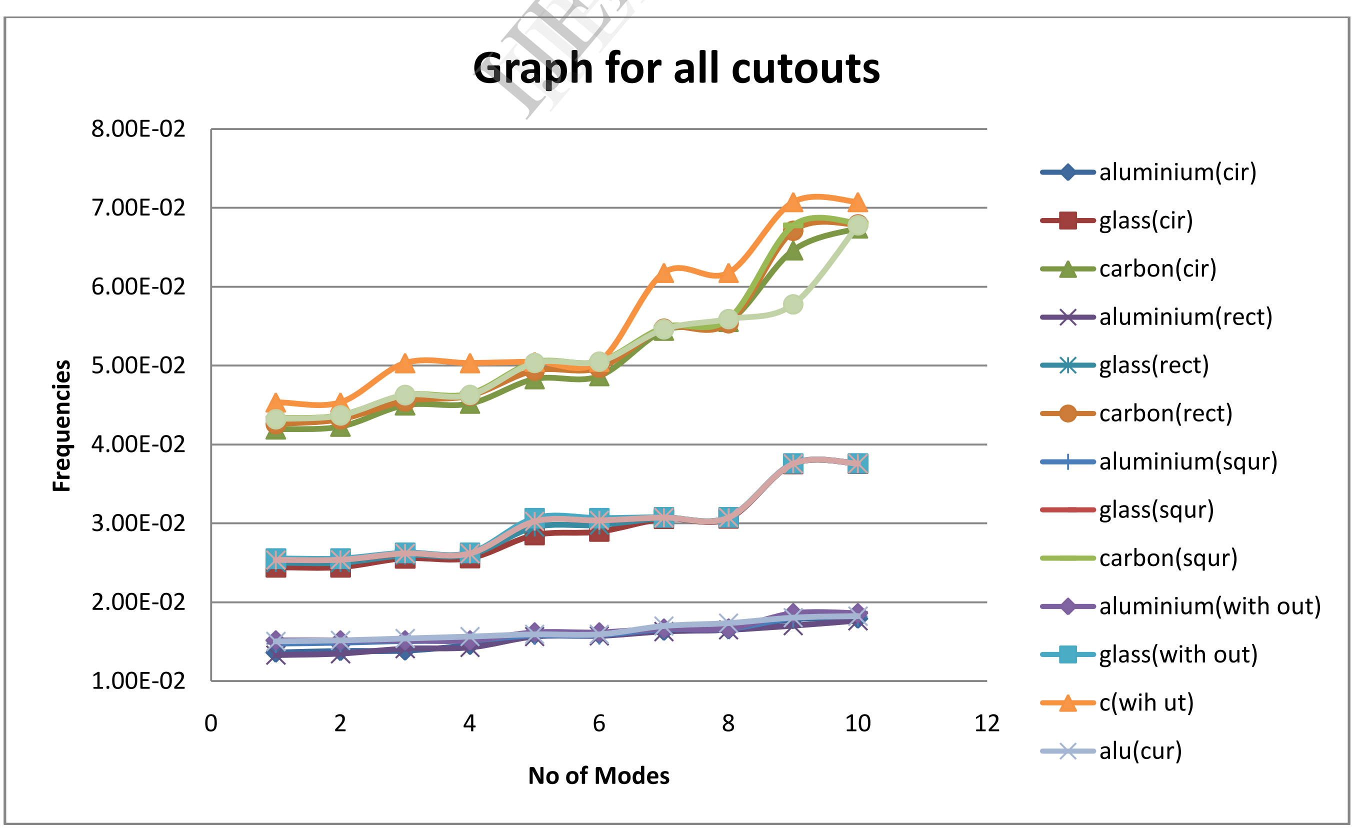
Table: shows the modes of cylinder with Circle cut-out frequencies of aluminum alloy, glass fabric, carbon epoxy without cot-outs



