

MHD Flow of A Non-Newtonian Fluid With Temperature Dependent Viscosity Past A Vertical Plate In The Presence Of Radiative Heat Flux And Chemical Reaction.

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Abstract

We investigated in this research a hydromagnetic heat and mass transfer in MHD flow of an incompressible, electrically conducting, Non-Newtonian viscous fluid past a continuously moving vertical infinite plate under suction velocity and heat flux. The non-linear coupled equations were solved analytically using an asymptotic series expansion. Numerical solution was also obtained using finite difference implicit scheme method to enable comparison with the approximate analytical solution. The results were displayed graphically and compared with the previous works in the literature. Our approximate analytical solution and Numerical solution are in good agreement.

Key words: MHD, temperature dependent viscosity, radiative heat flux, chemical reaction.

1.0 Literature review

Many research works have been done and a lot of studies are still going on, on the hydromagnetic heat and mass transfer in MHD flow of a viscous fluid. A number of researchers have considered works in relation to geophysics, astrophysics, engineering, soil sciences, and industrial processes.

Vast majority of these researchers such as Raptis and Massalas[1], Chamkha [2], Okedoye et al [3,4], Ibrahim and Makinde studied MHD flow of a viscous fluid on the supposition that viscosity was constant. However, this fluid property varies with temperature, time or the space variable. The results have significant applications in space technology, solar power technology, space vehicle re-entry, nuclear engineering and so on. Based on this, Hossain and Munir [5] in their research, analysed a two dimensional mixed convection flow of a viscous incompressible fluid of temperature dependent viscosity past a vertical plate. Fang [6] studied the influence of fluid property variation on the boundary layer of a stretching surface. Hossain et al [7] discussed the effect of radiation of free convection flow of a fluid with variable viscosity from a porous vertical plate. Okedoye et al [8] investigated the effects of variable viscosity on MHD boundary layer flow on a continuously moving vertical plate in the presence of radiation and a chemical reaction of order one. In all these studies the solutions were obtained numerically.

Most of the aforementioned references dealt with constant viscosity. In this paper, we propose the effect of variable viscosity (temperature dependent) and suction velocity on hydromagnetic heat and mass transfer in MHD flow.

1.1 Nomenclature

u : Velocity along x coordinate	T' : Non dimensional fluid temperature
v : Velocity along y coordinate	C' : Non dimensional species concentration
g : Acceleration due to gravity	T : Fluid temperature
U' : Non dimensional fluid velocity	γ : Reaction parameter
T_w : Ambient temperature	σ^* : Stefan- Boltzmann constant
C : Species concentration	B : Coefficient of mass expansion
C_w : Ambient species concentration	B_τ : Coefficient of thermal expansion
B_0 : Transverse magnetic field	ρ_w : Ambient density
τ : Skin-friction coefficient	σ : Electrical conductivity
q : Heat flux	ρ : Density of the fluid
k : Thermal conductivity	Sc : Schmidt number
c_p : Specific heat at constant pressure	R : Radiation parameter
q_r : Radiative heat flux	G_{rc} : Mass grashof number
v_0 : Normal velocity at the plate	G_{rr} : Thermal grashof number
k^* : Mean absorption coefficient	M : Hartmann number
δ : Delta, $0 \leq \delta_0 \ll 1$	ω : Angular velocity
Pr : Prandtl number	μ : Fluid viscosity
A : Pre-exponential factor	D : Molar diffusivity

2.0 Mathematical Formulation

A magnetohydrodynamic flow of viscous, incompressible, electrically conducting fluid past an infinite plate in a porous medium under suction velocity is considered. The x- axis is taken along the plate in the direction of the flow and y- axis normal to it. A uniform magnetic field is applied normal to the direction of the flow. It is assumed that the magnetic Reynolds number is less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field. We further assumed that all the fluid properties are constant except that of the influence of density variation with temperature. Thus, the basic flow in the medium is entirely due to buoyancy force caused by temperature difference between the wall and the medium. Initially at $t \leq 0$, the plate as well as fluid is assumed to be at the same temperature and the concentration of species is very low so that the Soret and Dofour effect are neglected [9]. When $t > 0$, the temperature of the plate is instantaneously raised (or lowered) to T_w' and the concentration of species is raised (or lowered) to C_w' .

