MATLAB SIMULINK Based Transient Exploration of RL Circuit

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Abstract:

Whenever a circuit is switched from one condition to another, either by change in applied voltage or change in circuit parameters, there is a transient period during which the branch current and element voltages change from their former values to new ones with a rate equal to the time constant. This period is called the transient period. After the transient has passed, the circuit is said to be in steady state. This paper epitomizes the results based upon the simulation of the RL model in MATLAB SIMULINK. The graphs obtained with various series resistances are compared and the results are discussed.

Introduction:

The above RL series circuit is connected across a constant voltage source, (the battery) and a switch. Assume that the switch, S is open until it is closed at a time $t = 0$, and then remains permanently closed producing a "step response" type voltage input. The current, $i$ begin to flow through the circuit but do not rise rapidly to its maximum value of $I_{max}$ as determined by the ratio of $V / R$ (Ohms Law).

This limiting factor is due to the presence of the self-induced emf within the inductor as a result of the growth of magnetic flux, (Lenz's Law). After a time the voltage source neutralizes the effect of the self-induced emf, the current flow becomes constant and the induced current and field are reduced to zero.

We can use Kirchhoff’s Voltage Law, (KVL) to define the individual voltage drops that exist around the circuit and then hopefully use it to give us an expression for the flow of current.

Kirchhoff’s voltage law gives us:

$V(t) = V_R + V_L = 0$

The voltage drop across the resistor, R is $IR$ (Ohms Law).
V_R = IR

The voltage drop across the inductor, L is by now our familiar expression \( L = \frac{di}{dt} \)

\[ V_L = L \frac{di}{dt} \]

Then the final expression for the individual voltage drops around the RL series circuit can be given as:

\[ V(t) = IR + L \frac{di}{dt} \]

**Mathematics Involved:**

As discussed above, the final expression for current is

\[ V(t) = IR + L \frac{di}{dt} \]

The differential equation in the form \( \frac{di}{dt} + Pi = Q \) has a solution:

\[ i(t) = e^{-Pt} \int Q e^{Pt} dt + K e^{-Pt} \]

Which gives

\[ i(t) = \frac{V}{R} \left( 1 - e^{R/L \cdot t} \right) \]

The voltage across the inductance is given by-

\[ V_L(t) = L \frac{di(t)}{dt} \]

So, \( V_L(t) = Ve^{-R/L \cdot t} \)

The above expression conclude that as the switch is closed, voltage initially across the inductor is \( V( \text{Battery Voltage}) \), but as time passes the voltage decays and finally at the steady state condition, Inductor behaves like a short circuit i.e voltage across the inductor becomes zero.

**Concept of time constant**

The L/R term in the above equation is known commonly as the **Time Constant**, \( \tau \) of the LR series circuit and \( V/R \) also represents the final steady state current value in the circuit. Once the current reaches this maximum steady state value at \( 5\tau \), the inductance of the coil has reduced to zero acting more like a short circuit and effectively removing it from the circuit. Therefore the current flowing through the coil is limited only by the resistive element in Ohms of the coils windings. A graphical representation of the current growth representing the voltage/time characteristics of the circuit can be presented as.

**Transient Curves for an LR Series Circuit**

Since the voltage drop across the resistor, \( V_R \) is equal to \( IxR \) (Ohms Law), it will have the same exponential growth and shape as the current. However, the voltage drop across the inductor, \( V_L \) will have a value equal to: \( Ve^{(R/L)} \). Then the voltage across the inductor, \( V_L \) will have an initial value equal to the battery voltage at time \( t = 0 \) or when the switch is first closed and then decays exponentially to zero as represented in the above curves.

The time required for the current flowing in the LR series circuit to reach its maximum steady state value is equivalent to about **5 time constants** or \( 5\tau \). This time constant \( \tau \), is measured by \( \tau = \frac{L}{R} \), in seconds, were R is the value of the resistor in ohms and L is the value of the inductor in Henries. This then forms the basis of an
RL charging circuit were $5\tau$ can also be thought of as "$5 \times L/R$" or the transient time of the circuit.

The transient time of any inductive circuit is determined by the relationship between the inductance and the resistance. For example, for a fixed value resistance the larger the inductance the slower will be the transient time and therefore a longer time constant for the LR series circuit. Likewise, for a fixed value inductance the smaller the resistance value the longer the transient time.

However, for a fixed value inductance, by increasing the resistance value the transient time and therefore the time constant of the circuit becomes shorter. This is because as the resistance increases the circuit becomes more and more resistive as the value of the inductance becomes negligible compared to the resistance. If the value of the resistance is increased sufficiently large compared to the inductance the transient time would effectively be reduced to almost zero.

From the above graph it is clear that as resistance R increases, the curves become steeper, indicating that the time constant is decreasing which means that the steady state condition (i.e. zero inductor voltage) is achieved at a much faster rate. When resistance becomes very high, the time constant becomes almost negligible.

**Expected Inductor voltage vs time characteristics:**

Graph representing the variation of inductor voltage with time for different resistance values:

Model simulated on MATLAB Simulink to verify the concept
Results:

1. $R = 500 \text{ ohm}$

2. $R = 1 \text{ K}$

3. $R = 2 \text{ K}$

4. $R = 3 \text{ K}$

5. $R = 5 \text{ K}$

6. $R = 20 \text{ K}$
Conclusion from simulated results:

The theoretical concepts discussed are verified by simulating the above model in MATLAB-SIMULINK. It can be seen from the simulated results, that as value of resistance $R$ increases, the curves become steeper i.e the time constant decreases. It means that zero inductor voltage is achieved at a much faster rate with higher resistance values. The results are more explicit when we compare $R=500$ ohm and $R=20K$ ohm values.

References


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