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Memristor: A New Step Towards System Stability and Performance

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***Abstract*— In this paper, for the first time, we propose a stability controller based on a new electrical element "Memristor" which will not only improve the system stability and performance, but also maintain that stability for a longer time, irrespective of different perturbation factors. Memristor changes its resistance on changing voltage or current through it which makes our proposed stability controller more flexible, compared to traditional stability controller. To illustrate this, a highly unstable Maglev train model is taken as an example which demands continuous stability for a long duration. The output results are demonstrated by simulation results and are verified by mathematical reasoning to support the unusual and unexpected characteristics.**

***Keywords*—memristor; stability analysis; controller; magnetic levitation; maglev train model**

I. INTRODUCTION

Generally in all kinds of systems, stability plays a major role and often used to provide safety to the whole system. Nowadays all systems are manufactured to fulfil this criteria but beyond making the system stable, it is also important that, these system remain in stable state for a long time. Many unavoidable perturbation factors in terms of temperature, varying gain or noise source etc., that come up at any time and can change the system stability and turn it into an unimaginable catastrophe [1]. Traditionally, to improve the stability of any system, gain of the system has to be increased. But by doing so, the system may go from stable to unstable state as well. Later, to improve the stability of a system without changing the gain, various models namely lead compensator, proportional, derivative or integrated controller were used widely. Each controller has its own limitations either in terms of frequency of operation, steady state response or transient response. Hence each and every controller design is dependent on the application, for which, it is constructed [2].

In 1971 Leon Chua proposed a new two terminal element named as "Memristor". Apart from R, L or C element, memristor was the fourth missing element which relates to flux (ϕ) and charge (q) passing through it as $M = (\phi / q)$ [2]. Basically memristor have a lot of unconventional properties which can be used for to modelling of several physical device and systems [3]. In May 2008 HP Lab's researcher realized a practical memristor device which had all the properties proposed by Leon Chua [4]. As shown in Fig. 1(a), memristor consist of a thin layer (Thickness ≈ 5nm) sandwiched between two Platinum contacts. This thin film layer is made up by aggregating TiO2 layer and a TiO_{2+} layer (Dopant with +2 charge) which makes doped and undoped regions respectively. Due to this the resistance of undoped region (Roff) is greater than doped region (Ron) which make it as a variable register element.

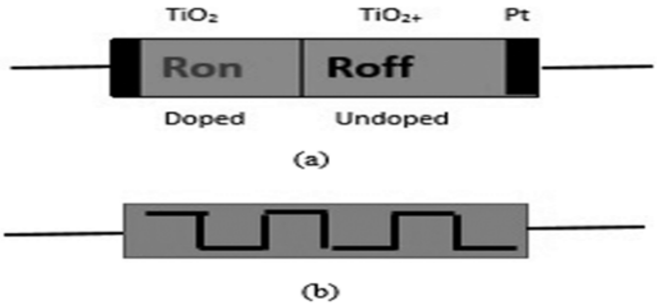


Fig. 1. (a) A Memristor element (b) Memristor Symbol

On applying voltage or current across its terminal the length of doped region will increase or decrease depending on the polarity of applied voltage or current. On applying current in one direction resistance of the memristor will decrease and on applying the current in opposite direction resistance of the memristor will decrease [3]. In our proposed controller this property adds, flexibility in the design of Maglev train model which will be discussed in section IV.

In case of our Maglev train model, corresponding pole and zero location are analysed through the characteristic equation of obtained transfer function. The root of this characteristic equation decides the location of pole and zero in s- plane. If all the poles of a system is in the left of s plane then our system will be stable otherwise it will be unstable. In our paper for analysing the system stability and its performance, we will be discussing the simulation results by using Root locus, Bode Plot and Step response between traditional stability controller and memristor based stability controller. Effect of each factor on stability, is reported through simulation results and mathematical verification.