Under the above assumptions and taking the usual Boussinesq's approximation into account, the governing equations for continuity, momentum, concentration and energy are presented below:

$$\frac{dv'}{dy'} = 0 \quad (2.1)$$

$$v' \frac{dU'}{dy'} = \frac{1}{\rho_\infty} \frac{d}{dy'} \left[\mu \frac{dU'}{dy'} \right] + g\beta^*(T' - T_\infty) + g\beta(C' - C_\infty) - \frac{\sigma B_0^2 U'}{\rho_\infty} \quad (2.2)$$

$$v' \frac{dC'}{dy'} = D \frac{d^2 C'}{dy'^2} - A(C' - C_\infty) \quad (2.3)$$

$$v' \frac{dT'}{dy'} = \frac{k_t}{\rho_\infty c_p} \frac{d^2T'}{dy'^2} - \frac{1}{\rho_\infty c_p} \frac{dq_r}{dy'} \quad (2.4)$$

The boundary conditions:

$$U' = v_0 \quad \frac{\partial T'}{\partial y'} = -\frac{q}{k} \quad C' = C_w \quad \text{at } y' = 0 \quad (2.5)$$

$$U' \rightarrow 0, \quad T' \rightarrow T_\infty \quad C' \rightarrow C_\infty \quad \text{as } y' \rightarrow \infty$$

From equation (2.1), we take $v'(y') = -v_0$

The minus sign indicates that the suction is towards the plate.

By using Rosseland approximation q_r takes the form [10]

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{dT^4}{dy} \quad (2.6)$$

The temperature difference within the fluid assumed sufficiently small such that T^4 may be expressed as a linear function of the temperature. Expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms, we have

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \quad (2.7)$$

Substituting equation (2.7) into (2.6), we obtain

$$q_r = -\frac{16\sigma^* T_\infty^3}{3k^*} \frac{dT^2}{dy^2} \quad (2.8)$$

Using the following non dimensional quantities:

$$y = \frac{v_0 y'}{\mathcal{G}} \quad U = \frac{U'}{v_0} \quad \theta = \frac{T' - T_\infty}{\left(\frac{q \mathcal{G}}{k v_0} \right)} \quad C = \frac{C' - C_\infty}{C_w - C_\infty} \quad G_{rr} = \frac{g \beta^* \frac{q \mathcal{G}}{k v_0}}{v_0^3}$$

$$G_{rc} = \frac{g \beta (C_w - C_\infty)}{v_0^3} \quad M = \frac{B_0}{v_0} \sqrt{\frac{\sigma \mathcal{G}}{\rho}} \quad \text{Pr} = \frac{\mu_\infty c_p}{k_t} \quad R = \frac{16 \sigma T_\infty^3}{3 k^* k} \quad \text{Sc} = \frac{\mathcal{G}}{D} \quad \gamma = A \frac{\mu_\infty}{v_0^2 \rho_\infty}$$

on equations (2.2)-(2.5), the dimensionless governing equations for momentum, energy and concentration and their boundary conditions are:

$$\frac{d^2 C}{dy^2} + \text{Sc} \frac{dC}{dy} - \gamma \text{Sc} C = 0 \quad (2.9)$$

$$\frac{d^2 \theta}{dy^2} + \frac{\text{Pr}}{1+R} \frac{d\theta}{dy} = 0 \quad (2.10)$$

$$\frac{d}{dy} \left[\frac{\mu}{\mu_\infty} \frac{dU}{dy} \right] + \frac{dU}{dy} - M^2 U = -G_{rr} \theta - G_{rc} C \quad (2.11)$$

