Mixed Convection Flow Through A Porous Medium Bounded By Two Vertical Walls With Slip Boundary Conditions

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Abstract

This paper investigates the steady fully developed mixed convection flow between two vertical walls filled with porous materials having slip boundary for the velocity and temperature. The analytic solutions for the velocity and temperature profiles has been obtained for different cases depending upon the values of the Darcy number, Rayleigh number and the ratio of the effective viscosity of the porous domain to the viscosity of the fluid. The effects of the various parameters entering into the problem, on the velocity and the temperature are depicted graphically and on the skin friction is depicted in tabular form, and discussed in detail.

1. Introduction

The study of flow and heat transfer through a porous medium has become of main interest in science and technology because of several engineering applications, particularly when the fluid flow is caused by shearing motion of a plate. The mechanism of mixed convection in the porous media has important applications in the utilization of geothermal energy. The recent books by Nield and Bejan [1] and Ingham and Pop [2] have extensively documented the works devoted in this area.


Jain and Sharma [9] and Jain and Gupta [10] have studied three dimensional couette flows with slip boundary conditions and suction velocity varies sinusoidaly. Sharma [11] investigate the effect of periodic heat and mass transfer on the unsteady free convection flow past a vertical flat plate in slip flow regime when suction velocity oscillates in time. Chaudhary and Jha [12] studied the effects of chemical reactions on MHD micropolar fluid flow past a vertical plate in slip-flow regime. Khaled [13] investigated the effect of slip condition on Stokes and Couette flows due to an oscillating wall. A more insight into the subject of the slip flow regime is given by Mahmoud, [14]. A series of investigations have been made on slip flow regime viz. Derek et.al [15].

In this paper, steady fully developed free and forced convection flow through porous bounded by two walls is considered. The boundary conditions for velocity and temperature are slip boundary conditions at both the walls. In which there is a uniform axial temperature variation along the walls and analytic solutions have been obtained for velocity and temperature profile for different cases and the effects of the pertinent parameters on the flow and temperature fields are examined and discussed.

2. Mathematical Analysis

Consider the fully developed laminar tree convection flow between two vertical walls filled with a fluid saturated porous medium under a constant pressure gradient. The walls are separated by a distance 2L apart and having an axial temperature variation. The x’-axis is taken along the vertical direction while y’-axis is perpendicular to it. For fully developed laminar flow, the velocity has only the vertical component and is a function of y’ only.

As a result of these assumptions, the equation of motion in x’ direction and energy equation are obtained as follows:
The boundary conditions for the velocity and temperature fields are

\[ u' = L \frac{du'}{dy}, \quad T' = T_0' + N x' + L \frac{dT'}{dy} \quad \text{at} \quad y' = L \]

\[ u' = L \frac{du'}{dy}, \quad T' = T_0' + N x' - L \frac{dT'}{dy} \quad \text{at} \quad y' = -L \]

(3)

Under the usual Bossinesq approximation the equation of state is assumed to be

\[ \rho = \rho_0 \left[ 1 - \beta (T' - T_0') \right] \]

(4)

After assuming the uniform axial temperature variation along the channel walls, the temperature of the fluid can be written as

\[ T' - T_0' = N x' + \theta'(y') \]

(5)

Using (1) and (5) and introducing dimensionless quantities

\[ y = \frac{y'}{L}, \quad u = \frac{u' L}{\alpha P_x}, \quad \theta = \frac{\theta'}{NLP_x} \]

\[ P_x = \frac{\rho L^3}{\alpha \mu_l} \left[ - \left( \frac{1}{\rho_0} \frac{\partial \rho'}{\partial x'} + g \right) \frac{\partial x'}{\partial y'} + \beta N x' \right] \]

The equation (1) and (2) in dimensionless form obtained as follows

\[ Rv \frac{d^2 u}{dy^2} - \frac{u}{Da} + Ra = -1 \]

(7)

\[ \frac{d^2 \theta}{dy^2} - u = 0 \]

(8)

In the above equations, Rv, Da and Ra are defined as

\[ Rv = \frac{\mu_{0ff}}{\mu_f}, \quad Da = \frac{k}{L^2}, \quad Ra = \frac{\rho g NBL^3}{\alpha \mu_f} \]

(9)

Where Rv= ratios of viscosities

Ra= Rayleigh Number

Da= Darcy Nimmer

h1= Velocity slip parameter

h2= Temperature slip parameter.

