Mathematical Theory of Tangent Bisector Iterative Integration Method for Iterative integration

Mayur Rajendra Waghchoure
Former Associate Manager ,
Commercial vehicle business unit, Tata Motors Ltd.,
Pune, India[1].

***Abstract* - In this present era of computers, we mostly rely on software packages to solve real life problems. All these software packages are best repleted with numerical methods to solve problems in iterative approach. As integration is a process of summing up all data element together and as numerical approaches haven't developed enough to get rid of errors while arriving at a solution because methods available can approximate the curve up to certain degrees only. So to develop generalized approach to solve integration, I have developed methodology that shall make use of tangents to approximate the curve. At the same time bisector drawn at intersection of tangents is used in such way that convergence is ensured. I have studied and solved different problems using this methodology and also developed different versions of it to solve them. I have also enlisted the scope, limitation and the condition of this new methodology.**

To conclude, I can say that this methodology will give convergence over 90% at the end of first iteration for solving mathematical integration and relatively give faster convergence than other methods for any degree of curve.

Keywords: Applied mathematics, Integration, Integral, Curve plotting, MATLAB, Tangent and bisector.

1. INTRODUCTION

As we know that, integration is process of finding area under the curve y=f(x) and which given by as follows

$$Y=\int_{x1}^{x2} f(x).dx$$

Integral of any function ever calculated by direct integration that is with the help of standard formulae directly and it may be following analytical or approximation approach for getting integral. Previously in existing methods, we were using data points which itself an localize property; hence data points will not approximate the curve accurately by interpolation due to presence of interpolation error. In existing method data points were interpolated over first, second or third degree curve and when higher degree curve is followed by data points then error will induce. As this method are either not generalized or not able to bear all kind of function given by data points in numerical integration problem. In previous existing method, we are not drawing tangent to curve at data points.
Tangent bisector iterative integration is itself an iterative approach of getting approximate integral and we will get close to actual answer or solution by performing number of iteration. Hence to minimize the error between curve & straight line and also to keep the nature of curve in account, we should place the data points in close affinity to curve; Hence in this method we are placing data points such that it should represent nature of curve as well as on joining by straight line it will give trapezoidal quadrature .
In this method to find area under any curve or to integrate any function which may follows any kind of nature, this method is using property of curve at data points in the form of tangents. As tangent is property of curve & it will approximate the nature of curve of any kind; hence any kind of curve mapping or function integration is possible in tangent bisector iterative integration.
Obtain from data points; we can use iterative tangent method.

2. CLASSIFICATION

Integration

A] Direct Integration
1. Definite integral
2. Indefinite integral
3. Numerical integration

B] Iterative Integration
1. Tangents method

3. Tangent Bisector Iterative Integration Method

A]. principle

As numerical integration is method of obtaining area under curve and data points are the localized property and if they are joined by straight line then the curve or relation for data points may be different than first degree polynomial hence error will get induced .hence for minimizing error we can use tangent method.
Tangent is a line parallel to the curve at data point and if tangent are drawn at multiple data points then they will all together approximate the curve in the form of element as straight lines and by finding their point of intersection, we will get new data points .then by applying principle for area of trapezoid, we will get area under curve by summation of all areas.

B]. procedure
Steps:
a) Initialization: To get equation of curve

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1) If there is a problem for which given a set of data points in two variable x and y, such that there will exist some relationship between two variable as y=f(x) and set of data points x may be placed at equal or unequal interval.
2) Firstly, we should draw Newton forward difference table for given data points.
3)from this table, we will get idea about the degree of polynomial that curve y=f(x) will followed as that column of Newton forward difference table will have single nonzero value or may have constant value throughout .
4) Once we get idea about the degree of polynomial, we can find the equation of curve by different method as follows:

a. if column of Newton forward difference table is constant throughout and step size is equal then we can use Newton forward method formulae to find equation of curve

$$y_g=y_0+u\Delta y_0+[\frac{u(u-1)}{2}]\ \Delta^2 y_0+\ldots\ldots$$

b. equation of curve with best fit can be obtain by different method as follows

Method of moments or method of least square, where we can use least square method for polynomial regression of high degree polynomial ,where number of equation in this method should equal to degree of polynomial plus one , we can get degree of curve from Newton forward difference table.