With the boundary conditions

$$U(y)=1 \quad C(y)=1, \quad \left. \frac{d\theta}{dy} \right|_{y=0} = -1 \quad \text{on } y=0 \quad (2.12)$$

$$U(y) \rightarrow 0 \quad C(y) \rightarrow 0 \quad \theta(y) \rightarrow 0 \quad \text{as } y \rightarrow \infty$$

The fluid viscosity $\mu(\theta)$ was assumed to obey the Reynold model [11].

$$\frac{\mu}{\mu_{\infty}} = e^{-\alpha\theta} \quad (2.13)$$

Using equation (2.13) in equation (2.11), we obtain

$$\frac{d}{dy} \left[e^{-\alpha\theta} \frac{dU}{dy} \right] + \frac{dU}{dy} - M^2 U = -G_{rr} \theta - G_{rc} C \quad (2.14)$$

$$U(y) = 1, \quad \text{on } y = 0 \quad U(y) \rightarrow 0 \quad \text{as } y \rightarrow \infty$$

3.0 Method of Solution

Solving equations (2.9) and (2.10), we have

$$C(y) = e^{my} \quad \text{and} \quad \theta(y) = \frac{1}{n} e^{ny} \quad (3.1)$$

To solve equation (2.14), we employ asymptotic technique as follows:

Let $\alpha = O(\delta)$ where $\delta \ll 1$

$$U = U_0 + \delta U_1 + \delta^2 U_2 + \dots + h.o.t \quad (3.2)$$

Putting equation (3.2) into equation (2.14) and collecting the order of δ , we have

Order δ^0

$$\frac{d^2 U_0}{dy^2} + \frac{dU_0}{dy} - M^2 U_0 = -G_{rr} \theta - G_{rc} C \quad (3.3)$$

$$U_0(y) = 1 \quad \text{on } y \rightarrow 0 \quad U_0(y) = 0 \quad \text{as } y \rightarrow \infty$$

Order δ

$$-\frac{d}{dy}\theta\frac{dU_0}{dy} + \frac{d^2U_1}{dy^2} + \frac{dU_1}{dy} - M^2U_1 = 0 \quad (3.4)$$

$$U_1(0) = 0 \quad U_1(\infty) = 0$$

Order δ^2

$$\frac{1}{2}\frac{d}{dy}\left(\theta^2\frac{dU_0}{dy}\right) - \frac{d}{dy}\left(\theta\frac{dU_1}{dy}\right) + \frac{d^2U_2}{dy^2} + \frac{dU_2}{dy} - M^2U_2 = 0 \quad (3.5)$$

$$U_2(0) = 0 \quad U_2(\infty) = 0$$

Solving equations (3.3) – (3.5), we have

$$U_0(y) = a_5e^{ry} + a_7e^{ny} - a_8e^{my} \quad (3.6)$$

$$U_1(y) = a_9e^{\beta y} - a_{11}e^{(n+r)y} - a_{12}e^{2ny} + a_{13}e^{(m+n)y} \quad (3.7)$$

$$U_2(y) = a_{14}e^{\alpha y} - a_{16}e^{(2n+r)y} - a_{17}e^{3ny} + a_{18}e^{(2n+m)y} - a_{19}e^{(\beta+n)y} \quad (3.8)$$