II. ANALYSIS OF AN UNSTABLE SYSTEM: A MAGLEV TRAIN MODEL

In this paper a maglev train model is taken as an example to fulfil stability criteria at high frequency of operation. However this model is complex in nature but we designed the whole system by observing stability as a major criteria and obtained the transfer function mathematically. A maglev train model is based on magnetic levitation system which demands continuous stability. A failure in continuous stability, gives catastrophic results. A maglev train is one of the fastest train all over the world. But its presence is limited to a few countries due to its complex design and modelling [1]. There are a lot of parameters that have to be decided before its designing but in our paper we will consider only those factors which will affect the stability of the system adversely [6]. In maglev train, the magnetic base of the train is kept to be hanging due to electromagnetic force against gravitational force. Acceleration of the magnetic base of train in downward or upward direction, will depend on difference between these two forces. When these two forces are equal and opposite in direction then the whole system is stable. The duration of this stability depends on different perturbation factors (viz. Temperature, Air gap, etc.) A Hall sensor can be placed to detect displacement between the magnetic base of train and electromagnetic source [6].Let

I = Current through magnet in Ampere
$\hat{I}$ = Current due to perturbation factor in Ampere
I_0= Current at steady state in Ampere
U= Vertical displacement of object from electro magnet in Meter
$\hat{U}$ = Displacement due to perturbation factors in Meter
U_0 = Displacement at steady state in Meter
L = Total inductance due to electro magnet in Henry
L_1= Additional inductance due to presence of object in Henry
L_0= Inductance due to absence of object in Henry ($L_1 >> L_0$)

Due to current flowing through electromagnetic coil, the electromagnetic force on the object is given as

$$F_e = \frac{\delta}{\delta U}\left(L(U) I^2\right) \tag{1}$$

Here $L(U) = L_1 + \frac{L_0}{1+\frac{U}{U_0}} = L_1 + \frac{L_0 U_0}{U}$ (For $U >> U_0$)

From (1), $Fe = \left(-\frac{I_0^2 L_0 U_0}{2U^2}\right)$

$$Fe = -C\left(\frac{I}{U}\right)^2 \tag{2}$$

Here $C = \frac{L_0 U_0}{2}$ = Constant

The gravitational force acting on a body is given as

$$Fg = Mg \tag{3}$$

The difference between electromagnetic and gravitational forces will decide acceleration of magnetic base in an upward or downward direction. The net force on the magnetic base is given by

$$F_{net}(t) = F_e\left(\hat{i}, \hat{U}\right) + F_g \tag{4}$$

At stable state, net force will be zero and the system will be in equilibrium state i.e. $F_{net} = 0$

From (4) $$C\left(\frac{I_0}{U_0}\right)^2 = Mg \tag{5}$$

To analyse the system using transfer function, all constraints should be linear in nature but from (4) it is clear that the net force on the magnetic base is nonlinear [7, 8]. Due to perturbation factor if change in displacement and current is $\hat{U}$ and $\hat{i}$ respectively where $\hat{i} = (I-I_0)$ and $\hat{U} = (U- U_0)$, to make net force linear, the Taylor Series expansion can be used as

$$F_e\left(\hat{i}, \hat{U}\right) = \left\{F_e\left(I_0, U_0\right)\right\} + \left\{\frac{\delta}{\delta U_0}\left[F_e\left(I_0, U_0\right)\right]\right\}\hat{U} + \left\{\frac{\delta}{\delta I_0}\left[F_e\left(I_0, U_0\right)\right]\right\}\hat{i}$$

From (2) $$F_e\left(\hat{i}, \hat{U}\right) = -C\left(\frac{I_0}{U_0}\right)^2 + 2C\left(\frac{I_0^2}{U_0^3}\right)\hat{U} - 2C\left(\frac{I_0}{U_0^2}\right)\hat{i} \tag{6}$$