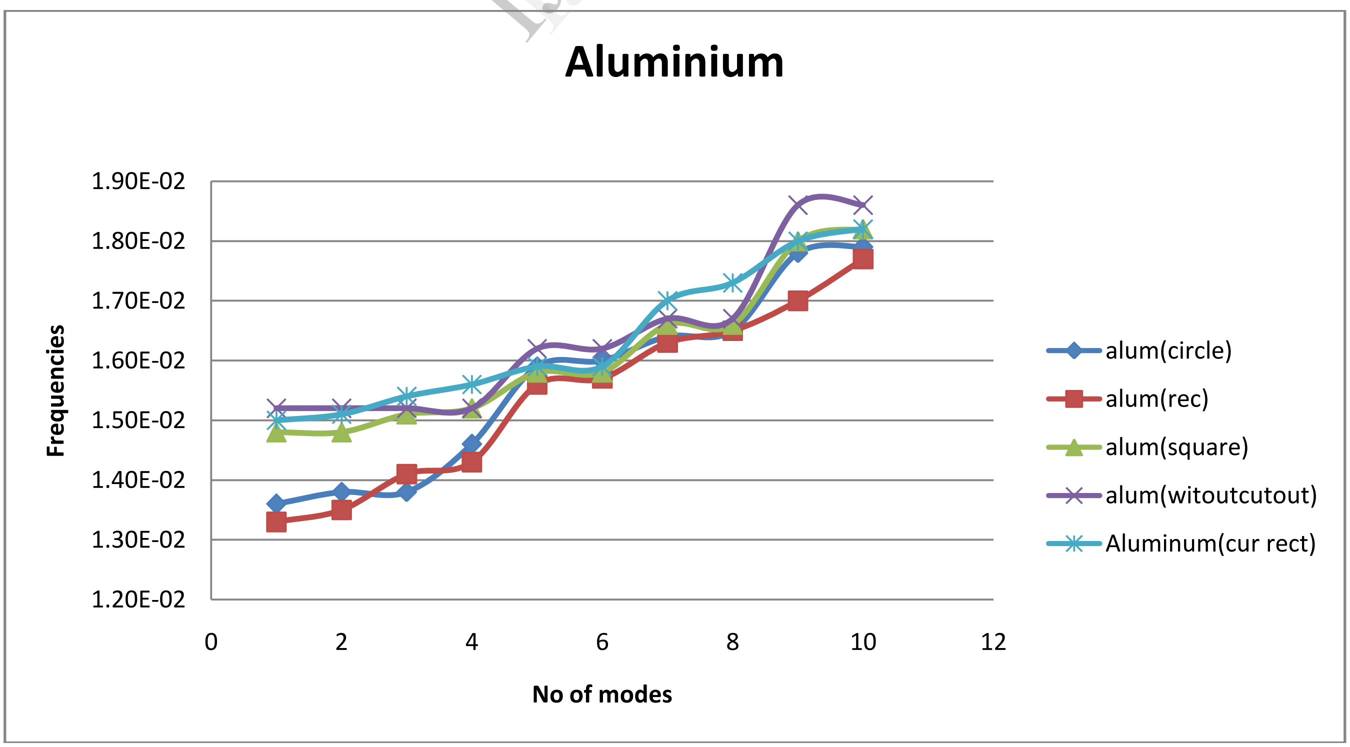
No of modes	Aluminium	Glass Fabric	Carbon Epoxy
1	1.48E-02	2.53E-02	4.33E-02
2	1.48E-02	2.54E-02	4.37E-02
3	1.51E-02	2.62E-02	4.63E-02
4	1.52E-02	2.62E-02	4.64E-02
5	1.58E-02	3.03E-02	5.04E-02
6	1.58E-02	3.03E-02	5.05E-02
7	1.66E-02	3.07E-02	5.48E-02
8	1.66E-02	3.07E-02	5.59E-02
9	1.80E-02	3.76E-02	6.78E-02
10	1.82E-02	3.76E-02	6.81E-02