c. Or also we can finding the relationship between x and y, we should use general equation for any degree of polynomial as follows

$$y=a_0+a_1x+a_2x^2+a_3x^3+\ldots\ldots\ldots\ldots+a_nx^n$$

where $a_0,a_1,a_2,a_3,\ldots\ldots\ldots a_n$=constant

5) for the degree, which we got from Newton forward difference table, we should restrict polynomial of n degree to the polynomial of that degree.
6) as for that degree of polynomial ,number of constant in polynomial equation should be equal to the ' degree of polynomial plus one' and hence number of equations required for getting value of that constant is equal to number of constant in equation. e.g. if degree of polynomial is 2 then equation of polynomial should be as follows

$$y=a_0+a_1x+a_2x^2$$

7) equations which are required for getting constant can be obtained from data points and by solving them by gauss elimination method ,we will get value of that constants.
8) After putting that constant value in equation, we will get final relation for data points and which is base for tangent method.

b) Core tangent method:

As base for tangent method is equation of curve and by finding that equation in case of numerical integration problem, we will get all kind of problem on equal base for tangent method.

9)once we get final equation as y=f(x) for curve , then procedure start for tangent method by drawing tangents and for getting tangent we should get slope at that point over the curve and then slope at that points is given as $m=(dy/dx)_{at\ x,y}$.
10)firstly, we should take points at extremities of curve from set of data points and then draw the tangents at a point $(x_0,y_0)$and$(x_1,y_1)$and we have to get their point of intersection as (x_1^1,y_1^1) ,which is given by solving two equation which are as follows

$y_1^1-y_0=m_0(x_1^1-x_0)$
$y_1-y_1^1=m_1(x_1-x_1^1)$

There point of intersection is given as follows for getting point of intersection of tangent 1 and tangent 2.

$$x^1=\frac{(y2-y1)+(m1*x1-m2*x2)}{(m1-m2)}$$
$$y^1=y1+m1*(\ x^1-x1);$$

Then modified for this conditions is as follows

$$X_1^1=\frac{(y1-y0)+(m0*x0-m1*x1)}{(m0-m1)}$$
$$y_1^1=y_0+m_0*(x_1^1-x_0)$$

11)after getting value of as x_1^1 and y_1^1 ,we have to keep the iteration in process till desired accuracy is obtain.
12) for getting next tangent over curve ,we should use angle bisector from point (x_1^1,y_1^1) over the curve ,

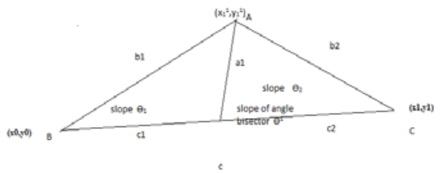

Fig.1. triangle form by intersecting tangent

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Where angle of first tangent $\Theta_1 = \tan^{-1}(m0)$
Angle of second tangent $\Theta_2 = \tan^{-1}(m1)$
$\Theta^1 = (\Theta_1 + \Theta_2)/2$, $m^1 = \tan \Theta 1$
Important conditions regarding angle bisector: As when slope of two tangents is obtain, let it have any combination like both slope positive or both slope negative or one slope is positive and other one is negative
Then for any single combination of slopes, angle made by angle bisector will always have two options , where first is angle bisector lie in same opposite two quadrants in which tangents lie and its angle is given as follows
$\Theta^1 = (\Theta_1 + \Theta_2)/2$, $m^1 = \tan \Theta 1$
And second option will arise when angle bisector diverges away from curve then angle made by angle bisector will lie in another two quadrant and its angle is obtain by
$\Theta^{11} = \Theta^1 + 90$, $m^1 = \tan \Theta 1\ 1$
In these way when two tangent are intersecting each other at some points and if at that point angle bisector are drawn then we will get two possibilities of angle bisector slopes or four angle bisector in four direction, we have to choose right one angle bisector to continue further operation out of that four angle bisector based on criterion that "whether any angle bisector is not touching curve or touching curve out of limits of integration or covering same region which is covered earlier".
If all possible four angle bisector are cutting the curve at some uncovered region of curve then these all angle bisector should be drawn independently at a same time over a curve and draw their respective tangents at that points and finally find point of intersection of all tangents and this whole operation fall under single iteration of process.
13) Hence equation for angle bisector should be as follows
$y - y_1^1 = m^1(x - x_1^1)$
14) As we know the equation of the curve y=f(x), we should find point of intersection of angle bisector and curve, which will give data point (x_2, y_2).
where length of angle bisector is as follows
$h = ((y_2 - y_1^1)^2 + (x_2 - x_1^1)^2)^\wedge(1/2)$