In stable state, at time 't', the net force on the magnetic base, due to perturbation factor is given by

$$F_{net}(t) = F_e\left(\hat{i}, \hat{U}\right) + F_g$$

From (5) and (6)

$$F_{net}(t) = 2C\left(\frac{I_0^2}{U_0^3}\right)\hat{U} - 2C\left(\frac{I_0}{U_0^2}\right)\hat{i} \tag{7}$$

At the same time 't', for an electromagnetic coil the voltage across coil is given by

$$V_m(t) = \left[RI(t) + L(U)\frac{\partial}{\partial t}\left(I(t)\right)\right] \tag{8}$$

Here L(U) depends on magnetic base position which make $V_m(t)$ nonlinear. To make it linear let us assume that magnetic base is placed close to its stable state i.e. $U \simeq U_0$. Hence for ($L_1 >> L_0$)

$$V_m(t) = \left[RI(t) + L_1\frac{\partial}{\partial t}\left(I(t)\right)\right] \tag{9}$$

If M is the mass due to magnetic base and U is its displacement from the magnetic coil then from Newton's law of motion

$$F = M\left(\frac{\partial^2 U}{\partial t^2}\right) \tag{10}$$

The voltage across the sensor is proportional to the position of the magnetic base below the magnetic coil hence

$$V_s(t) \propto U$$

$$V_s(t) = \lambda U \tag{11}$$

Here λ = Gain of sensor. From Laplace transformation the above equations (7), (9), (10) and (11) can be re written as

$$F_{net}(s) = 2C\left(\frac{I_0^2}{U_0^3}\right)U(s) - 2C\left(\frac{I_0}{U_0^2}\right)I(s) \tag{12}$$

$$V_m(s) = \left[RI(s) + sL_1 I(s)\right] \tag{13}$$

$$F(s) = s^2 MU(s) \tag{14}$$

$$V_s(s) = \lambda U(s) \tag{15}$$

Now for the given Maglev train model the transfer function is given by

$$G(s) = U(s) / I(s)$$

From (14), (15) $V_m(s) \propto I(s)$ And $V_s(s) \propto U(s)$

Hence $$G(s) = \frac{V_s(s)}{V_m(s)} \tag{16}$$

Substituting (12), (13), (14) and (15) on (16)

$$G(s) = \frac{\left(\frac{2CI_0\lambda}{MU_0^2 L_1}\right)}{\left(S + \frac{R}{L_1}\right)\left(S^2 - \frac{2CI_0^2}{MU_0^3}\right)} \tag{17}$$