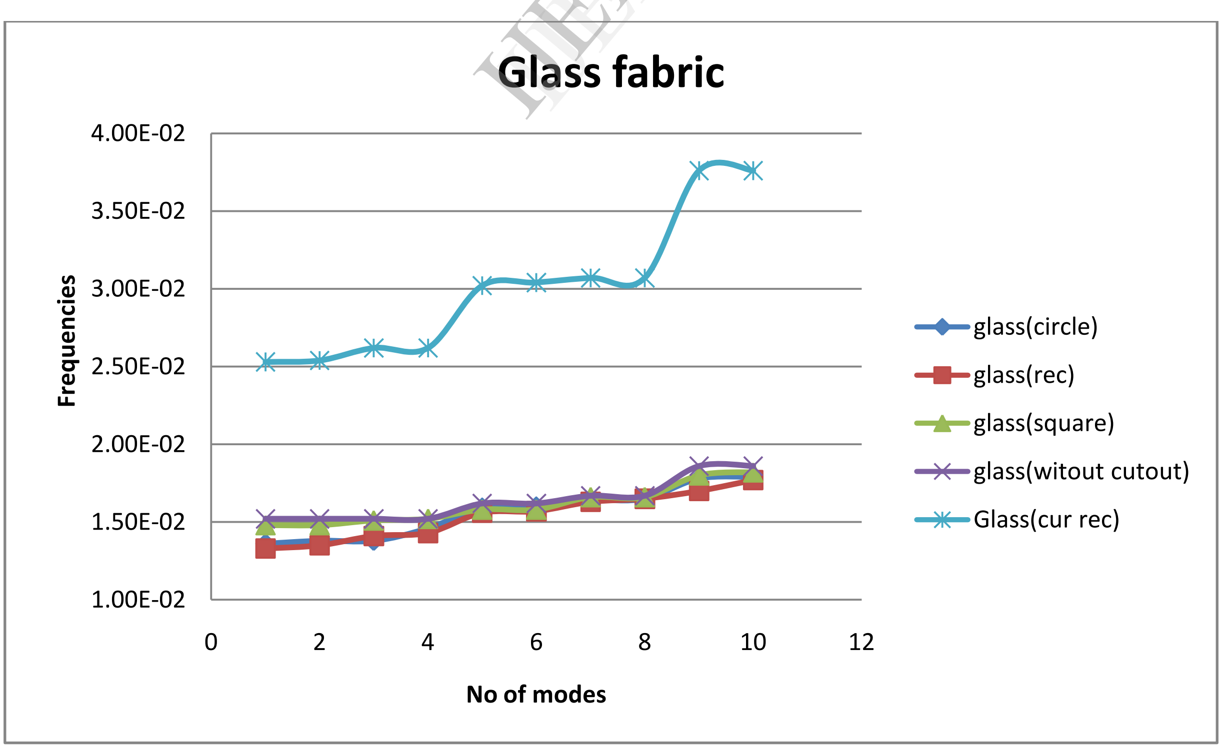
No of modes	aluminium(cir)	glass(cir)	carbon(cir)	aluminium(rect)	glass(rect)	carbon(rect)	aluminium(squr)	glass(squr)	carbon(squr)	aluminium(with out)	glass(with out)	c(wih ut)	alu(cur)	glass(cur)	carbon(cur)
1	1.36E-02	2.44E-02	4.19E-02	1.33E-02	2.50E-02	4.26E-02	1.48E-02	2.53E-02	4.33E-02	1.52E-02	2.55E-02	4.53E-02	1.50E-02	2.53E-02	4.32E-02
2	1.38E-02	2.44E-02	4.23E-02	1.35E-02	2.50E-02	4.32E-02	1.48E-02	2.54E-02	4.37E-02	1.52E-02	2.55E-02	4.53E-02	1.51E-02	2.54E-02	4.37E-02
3	1.38E-02	2.56E-02	4.50E-02	1.41E-02	2.60E-02	4.55E-02	1.51E-02	2.62E-02	4.63E-02	1.52E-02	2.62E-02	5.03E-02	1.54E-02	2.62E-02	4.62E-02
4	1.46E-02	2.56E-02	4.52E-02	1.43E-02	2.60E-02	4.62E-02	1.52E-02	2.62E-02	4.64E-02	1.52E-02	2.62E-02	5.03E-02	1.56E-02	2.62E-02	4.63E-02
5	1.59E-02	2.85E-02	4.83E-02	1.56E-02	2.95E-02	4.94E-02	1.58E-02	3.03E-02	5.04E-02	1.62E-02	3.06E-02	5.05E-02	1.59E-02	3.02E-02	5.03E-02
6	1.60E-02	2.89E-02	4.87E-02	1.57E-02	2.98E-02	4.97E-02	1.58E-02	3.03E-02	5.05E-02	1.62E-02	3.06E-02	5.05E-02	1.59E-02	3.04E-02	5.05E-02
7	1.64E-02	3.06E-02	5.44E-02	1.63E-02	3.07E-02	5.47E-02	1.66E-02	3.07E-02	5.48E-02	1.67E-02	3.08E-02	6.18E-02	1.70E-02	3.07E-02	5.46E-02
8	1.65E-02	3.06E-02	5.55E-02	1.65E-02	3.07E-02	5.54E-02	1.66E-02	3.07E-02	5.59E-02	1.67E-02	3.08E-02	6.18E-02	1.73E-02	3.07E-02	5.59E-02
9	1.78E-02	3.75E-02	6.46E-02	1.70E-02	3.75E-02	6.71E-02	1.80E-02	3.76E-02	6.78E-02	1.86E-02	3.76E-02	7.07E-02	1.80E-02	3.76E-02	5.78E-02
10	1.79E-02	3.76E-02	6.74E-02	1.77E-02	3.76E-02	6.79E-02	1.82E-02	3.76E-02	6.81E-02	1.86E-02	3.76E-02	7.07E-02	1.82E-02	3.76E-02	6.78E-02