Recall

$$U(y) = U_0 + \delta U_1 + \delta^2 U_2 \quad (3.2)$$

Based on the solution obtained in equations (3.6)-(3.8), we finally calculate equation (3.2) to be

$$U(y) = a_5e^{ry} + a_7e^{ny} - a_8e^{my} + \delta[a_9e^{\beta y} - a_{11}e^{(n+r)y} - a_{12}e^{2ny} + a_{13}e^{(m+n)y}] \\ + \delta^2[a_{14}e^{\alpha y} - a_{16}e^{(2n+r)y} - a_{17}e^{3ny} + a_{18}e^{(2n+m)y} - a_{19}e^{(\beta+n)y}] \quad (3.9)$$

$$n = -\frac{Pr}{1+R} \quad m = -\frac{1}{2} \left[Sc + \sqrt{Sc^2 + 4\gamma Sc} \right] \quad r = -\frac{1}{2} \left[1 + \sqrt{1 + 4M^2} \right] \quad \omega = -\frac{1}{2} \left[1 + \sqrt{1 + 4M^2} \right]$$

$$\beta = -\frac{1}{2} \left[1 + \sqrt{1 + 4M^2} \right] \quad a_7 = \frac{1}{n} \frac{G_{rr}}{(n^2 + n - M^2)} \quad a_8 = \frac{G_{rc}}{(m^2 + m - M^2)} \quad a_5 = 1 - a_7 + a_8$$

$$a_{11} \left(\frac{a_3 r}{(n+r)^2 + (n+r) - M^2} \right) \left[1 + \frac{r}{n} \right] \quad a_{12} = \frac{2a_7 n}{(4n^2 + 2n - M^2)} \quad a_{13} = \left(\frac{a_8 m}{(m+n)^2 + (m+n) - M^2} \right) \left[1 + \frac{m}{n} \right]$$

$$a_9 = a_{11} + a_{12} - a_{13} \quad a_{16} = \left[\frac{a_5 r^2 + 2na_5 r - 2na_{11}(n+r)^2 - 2n^2 a_{11}(n+r)}{2n^2((2n+r)^2 + (2n+r) - M^2)} \right] \quad a_{17} = \left[\frac{3a_7 - 12na_{12}}{2(9n^2 + 3n - M^2)} \right]$$

$$a_{18} = \left[\frac{a_8 m^2 + 2na_8 m - 2na_{13}(m+n)^2 - 2n^2 a_{13}(m+n)}{2n^2((2n+m)^2 + (2n+m) - M^2)} \right] \quad a_{19} = \left[\frac{\beta^2 a_9 + n\beta a_9}{n((\beta+n)^2 + (\beta+n) - M^2)} \right]$$

$$a_{14} = a_{16} + a_{17} - a_{18} + a_{19}$$

The physical quantity of most interest in such problem is the skin-friction coefficient which is

defined by $\tau = \left(\frac{\partial U}{\partial y} \right)_{y=0}$

From equation (3.9) we calculate τ as follows:

$$\tau = ra_5 + na_7 - ma_8 + \varepsilon [\beta a_9 - (n+r)a_{11} - 2na_{12} + (m+n)a_{13}] + \varepsilon^2 [\omega a_{14} - (2n+r)a_{16} - 3na_{17} + (2n+m)a_{18} - (\beta+n)a_{19}] \quad (3.10)$$

4.0 Discussion of Results and Conclusion

For the purpose of discussing the effect of various parameters on the flow profiles and the temperature distribution within the boundary layer, analysis had been carried out for various values of $M, G_{rr}, G_{rc}, \alpha, \gamma$ and R with fixed values of Pr , and Sc . The values of Pr , and Sc were taken to be 0.71 and 0.6 respectively for plasma. These parameters were assigned the values

$M = 1, G_{rr} = 5.0, G_{rc} = 1.0, \gamma = 0.1, R = 0.5, \delta = 0.1$ except where stated otherwise. It should be noted that increase in α viscosity parameter leads to decrease in viscosity as given by the relation in equation (2.13). Also, $\gamma < 0, \gamma = 0$ and $\gamma > 0$ indicate generative, no reaction and destructive chemical reaction respectively. The variation of the skin-friction coefficient τ for various values of α, M, R and G_{rr} with $Pr = 0.71$ is shown in table 4.1. It can be seen from this table that the skin-friction coefficient increases as the viscosity parameter, mass Grashof number or the thermal Grashof number increases. Increasing of the magnetic parameter, the radiation parameter or reaction parameter leads to a decrease in the skin-friction.