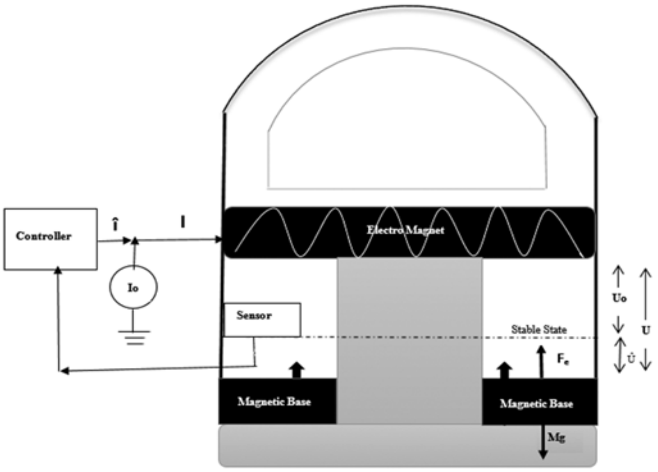
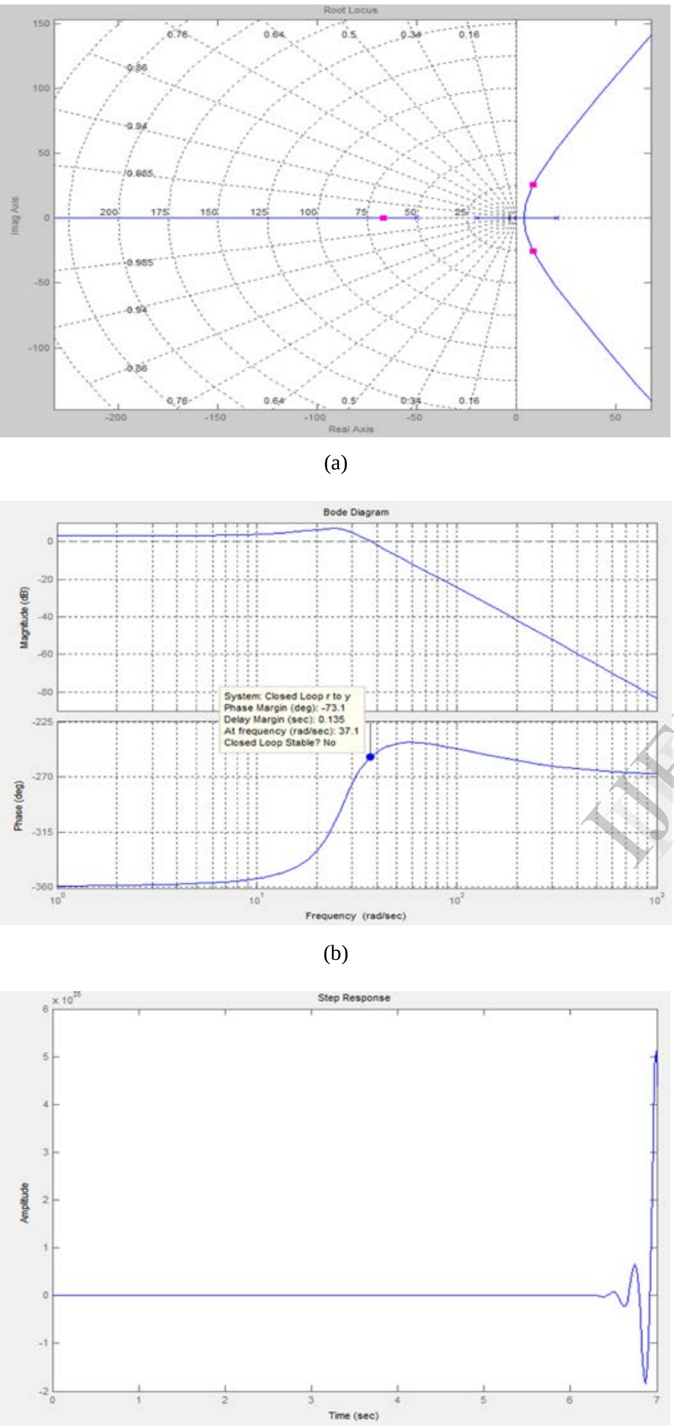


Fig. 2. A Maglev train model with stability and performance as a major criteria

On experimenting with various values, the following values are found to be suitable for the given system to make it unstable initially, for better stability analysis : $C=2.22*10^{-5}\ NM^2A^{-2}$; $U_0=0.01$ M; $I_0= 1.5$Amp ; M= 0.250 Kg ; R=1 Ohm ; L1= 0.02H ; $\propto = 511.4$ V/m [7].



(a)

(b)

(c)

Fig. 3 Analysis of Maglev train model to comment on stability (a) Root Locus plot. Pink Squares (closed loop poles) which are in the right side of s plane ensure unstable system (b) Bode plot. Unstable system due to negative phase margin (c) Step response. Infinite rise and fall time assure response will reach at a steady state in infinite time

Hence for a given maglev train model, transfer function is given as

$$G(s)=\frac{68,818}{(S+50)(S^2-400)} \qquad (18)$$

Mathematically, from given open loop transfer function (18) it is clear that one open loop pole lies in right half of s plane hence for assumed parameters, Maglev train model is unstable. The system un-stability along with system performance is verified by simulation results in Fig. 3. In next section to make Maglev system stable, first we will cascade the Maglev train model with traditional controller followed by proposed memristor based stability controller and analyse the performance by comparing the simulation results.

