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MODAL ANALYSIS OF PLANE FRAMES

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ABSTRACT

In the modal analysis of plane frames the main aim is to determine the natural mode shapes and frequencies of an object or structure during free vibration. The dynamic analysis of frames requires inclusion of axial effect in the stiffness and mass matrices. It also requires a co-ordinate transformation of the nodal or local co-ordinates to global co-ordinates. The analysis is performed using Modal Analysis. The physical interpretation of the eigenvalues and eigenvectors which come from solving the system are that they represent the frequencies and corresponding mode shapes. The objective of this paper is to study the vibration, frequency and mode shape of plane frames. Formulation of stiffness matrix and mass matrix are to be done using direct stiffness method. Programs are to be developed using ANSYS and MATLAB codes.

1. Introduction

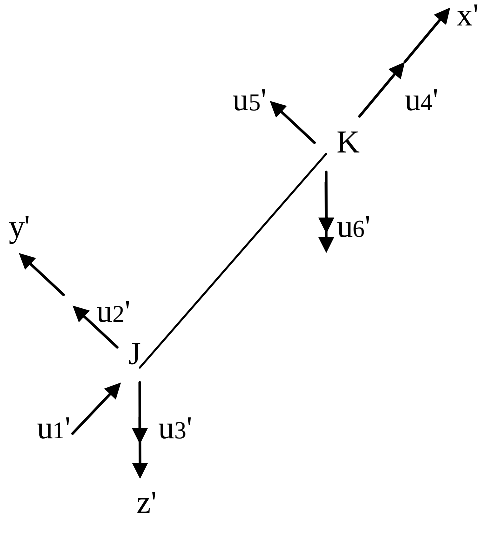
Modal analysis is the study of the dynamic properties of structures under vibrational excitation. When a structure undergoes an external excitation, its dynamic responses are measured and analysed. This field of measuring and analysing is called modal analysis. In structural engineering, modal analysis is applied to find the various periods that the structure will naturally resonate at, by using the structure's overall mass and stiffness. The modal analysis is very important in earthquake engineering, because the periods of vibration evaluated helps in checking that a building's natural frequency does not coincide with the frequency of earthquakes prone region where the building is to be constructed. In case a structure's natural frequency coincidently equals an earthquake's frequency, the structure suffers severe structural damage due to resonance. The frequency and mode shape of a model is determined by modal analysis. When the models are subjected to cyclic or vibration loads, the dynamic response of structures due to these external loads acting, which include resonance frequencies (natural frequencies), mode shape and damping, are estimated.

1.1 Formation of Stiffness Matrix

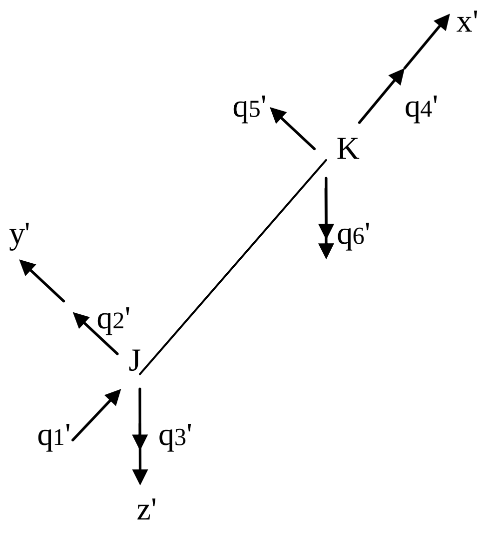
In Fig. 1a, we assume a member of a plane frame in the member co-ordinate system x'y'z' whose global orthogonal set of axes are xyz. Some of the assumptions to be considered are:

1) The plane in which the frame lies is x-y plane.
2) All the members of the plane frame should have uniform flexural rigidity EI and uniform axial rigidity EA.

This modal analysis takes the axial deformation of member into consideration. The possible displacements at each node of the member are: translation in x'- and y'- direction and rotation about z'- axis.