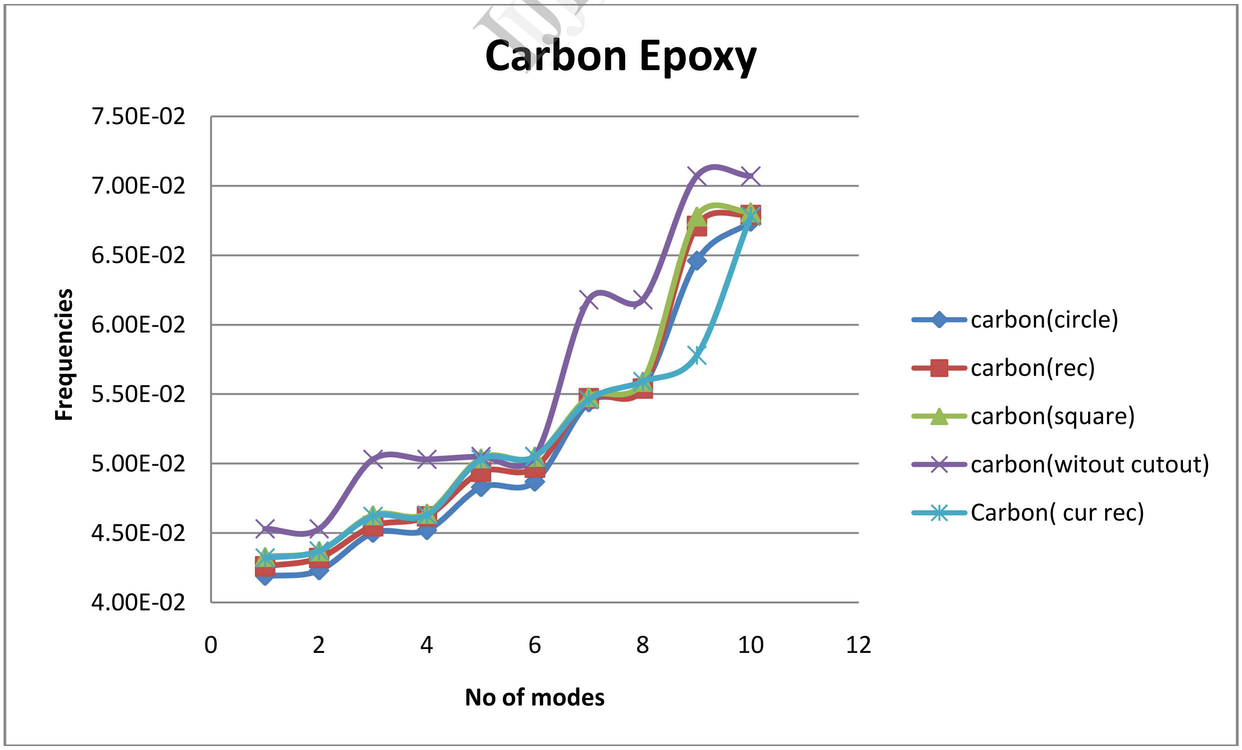
No of modes	Aluminum (circle)	Aluminum (rectangle)	Aluminum (square)	Aluminum(without cutout)	Aluminum (curved rect)
1	1.36E-02	1.33E-02	1.48E-02	1.52E-02	1.50E-02
2	1.38E-02	1.35E-02	1.48E-02	1.52E-02	1.51E-02
3	1.38E-02	1.41E-02	1.51E-02	1.52E-02	1.54E-02
4	1.46E-02	1.43E-02	1.52E-02	1.52E-02	1.56E-02
5	1.59E-02	1.56E-02	1.58E-02	1.62E-02	1.59E-02
6	1.60E-02	1.57E-02	1.58E-02	1.62E-02	1.59E-02
7	1.64E-02	1.63E-02	1.66E-02	1.67E-02	1.70E-02
8	1.65E-02	1.65E-02	1.66E-02	1.67E-02	1.73E-02
9	1.78E-02	1.70E-02	1.80E-02	1.86E-02	1.80E-02
10	1.79E-02	1.77E-02	1.82E-02	1.86E-02	1.82E-02