III. TRADITIONAL STABILITY CONTROLLER

A stability controller should be designed in such a way so that zero of the controller is placed between the origin and first pole of the system and the pole should be placed deeper than the deepest pole of the system, from the left hand side of the plane. For the given stability controller transfer function is given as

$$\frac{V_o(s)}{V_i(s)}=\frac{K\left(s+\frac{1}{T}\right)}{\left(s+\frac{1}{\eta T}\right)}$$

Here $T = (R_A C_A)$,$\eta = R_B / (R_A + R_B)$,$K = 1+ (R_F / R_1)$ The value of all parameters should be selected in such a way that the system will move from unstable **state** to stable state i.e. all poles should be in the left half of s plane. The location of pole and zero is given by $s = (-1/\eta T)$ and $s = (-1/T)$ respectively. On selecting $R_A = 2.987\Omega$, $R_B = 1.3483\Omega$, $C_A= 0.018F$, transfer function of the tradition stability controller is given by

$$G_C(s)=\frac{1.0498(s+18)}{(s+57.8)}$$

To achieve gain K of the traditional controller, R_F and R_1 selected as 0.0996 Ω and 2Ω respectively. From the Figure 5(b) the gain and phase margin of the stable Maglev system are 0.254 and 0.635 respectively. Hence practically, environmental perturbation can make these margin negative and due to this the system will not maintain its stability longer. After implementation of the controller it is very hard to change its parameters to maintain its stability longer. To overcome all these limitations a memristor based stability controller is proposed in the next section which maintains its stability for longer duration. Even after system implementation, it is possible to attain stability without replacing components inside the system.

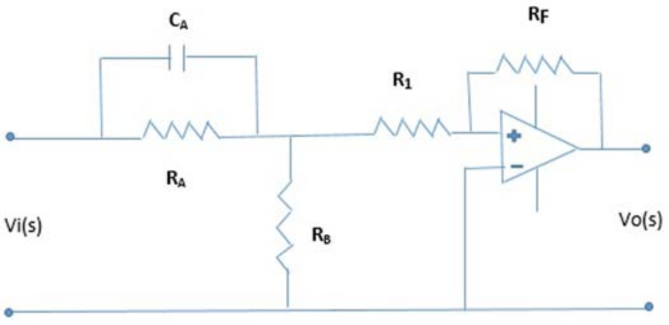
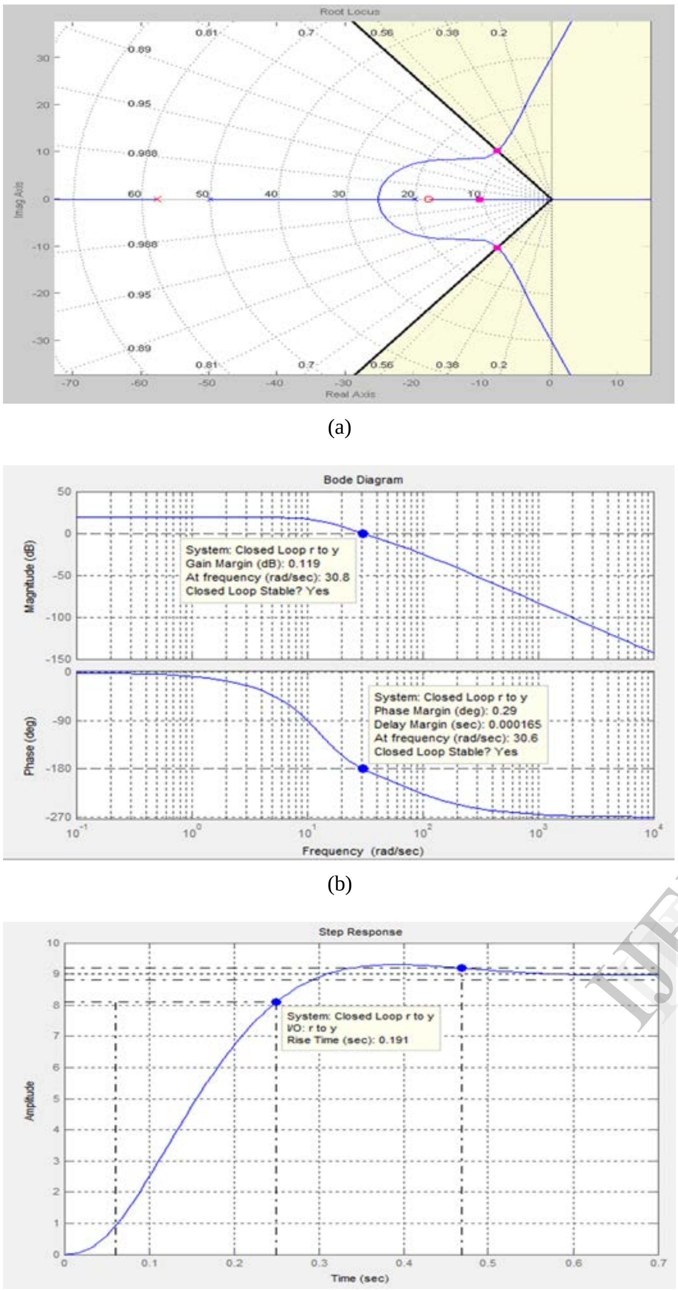