(a)Member displacement



(b) Member Forces

From the above Fig. 1a we can see that the frame members have six (6) degrees of freedom. Fig. 1b shows the member being subjected to forces at nodes j and k. By merging stiffness matrix for axial effects and the stiffness matrix for flexural effects into a single matrix, the stiffness matrix at the co-ordinates of each node of the frame member is obtained. Thus, the local co-ordinate axes for the plane frame can be represented by the load-displacement relation as shown in Fig a, b as, (eq. 1a)

$$\begin{Bmatrix} q1' \\ q2' \\ q3' \end{Bmatrix} = \begin{bmatrix} \frac{AE}{L} & 0 & 0 \\ 0 & \frac{12EI}{L} & \frac{6EI}{L2} \\ 0 & \frac{6EI}{L2} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} u1' \\ u2' \\ u3' \end{Bmatrix} \text{...............} \quad (1a)$$

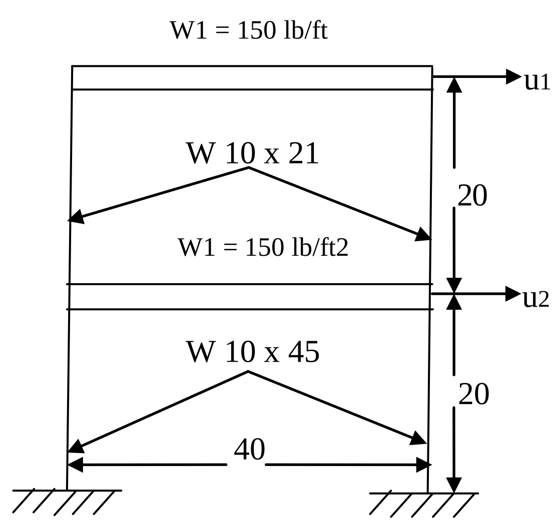
This may be compactly written as (eq. 1b),

$\{q'\} = [k'] \{u'\}$ .... (1b)

Where $[k']$ is the member stiffness matrix. Another method of obtaining the member stiffness matrix is calculating resultant restraint actions by applying one at a time unit displacement along each possible displacement degree of freedom.

2. ILLUSTRATIVE EXAMPLE

A two-storey steel rigid frame is to be analyzed whose weights of the floors and walls inclusive of the structural weights are indicated in the figure. The frames of the building are 15ft apart. The structural properties along the length of the structure are assumed to be uniform.



Natural Frequencies and Modal Shapes:
According to the lumped mass system, the concentrated weights are the total of floor weight and the weight of tributary walls.

W1 = 150x40x15 + 20x20x15x2 = 102,000 lb

m1 = 265 lb.sec^2/in

W2 = 75x40x15 + 20x10x15x2= 51,000 lb

m2 = 132 lb.sec2/in

For each storey the stiffness coefficient is calculated as,

$$k1 = \frac{12E(2I)}{L^3}$$

$$k1 = \frac{12x30x10^6x248x2}{(20x12)^3} = 13{,}000\ lb/in$$

$$k2 = \frac{12x30x10^6x106.3x2}{(20x12)^3} = 56{,}00\ lb/in$$

Using dynamic equilibrium on every element of the system which is free from external vibration, the equations of motion are obtained as already discussed which when solved give the natural frequencies as,

ω1 = 7.542 rad/sec

ω2 = 20.334 rad/sec

in cycles per second

f1 = ω1/2□ = 1.2 cps

f2 = ω2/2□ = 3.236 cps

The time periods are,

T1 = 1/ f1 = 0.833 sec

T2 = 1/ f2 = 0.309 sec

Substituting ω1 = 7.542 rad/sec in the matrix equation,

3526 a_{11} – 5600 a21 = 0

$$\frac{a21}{a_{11}} = 0.63$$

By assigning a unit value to one of the amplitudes,

a_{11} = 1

a_{21} = 0.63

Likewise putting ω2 = 20.334 rad/sec, we get the second normal modes,

a_{12} = 1

a_{22} = -6.42

Normalized Modal Shapes of Vibration

Normalized modes, $Øij = \frac{aij}{\sum_{k=1}^{n}(mk)(a^2kj)}$

By putting the values of amplitudes already calculated and the masses,

$$\sqrt{(265)(1.00)^2) + (132)}(1.263)^2 = \sqrt{317.4}$$

$$\sqrt{(265)(1.00)^2) + (132)}(1 - 6.42)^2$$
$$= \sqrt{5671.7}$$

Normalized modes are,

$Ø11 = \frac{1.000}{\sqrt{317.4}} = 0.056; \quad Ø12 = \frac{1.000}{\sqrt{5671.7}} = 0.0132$