No of modes	glass(circle)	glass(rect)	glass(square)	glass(without cutout)	glass(cur rec)
1	1.36E-02	1.33E-02	1.48E-02	1.52E-02	2.53E-02
2	1.38E-02	1.35E-02	1.48E-02	1.52E-02	2.54E-02
3	1.38E-02	1.41E-02	1.51E-02	1.52E-02	2.62E-02
4	1.46E-02	1.43E-02	1.52E-02	1.52E-02	2.62E-02
5	1.59E-02	1.56E-02	1.58E-02	1.62E-02	3.02E-02
6	1.60E-02	1.57E-02	1.58E-02	1.62E-02	3.04E-02
7	1.64E-02	1.63E-02	1.66E-02	1.67E-02	3.07E-02
8	1.65E-02	1.65E-02	1.66E-02	1.67E-02	3.07E-02
9	1.78E-02	1.70E-02	1.80E-02	1.86E-02	3.76E-02
10	1.79E-02	1.77E-02	1.82E-02	1.86E-02	3.76E-02



No of modes	carbon(circle)	carbon(rec)	carbon(square)	carbon(witout cutout)	Carbon(cur rec)
1	4.19E-02	4.26E-02	4.33E-02	4.53E-02	4.32E-02
2	4.23E-02	4.32E-02	4.37E-02	4.53E-02	4.37E-02
3	4.50E-02	4.55E-02	4.63E-02	5.03E-02	4.62E-02
4	4.52E-02	4.62E-02	4.64E-02	5.03E-02	4.63E-02
5	4.83E-02	4.94E-02	5.04E-02	5.05E-02	5.03E-02
6	4.87E-02	4.97E-02	5.05E-02	5.05E-02	5.05E-02
7	5.44E-02	5.47E-02	5.48E-02	6.18E-02	5.46E-02
8	5.55E-02	5.54E-02	5.59E-02	6.18E-02	5.59E-02
9	6.46E-02	6.71E-02	6.78E-02	7.07E-02	5.78E-02
10	6.74E-02	6.79E-02	6.81E-02	7.07E-02	6.78E-02



CONCLUSION

The frequency values of carbon Epoxy are higher than Aluminum, Glass Fabric, where as the glass fabric values are in between aluminum and carbon epoxy in case of cylinder without cutout. The above results are repeated for cylinder with circular cutout, square cutout, rectangle cutout, rectangle with rounded corners cutout.

If we observe each and every graph with same material for different cutouts the frequencies are high with the cylinder without cutout but cutouts are necessary for aircraft fuselage section to accommodate windows, doors, etc. Therefore from above cases the next higher frequencies are observed for the cylinder with rectangle with rounded corners cutout.

If we observe each and every graph of any type of cutout the frequencies are high for carbon epoxy material. By comparing the results of carbon epoxy material for all cutouts the results are high in case of the cylinder with rectangle with rounded corners cutout, indicating that it has a higher structural stiffness. Therefore rectangle with rounded corners cutout with the Carbon Epoxy material is preferred for aircraft fuselage section.

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