Fig. 4. A traditional stability controller



(a)

(b)

(c)

Fig. 5. Performance of Traditional Stability Controller (a) Root Locus. Pink squares (closed loop poles) are in the left hand of s plane which ensure system is stable (b) Bode Plot. Positive gain and phase margin make system stable but due to lower phase and gain margin, system cannot maintain its stability longer. (c) Step response. System give slow response due to large rise time

IV. MEMRISTOR BASED STABILITY CONTROLLER

In our proposed controller we are using charge controlled memristor to achieve stability criteria. In charge controlled memristor, flux will be the function of charge passing through it [3]. In a Charge controlled memristor

$$\phi = f(q)$$

$$\frac{d\phi}{dt} = M(q)\left(\frac{dq}{dt}\right)$$

$V(t) = M(q)I(t)$ Here $M(q) = \frac{d\phi}{dq}$ =Memristance (Ω)

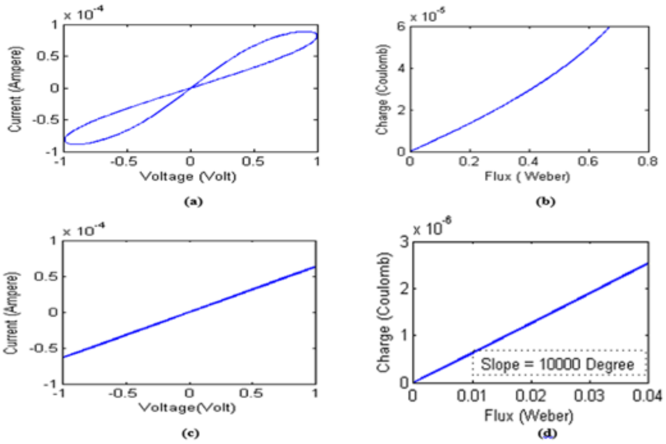


Fig. 6. V-I Hysteresis Curve for (a) w=3 rad/sec (b) w >>3 rad/sec; Φ-q curve for (c) w=3 rad/sec (d) w>>3 rad/sec

The physical realization of memristor proposed by HP labs is based on Chua's theoretical model. Simulation results describe its behavior and adaptability for other applications. As shown from V-I characteristic of memristor in Fig. 6(a), on increasing frequency, the hysteresis width will decrease. In Figure 6(b), at high frequency hysteresis width is very less which makes memristor characteristic analogous to register's transfer characteristic [4].