$Ø21 = \frac{1.263}{\sqrt{317.4}} = 0.0354; \quad Ø22 = \frac{-6.42}{\sqrt{5671.7}} = -0.0852$

To satisfy the orthogonality equation,

$[Ø]^T [M] [Ø] = [I]$

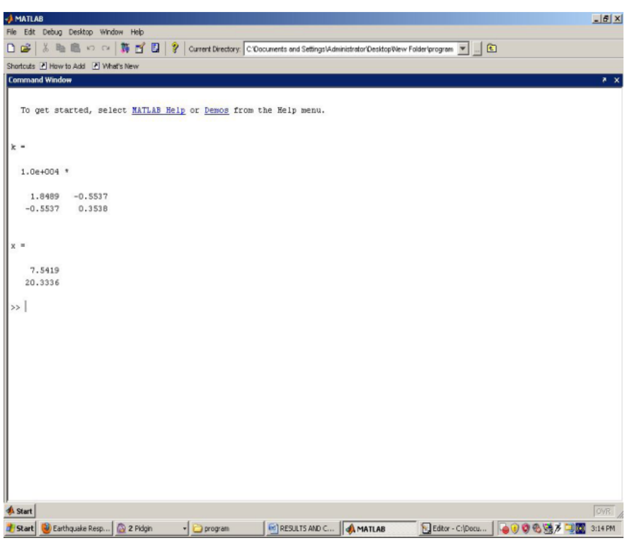
$$[Ø] = \begin{bmatrix} 0.056 & 0.0132 \\ 0.0354 & -0.0852 \end{bmatrix}$$

$$= \begin{bmatrix} 0.056 & 0.0354 \\ 0.0132 & -0.0852 \end{bmatrix} \text{X}$$
$$\begin{bmatrix} 265 & 0 \\ 0 & 132 \end{bmatrix} \begin{bmatrix} 0.056 & 0.0132 \\ 0.0354 & -0.0852 \end{bmatrix}$$

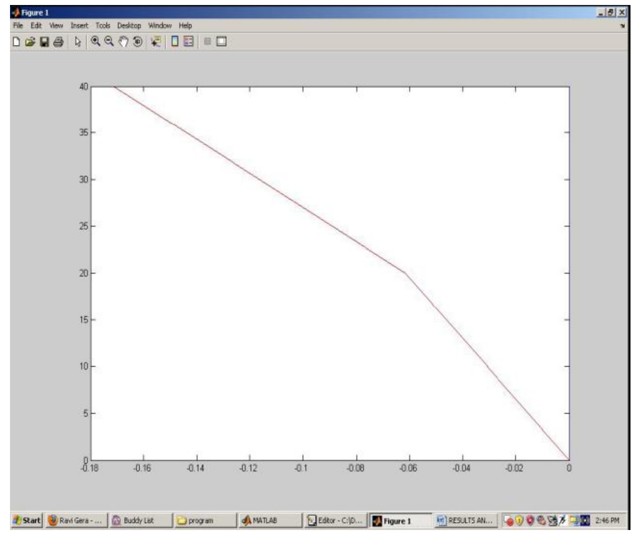
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