The effect of α on the dimensionless velocity U is illustrated in table 4.2. From the table, we can see that increase in viscosity parameter increased the fluid velocity near the plate only while reverse is the case as we move away from the plate.

In figure 4.1, we show the distribution of velocity for various values of delta. It could be seen that increase in delta increases velocity. In figure 4.2, we show that generative chemical reaction leads to increase in fluid velocity while increase in destructive chemical reaction lowers the velocity.

Also, in figure 4.3, we show the distribution of velocity for various values of radiation parameter. It could be seen that increase in radiation parameter increases the velocity. We displayed in figure 4.4 and 4.5 the effect of thermal and mass Grashof number respectively. It is observed that an increase in the values of both parameters leads to increase in the velocity and vice versa. The velocity U of the fluid decreases as the magnetic parameter M increases as shown in figure 4.6. It is observed that the velocity increased to its maximum value near the plate and then decreased to zero. Figure 4.7 shows the temperature θ profile for various values of

radiation parameter. It could be seen that increase in radiation parameter R reduces temperature of the fluid. Also it is noticed that a decrease in the fluid temperature with maximum value at the plate and minimum at a distance away from the plate. The effect of reaction parameter on the concentration of chemical species is shown in figure 4.8. We noticed that increase in reaction parameter reduces the concentration of the chemical species.

Table 4.1: The values of skin-friction τ when $\alpha \neq 0$

α	M	R	Grt	Grc	γ	τ
-0.2	1.0	0.5	5	1	0.1	6.2229
-0.1	1.0	0.5	5	1	0.1	7.3786
0.0	1.0	0.5	5	1	0.1	8.8271
0.1	1.0	0.5	5	1	0.1	10.5683
0.2	1.0	0.5	5	1	0.1	12.6023
0.1	0.0	0.5	5	1	0.1	28.8078
0.1	0.5	0.5	5	1	0.1	18.8078
0.1	1.0	0.1	5	1	0.1	6.0487
0.1	1.0	0.5	5	0	0.1	9.6502
0.1	1.0	0.5	5	2	0.1	11.4864
0.1	1.0	0.0	5	4	0.1	7.6376
0.1	1.0	0.1	5	1	-0.12	6.2558
0.1	1.0	0.5	5	1	0.0	10.6327
0.1	1.0	0.5	5	1	0.1	10.5683
0.1	0.5	0.5	5	1	0.2	18.7082
0.1	1.0	0.5	8	1	0.1	17.5009
0.1	1.0	0.5	10	1	0.1	22.1227
0.1	1.0	0.0	10	1	0.1	11.0303

Table 4.2: Velocity U distribution for various values of α

y	$U(-0.2)$	$U(-0.1)$	$U(0.0)$	$U(0.1)$	$U(0.2)$
0	1	1	1	1	1
1	3.6507	3.8519	4.0395	4.2136	4.3742
2	3.1087	3.1418	3.1637	3.1743	3.1737
3	2.1099	2.0981	2.084	2.0676	2.0491
4	1.3358	1.3244	1.313	1.3016	1.2902
5	0.8287	0.8228	0.817	0.8115	0.8061
6	0.5119	0.5093	0.5068	0.5044	0.5021
7	0.3164	0.3153	0.3143	0.313	0.3124
8	0.1959	0.1954	0.195	0.1947	0.1943
9	0.1214	0.1213	0.1211	0.1209	0.1208
10	0.0754	0.0753	0.0752	0.0752	0.0751