According to Chua's theory memristor can be used as a passive element if its Memristance is positive i.e. $M(q) \geq 0$ and the slope of its Φ-q curve will decide the equivalent Memristance [1]. Mathematically from HP model the equivalent Memristance can be given by [5]

$$M(q) = \frac{R_{off}\left(1 - R_{on}q(t)\right)}{\beta} \qquad \text{Here} \quad \beta = \frac{D^2}{\mu_D}$$

On selecting D (Width of TiO_2 layer) =10nm, μ_D (Dopant mobility) = 10^{-10} $cm^2s^{-1}v^{-1}$, Ron = 100Ω, Roff =16kΩ and Rinit = 80kΩ, the resultant slope is calculated as 10,000 degree. In Figure. 7 proposed stability controller uses high frequency of operation. Hence in Figure. 4 on replacing R_A by M (q), without affecting the other parameter, the transfer function of proposed stability controller is given by

$$G_M(s) = \frac{K\left(s + \frac{1}{T}\right)}{\left(s + \frac{1}{\eta T}\right)}$$

Here $T = M(q)C_A$, $\eta = \frac{R_B}{R_B + M(q)}$, $K = 1 + \frac{R_F}{R_1}$

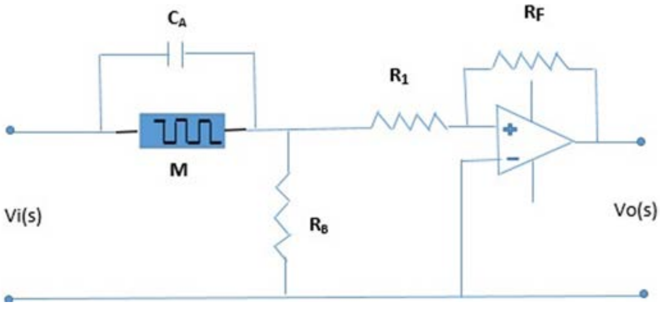


Fig. 7. Memristor based Stability Controller

For better comparative analysis the value of all parameters is kept same as that of traditional stability controller. On substituting all the parameter along with Memristance M (q) =10000, transfer function changed as