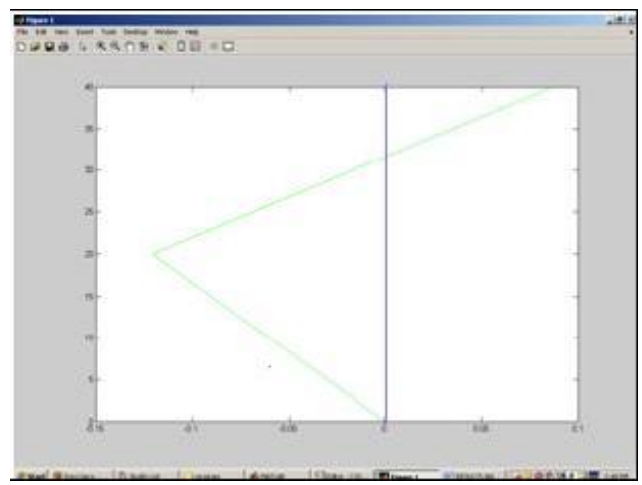
MATLAB Results:

These values are also confirmed by the MATLAB results which gave the following values:

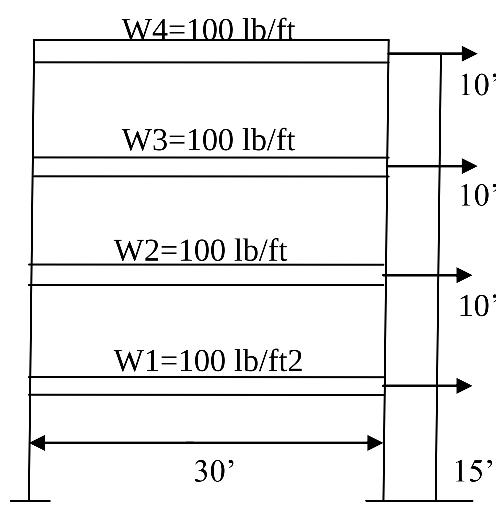


Figures below shows the modal shapes for the two modes of the shear building





For four-storey shear building



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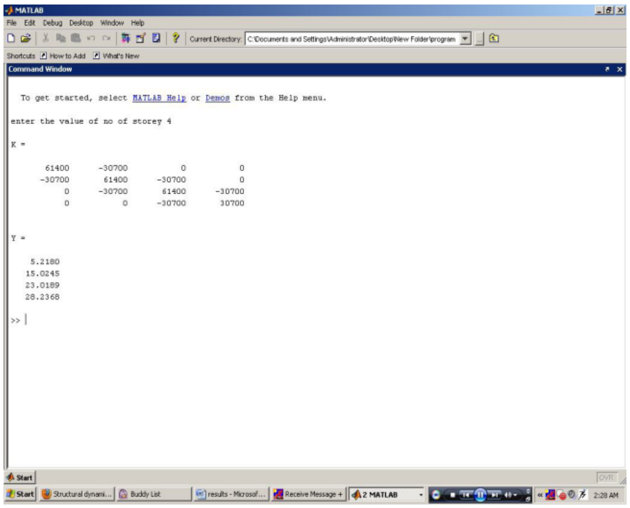
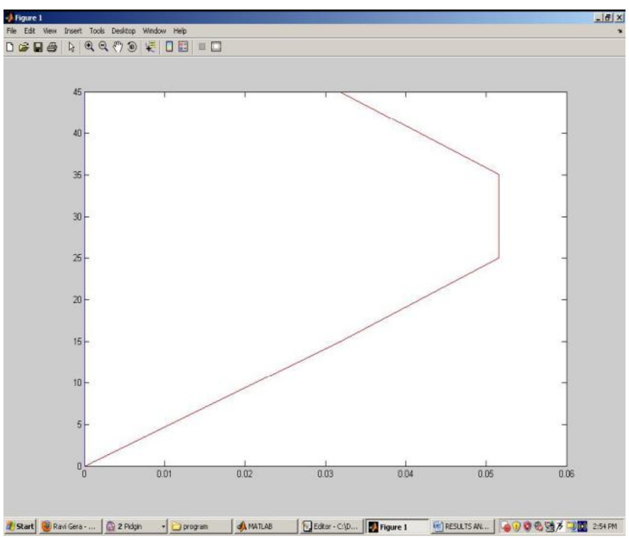
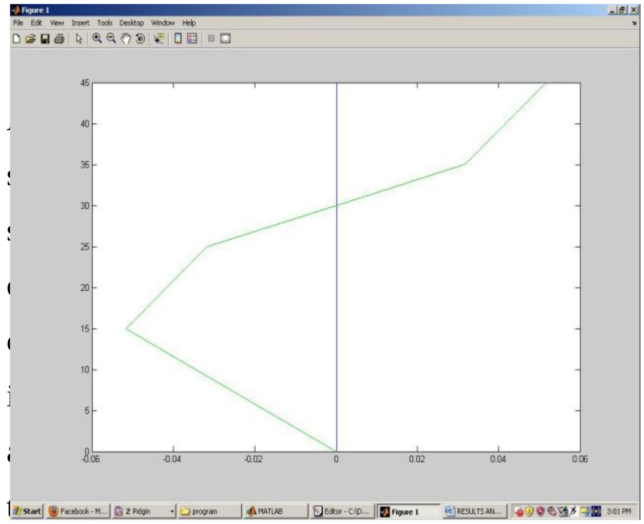
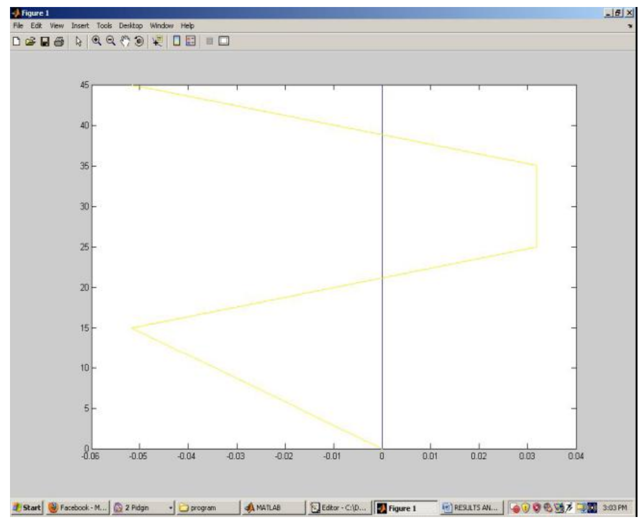
In the similar way as in the case of two-storey shear building, the natural angular frequencies of the considered system computed by MATLAB were