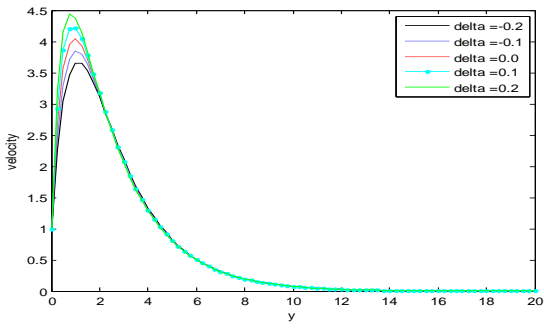


Fig 3.1: velocity $U(y)$ distribution for various values of δ

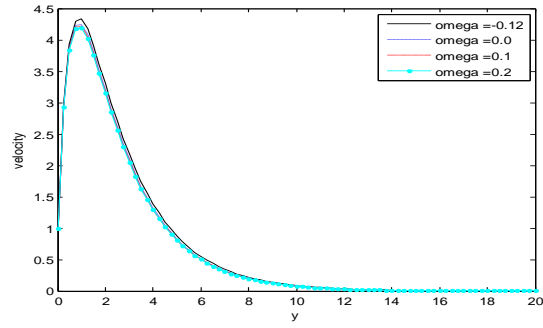


Fig 3.2: velocity $U(y)$ distribution for various values of γ

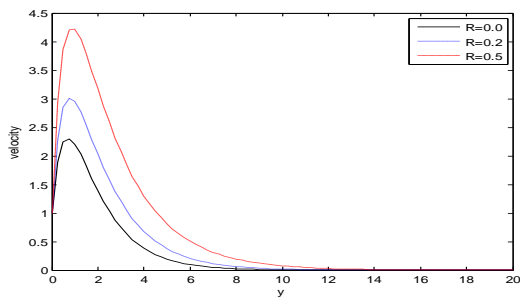


Fig 3.3: velocity $U(y)$ distribution for various values of R

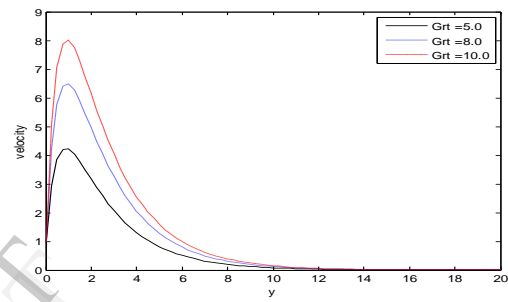


Fig 3.4: velocity $U(y)$ distribution for various values of G_{rr}

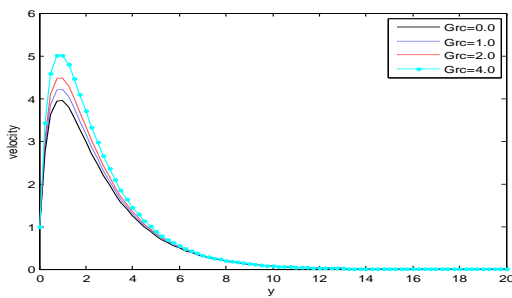


Fig 3.5: velocity $U(y)$ distribution for various values of G_{rc}

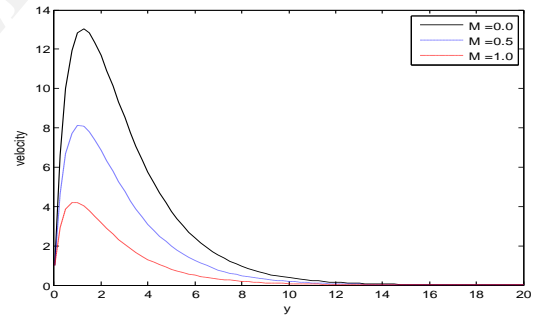


Fig 3.6: velocity $U(y)$ distribution for various values of M

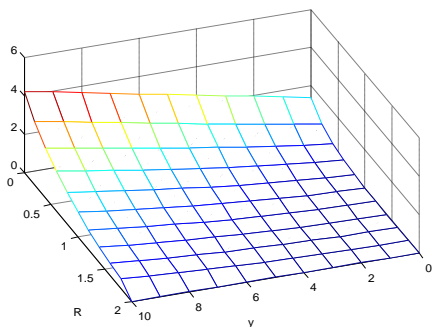


Fig 3.7: temperature $\theta(y)$ distribution for various values of R

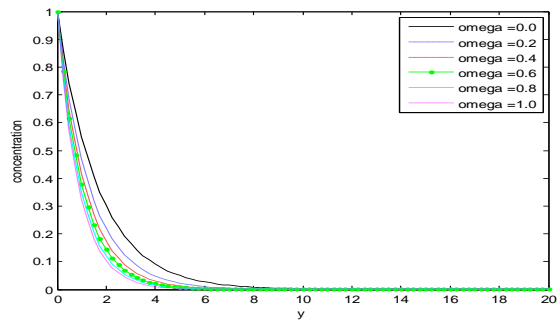


Fig 3.8: Conc. $C(y)$ distribution for various values of γ

5.0 Numerical Approach

Applying Crank-Nicolson formula to equations (2.9), (2.10) and (2.14)

$$P_3 C_{i-1} - Q_3 C_i + R_3 C_{i+1} = 0 \quad (5.1)$$

$$P_2 \theta_{i-1} - Q_2 \theta_i + R_2 \theta_{i+1} = 0 \quad (5.2)$$

$$P_1 U_{i-1} - Q_1 U_i + R_1 U_{i+1} = -D_{1i} \quad (5.3)$$

With corresponding boundary conditions (2.12) becoming

$$U_i(0) = 1 \quad \left[\frac{d\theta}{dy} \right]_{y=0} = 1 \quad C_i(0) = 1$$

$$U_i(\infty) = 0 \quad \theta_i(\infty) = 0 \quad C_i(\infty) = 0 \quad (5.4)$$

Where

$$P_3 = \left[1 - \frac{(\Delta y) S_c}{2} \right] \quad Q_3 = \left[2 + (\Delta y)^2 \gamma S_c \right] \quad R_3 = \left[1 + \frac{(\Delta y) S_c}{2} \right] \quad P_2 = \left[1 - \frac{(\Delta y) Pr}{2(1+R)} \right] \quad Q_2 = 2$$