The boundary conditions for the model are as follows:

\[ u = h_1 \frac{du}{dy}, \quad \theta = h_2 \frac{d\theta}{dy} \quad \text{at} \quad y = 1 \]

\[ u = -h_1 \frac{du}{dy}, \quad \theta = -h_2 \frac{d\theta}{dy} \quad \text{at} \quad y = -1 \]

(10)

Now by using (7) and (8) resulted into a fourth order differential equation in u as

\[ Rv \frac{d^4 u}{dy^4} - \frac{1}{Da} \frac{d^2 u}{dy^2} + Ra \frac{du}{dy} = 0 \]

(11)

The auxiliary roots \( m_1, m_2, m_3 \) and \( m_4 \) of the above equations are as follows:

\[ m_1, m_2 = \frac{1 \pm A}{2 Da Rv} \]

(12)

\[ m_3, m_4 = -\frac{1 \pm A}{2 Da Rv} \]
Where,

\[
A = \sqrt{1 - 4RaRvDa^2}
\]  

(13)

The Auxiliary equation roots given by the equations (12) and shows that the solution for \( u \) and \( \theta \) depends on the values of \( Da^2, Ra \) and \( Rv \). Thus three different cases arise and the solutions have been obtained as under.

Case I: When \( 0 < A < 1 \).

The solution for \( u \) and \( \theta \) by solving (7) and using (8) and (10) is obtained as:

\[
u(y) = C_1 \cosh(m_1 y) + C_2 \cosh(m_2 y)
\]  

(14)

\[
\theta(y) = -\frac{1}{Ra} + A_1 C_1 \cosh(m_1 y) + A_2 C_2 \cosh(m_2 y)
\]  

(15)

where \( A_1 = \frac{1}{RaDa} - \frac{Ra}{Rv} m_1^2 \)

\( A_2 = \frac{1}{RaDa} - \frac{Ra}{Rv} m_2^2 \)

\( A_3 = m_1 h_1 \cosh(m_2 y) \sinh(m_1 y) - m_2 h_2 \sinh(m_2 y) \cosh(m_1 y) \)

\( A_4 = h_1 m_1 \sinh(m_2 y) \cos(m_2 y) - h_2 m_1 \sinh(m_1 y) \cos(m_1 y) \)

\( A_5 = (m_1^2 - m_2^2) \cosh(m_2 y) \sinh(m_1 y) + m_1 m_2 h_1 h_2 (m_1^2 - m_2^2) \sinh(m_1 y) \sinh(m_2 y) \)

\( A_6 = (m_1 h_2 \sinh(m_1 y) \cosh(m_2 y) + m_2 h_1 \sinh(m_2 y) \cosh(m_1 y)) \)

\( A_7 = m_1 h_2 \sinh(m_1 y) \cosh(m_2 y) + m_2 h_2 \sinh(m_2 y) \cosh(m_1 y) \)

\( A_8 = \frac{1}{Ra} (A_1 + A_4) \)

\( A_9 = Ra A_3 + m_1 A_6 - m_2 A_5 \)

\( A_{10} = \cosh(m_1 y) + m_1 \sinh(m_1 y) \)

\( A_{11} = A_4 + A_5 \)

\( A_{12} = \cosh(m_2 y) + m_2 \sinh(m_2 y) \)

\( C_1^2 = \frac{A_{12}}{A_{11}} \quad C_2^2 = \frac{A_{12}}{A_{11}} \)

Case II: When \( A = 1 \).

The velocity and temperature profiles for this case by using the boundary conditions are as follows:

\[
u(y) = B_1 \cosh(Ey) + B_2
\]  

(16)

\[
\theta(y) = \frac{-1}{Ra} + B_1 E_1 \cosh(Ey) + \frac{B_2}{RaDa}
\]  

(17)

where \( E = \frac{1}{\sqrt{DaRv}} \) , \( E_1 = \frac{1}{RaDa} - \frac{Ra}{Rv} E^2 \)

\( E_4 = (RaDaE_1 - 1) \cosh(E) \), \( E_5 = (RaDaE_1 - 1) \cosh(E) \)

\( B_3 = \frac{-DaE_4}{(E_1 + E_5)} \quad B_4 = \frac{Da}{(E_1 + E_5)} \)

Case III: When \( A > 1 \).