$$G_M(s) = \frac{12.488(s+16.8)}{(s+506)}$$

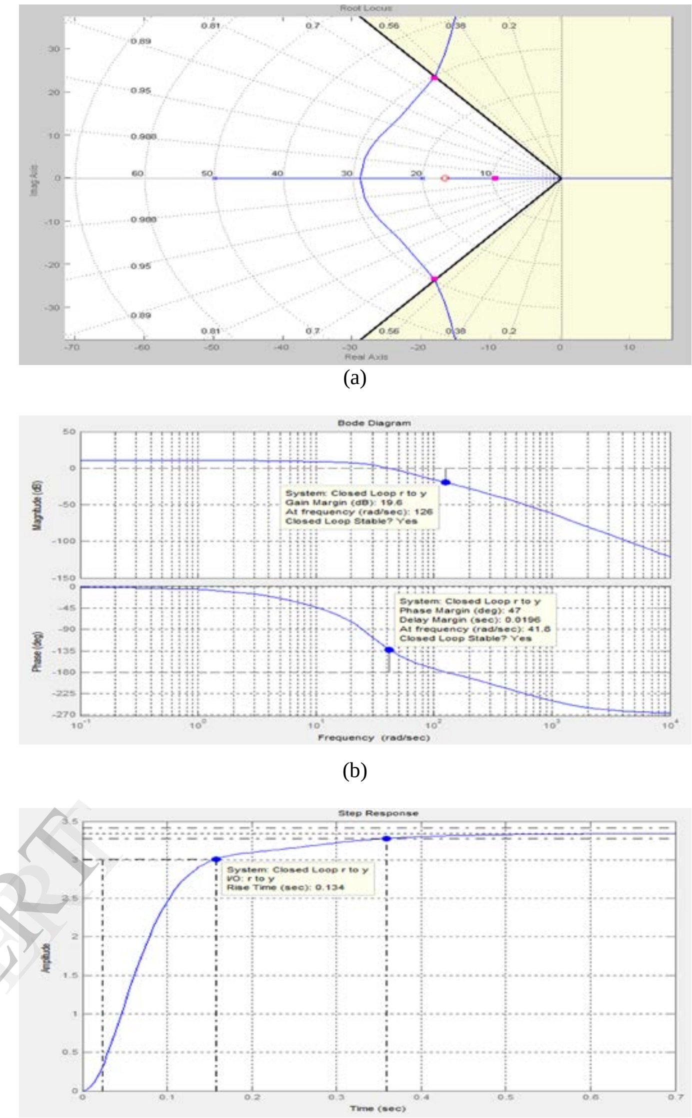
The overall gain K of the controller can be achieved by substituting R_1=0.099Ω and R_F=2Ω externally. In Figure. 8(a), adding pole on more negative part of the s plane, moves the locus of closed loop poles from right to left. Hence the stability will improve and due to which perturbation factor cannot change the stability of the system. From the root locus it is clear that overshoot of proposed controller is lower than the traditional controller by 59%, ensure less distorted output. From Bode plot in Figure 8(b), the gain margin and phase margin increased radically, for the same system parameter as used in traditional controller. These high margins ensure the stability for longer duration. In Figure8(c), step response of proposed controller also shows lower rise time compared to the traditional controller which leads to a faster response [2]. Hence overall system performance is improved by the Memristor based stability controller as compared to a traditional stability controller. A comparative analysis of maglev train model with traditional and memristor based controller is summarized in table 1.

TABLE I. COMPARATIVE ANALYSIS OF MAGLEV TRAIN MODEL WITH TRADITIONAL & PROPOSED STABILITY CONTROLLER

Performance Parameter of Maglev Train Model	**With Traditional Stability Controller**	**With Memristor Based Stability Controller**	**Remarks**
Rise time	0.191 sec	0.147 sec	Fast system response
Settling time	0.500 sec	0.371 sec	Lower time constant
Gain margin	0.0949 dB	19.7 dB	Improved and Longer stability
Phase Margin	0.231°	47.3 °	
Phase cross over frequency	30.8 rad/sec	126 rad/sec	
Gain cross over frequency	30.6 rad/sec	41.3 rad/sec	
Max Overshoot	13.33%	7.85%	Lower Distortion
Max natural Frequency	11.3 rad/sec	27.1 rad/sec	Improved Frequency range of operation

V. CONCLUSION

In this paper we addressed some possible research gap in the area of stability and demonstrated that further device and circuit modelling are required to avoid catastrophic situation. Memristor based stability controller has been proposed and compared to a traditional stability controller. It is found that proposed memristor based stability controller shift the locus of closed loop pole to extreme negative left of s plane due to which it maintain stability for longer duration without getting affected by perturbation factors. It is reported that the Memristor has variable resistance which depends on the direction of current. This property makes our proposed controller more flexible than traditional controller even after implementation of the whole system. Hence a memristor based controller is far better than traditional controller not only in terms of stability but also for overall performance as well. Results are also verified using mathematical reasoning and simulation results corresponding to Bode plot, Root locus and step response.



(a)

(b)

(c)

Fig. 8. Comments on performance of Memristor based Stability Controller (a) Root Locus shift comparatively more than traditional controller in left of s plane which ensure better stability (b) Bode Plot. Gain and Phase margin comparatively more positive ensure longer stability (c) Step response. Minimum rise time ensure comparatively faster response.

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