$$R_2 = \left[1 + \frac{(\Delta y) Pr}{2(1+R)} \right] \quad P_1 = \left[1 - \frac{a_6 (\Delta y)}{2} \right] \quad Q_1 = \left[2 + a_1 (\Delta y)^2 M^2 \right] \quad R_1 = \left[1 + \frac{a_6 (\Delta y)}{2} \right]$$

$$D_{1i} = (\Delta y)^2 a_1 a_5 \quad a_1 = e^{\alpha \theta} \quad a_2 = \frac{d\theta}{dy} \quad a_3 = G_{rr} \theta \quad a_4 = G_{rc} C \quad a_5 = a_3 + a_4 \quad a_6 = a_1 - \alpha a_2$$

Thus, Δy is a constant mesh size along y direction respectively. We need a scheme to find single values at next level time in terms of known values at an earlier time level. A central difference approximation for the first order derivatives of C, θ and U with respect to y and a central difference approximation for the second order derivative of C, θ and U with respect to y are used.

5.1 Tables and Graphical Presentations: Numerical Solution

To ensure the validity of our analytical solutions, we have compared our numerical solutions with the exact solutions obtained for concentration, temperature and velocity for some varying fluid parameters.

Table 5.1 to 5. 3 show comparison between analytical and numerical values for concentration, temperature and velocity respectively $M = 1.0, G_{rr} = 5.0, G_{rc} = 1.0, \gamma = 0.1, R = 0.5, \varepsilon = 0.1$. It was clearly seen from these tables that the error decreases as the value of y approaches 10. Hence the results are in good agreement. Also, figure 5.1 to 5.3 for concentration, temperature and velocity below show that the curves corresponding to exact and numerical solutions are lying very close to each other which further confirm the accuracy of our model.