The velocity and temperature profiles for this case by using the boundary conditions are as follows:

\[
u(y) = C_1 \cosh(m_1 y) + C_2 \cos(m_2 y)
\]  

(18)

\[
\theta(y) = \frac{-1}{Ra} + D_1 C_1 \cosh(m_1 y) + D_2 C_2 \cos(m_2 y)
\]  

(19)

Where \( D_1 = \frac{1}{RaDa} - m_1^2 \frac{Rv}{Ra} \)

\( D_2 = \frac{1}{RaDa} + m_2^2 \frac{Rv}{Ra} \)

\( D_3 = D_2 \cosh(m_2 y) - h_2 D_2 m_2 \sinh(m_2 y) \)

\( D_4 = \cosh(m_1 y) + m_1 h_1 \sinh(m_1 y) \)

\( D_5 = \cosh(m_2 y) - h_2 m_2 \sinh(m_2 y) \)

\( D_6 = D_1 \cosh(m_1 y) + h_2 D_1 m_2 \sinh(m_1 y) \)

\( D_7 = Ra(D_2 - D_5 D_6) \)

\( C_3 = \frac{D_3}{D_7} \quad C_4 = \frac{D_4}{D_7} \)

The expressions for the skin friction in non-dimensional form for the different cases are obtained by using the relation

\[
\tau = \left. \frac{du}{dy} \right|_{y=1}
\]

They are as follows:

Case I-

\[
\tau = C_1 m_1 \sinh(m_1 y) + C_2 m_2 \sinh(m_2 y)
\]
Case II

\[ \tau = B E_1 \sinh(E_1) \]

Case III

\[ \tau = C_1 m_1 \sinh(m_1) + C_2 m_2 \sin(m_2) \]

3. Result and Discussion

In order to point out the effects of different parameters on velocity \( u \), following discussions are set out. Numerical calculations are carried out for different values of the Darcy number \( (Da) \), ratio of viscosities \( (R_v) \), Rayleigh number \( (Ra) \), velocity and temperature slip parameters \( (h_1, h_2) \). The results have been shown graphically for the three cases \( A > = < 1 \).

The Figure 1 illustrates the physical configuration of the model. Both the walls are separated by distance \( 2L \) and other parameters are explained there.

The velocity profiles for different values of \( Ra(10, 15) \) and \( R_v (1.0, 1.5, 2.0) \) are shown in figure 2, when \( Da=10^{-1} \), for case I and figures 3 and 4 for case II and case III respectively, when \( Da=10^{-1}, 10^2 \) and \( 10^3 \) and \( h_1 \) and \( h_2 \) is fixed. As expected the flow is symmetrical about \( y=0 \). These figures demonstrate that the flow is parabolic type up to certain values of Rayleigh number. As usual, the velocity increases with increase of Darcy number \( (Da) \) due to fact that Darcy number is directly proportional to permeability \( (K) \) of the medium, but the reverse situation occurs in the case of ratio of viscosities \( (R_v) \). It is also evident that velocity decreases with the increase of Rayleigh number.

In figures 5 and 6 the impact of \( (h_1, h_2) \) on the velocity profiles have been shown. Figures 5 and 6 illustrates the velocity profiles for the case I and case III respectively when \( Da=10^{-1}, Ra=, h_1 =0.4, 0.8, 1.2 \) and \( h_2 = 0.2, 0.4, 0.6 \). It noticed that the velocity increases with the increase of slip parameter \( h_1 \), which, represents that the increase in the slip parameter has the tendency to reduce the friction forces which increases the fluid velocity. While reverse phenomena occurs in the case of temperature slip parameter \( h_2 \).

4. Conclusion

The present study investigates the fully developed mixed convection flow of an incompressible viscous fluid between two vertical walls filled with porous medium saturated by the same fluid. The Brinkman Darcy model is used to analyse the porous domain. The velocity profiles increase with the Darcy number while decreases with ratios of viscosities. The velocity slip parameter promotes the velocity profiles while temperature slip parameter has reverse impact on velocity profiles.
**Figure 2:** Velocity Profiles for Case I for different values of Ra and Rv, when Da=10^{-1}.

**Figure 3:** Velocity Profiles for Case II for different values of Rv, when Da=10^{-1}, 10^{-2} and 10^{-3}

**Figure 4:** Velocity Profiles for Case III for different values of Rv, when Da=10^{-1}, 10^{-2} and 10^{-3}

**Figure 5:** Velocity Profiles for Case I for different values of h_1 and h_2, when Da=10^{-1}
Figure 6: Velocity Profiles for Case III for different values of $h_1$ and $h_2$, when $Da=10^{-1}$

5. References