15)then we draw the third tangent at point $(x_2, y_2)$and which intersect the tangent 1 and tangent 2 at point (x_2^1, y_2^1) and (x_3^1, y_3^1) respectively.
16) Point (x_2^1, y_2^1) can be obtain by finding point of intersection of tangent 1 and tangent3

$y_2^1 - y_0 = m_0(x_2^1 - x_0)$

$y_2 - y_2^1 = m_2(x_2 - x_2^1)$

Finally point of intersection is given as above used formula.
17)point (x_3^1, y_3^1)can be obtain by finding point of intersection of tangent 2 and tangent 3 and second branch proceed further by iteration method.
18) In these way ,first draw two tangent at extremities ,then finding their point of intersection ,then draw angle bisector and draw third tangent at which curve intersect angle bisector ,then find point of intersection of tangent 1 and tangent3 and also point of intersection of tangent 2 and tangent3.by completing above step we will get first iteration finished .
19)then considering tangent 1 and tangent3 as one branch and tangent 2 and tangent 3 as second branch ,we should continues iteration further till desired accuracy is obtain.
20)In similar way number of tangents can be drawn over the curve and after finding their point of intersection we will get data points which will approximates the curve by joining number of tangent together and finally it will minimizes the error.
21) An iterative tangent bisector approach can be used for iterative integration and iterative curve plotting as final points of intersection of all tangents and points of intersection of all angle bisector with curves are obtain by iterative tangents method.
a) Iterative integration
22) From all points of intersection of tangents with each other, we are able to find area under any curve or integration for any curve by summation of area of variable base width trapezoid.
23) After that basic principle for area of trapezoid for new points of intersection of tangent as follows
$A = [\frac{(x1 - x0)}{2}][y_0 + y_1]$
24) Finally we will take summation of all trapezoid, which will give complete area under curve y=f(x)
$AREA = \sum A$
b) Curve plotting
25) Generally, when we are given with some sort of data points or any sort of equation and we have to draw the curve for same with finite accuracy and surety, at that time iterative tangent method will best approach to get it done.
26) From points of intersection of all tangents and points of intersection of all angle bisector with curve, we are able to plot the curve which follows nature of curve by joining all points by straight line and it will give exact nature of curve as it is tangent driven approach with minimum number of data points.
27) As it is tangent driven approach and tends to convergent at each point; hence it will gives all points only on curve or in close affinity to curve and with minimum points curve can be plotted with finite surety and accuracy in any software by conversion of non parametric equation to parametric form.
28) This method will give good global control over curve plotting as it uses tangents to be drawn all over the curve for any degree of curve.