Table 5.1: Comparison between exact and numerical values for concentration

y	Exact	Numerical	Error
0	1	1	0
2	0.2529	0.2524	0.0005
4	0.064	0.0637	0.0003
6	0.0162	0.0161	0.0001
8	0.0041	0.0041	0
10	0.001	0.001	0

Table 5.2: Comparison between exact and numerical values for temperature

y	Exact	Numerical	Error
0	2.1127	2.1127	0
2	0.8198	0.8188	0.001
4	0.3181	0.3173	0.0008
6	0.1234	0.1229	0.0005
8	0.0479	0.0475	0.0004
10	0.0186	0.0183	0.0003

Table 5.3: Comparison between exact and numerical values for velocity

y	Exact	Numerical	Error
0	1	1	0
2	3.1743	3.5772	0.4029
4	1.3016	1.4618	0.1602
6	0.5044	0.5654	0.0610
8	0.1947	0.2174	0.0227
10	0.0752	0.0834	0.0082

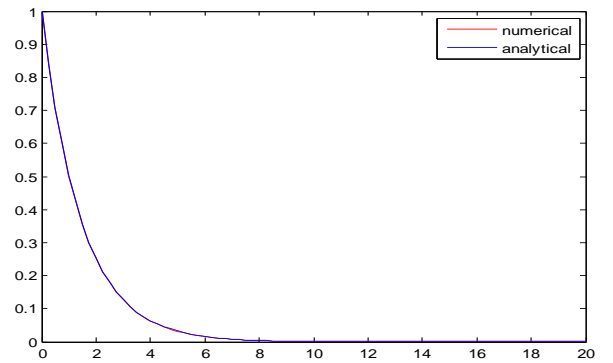


Figure 5.1: Concentration profiles

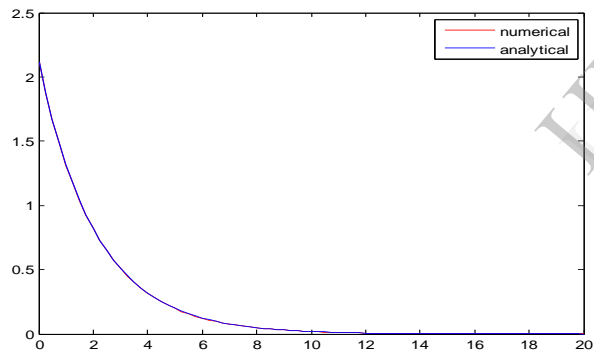


Figure 5.2: Temperature profiles

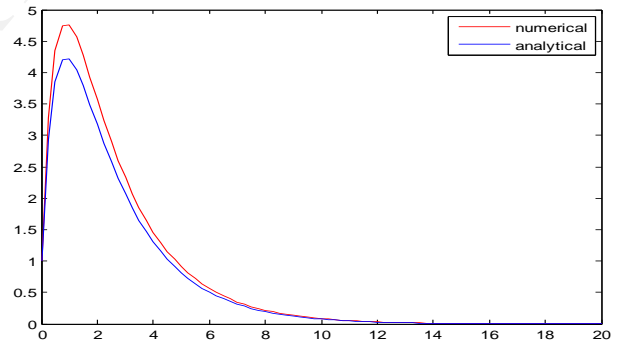


Figure 5.3: Velocity profiles

Concluding remarks

In this work, the problem of boundary layer flow of a steady viscous, incompressible electrically conducting fluid with variable viscosity over a continuously moving vertical porous plate in the presence of magnetic field and radiation had been investigated. An asymptotic method was employed to solve the resulting coupled differential equations. The obtained results were compared with Crank-Nicolson finite difference method of implicit scheme and found to be in good agreement. Also, graphical illustrations presented further revealed the relationship between varying parameter affecting fluid behaviour. Above all, the result is an improvement over the previous work by other authors in that it caters for temperature dependent viscosity rather than constant viscosity which is simple to comprehend.

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