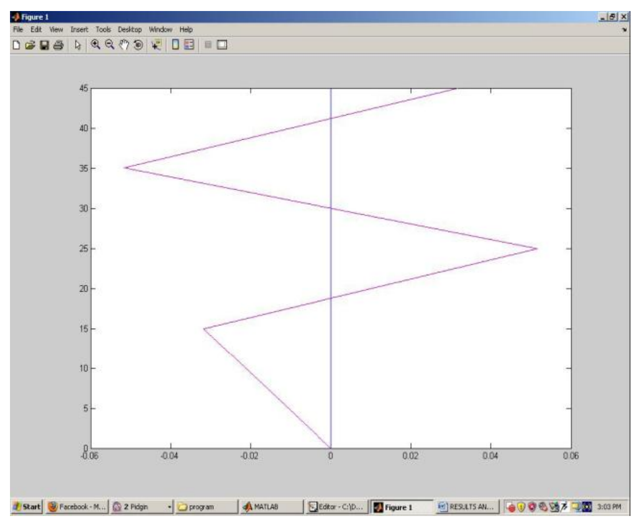


Figures below shows the modal shapes for the four modes of the shear building









3. CONCLUSION

After obtaining the natural frequencies and the Eigen vectors for two-storey shear building manually, we compared it with the results of the MATLAB code and the natural frequencies in both cases were found out to be 7.542 rad/sec and 20.334 rad/sec.

After getting the same results, we created a generalized MATLAB code for n-storey shear building and obtained the natural frequencies for four-storey building to be 5.218 rad/sec, 15.0245 rad/sec, 23.0189 rad/sec and 28.236 rad/sec. With the same MATLAB code we obtained the mode shape for each degree of freedom for specific natural frequency already calculated.

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