C]. Methods

For getting accuracy and high degree of convergence, we can use four methods as follows
a) General iterative tangent method
b) Iterative tangent method with regular stepping
c) Graphical iterative tangent method
d) considering error as per degree of curve

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a) general iterative tangent method:

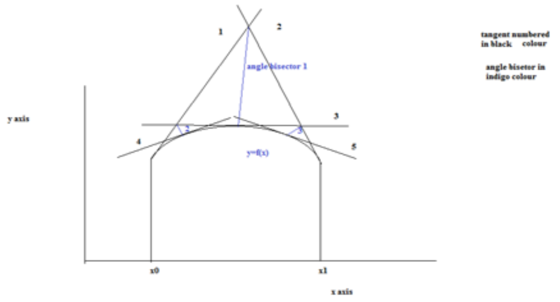

Fig.2. iterative tangent method principle

1) This method found to be more suitable for low degree polynomials and degree of convergence is fast but large number of iteration is required to be performed.
2) As explained above, tangents are drawn all over the curve, so that they will approximate the whole curve.
3) By above procedure we have drawn tangent all over the curve iteration by iteration and get their point of intersection, finally from that point we can find area under curve as explained in procedure.
4) There are chances of divergence for complex curve if angle bisector do not touch curve anywhere or in between limits of curve for which integration is to be find, then in this case we should proceed by drawing tangent at midpoint for that iteration or we may proceed to use iterative tangent method with stepping.
b) Iterative tangents method with regular stepping:

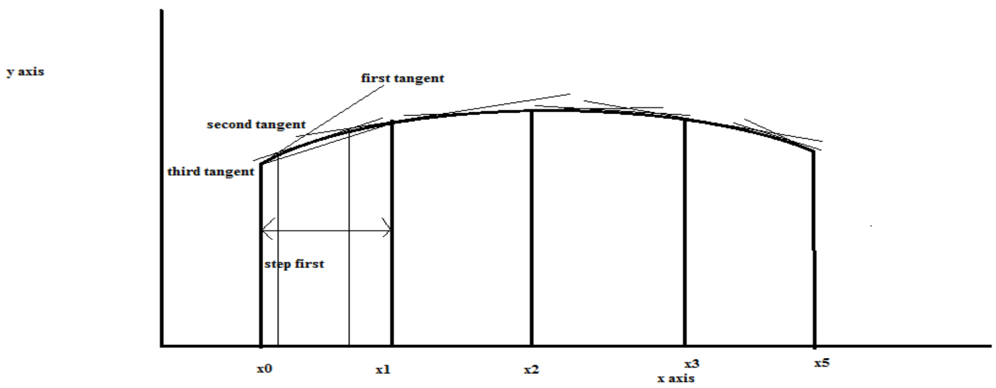

Fig.3.Iterative (three) tangent method with stepping

1) These method is generally for high degree polynomial, for getting very high accuracy and get good convergence.
2) In these method divide curve along x axis into equal or unequal interval, at all given data points tangents are drawn and in between each two data points two tangent intersect with each other, then by angle bisector procedure draw third tangent over the curve and find the point of intersection between tangent 1 and tangent3 and also between tangent 2 and tangent 3.
3) After getting these three points between two fixed points over the curve, we should find area for that single step by basic principle of area of trapezoid. This process is further proceeded to get very high accuracy with increased iteration and for each step.
4) After getting summation of area of all steps, we will get whole area under the curve given by y=f(x).
C) Graphical iterative tangent method:

As per above procedure of general iterative tangent method or iterative tangent method with stepping, instead solving problem analytically; we can also solve problem graphically with more ease. Once tangents are drawn all over the curve ,we just have take out points of intersection of tangents and by using formula for area of trapezoid, we can get area under the curve.
D) Considering error as per degree of curve:

1) In previous method, data points are joined by straight line and area under the straight is given by integration is as follows

Area1=$\int_{x1}^{x2} x\,dx$

2) But curve may be of higher degree than single and hence area under that curve by integration is as follows

Area2=$\int_{x1}^{x2} (x.x)\,dx$

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3) Error in area calculation is equal difference in area under curve and area under straight line; hence error is given as follows
Theoretical error=Area2-Area1
4) As both line and curve involving some sort of constant into the equations hence for getting actual error, theoretical error should be multiplied by some constant and hence actual error is as follows
Actual error= K*(theoretical error)

c. Area under the curve is equal to sum of area1 and actual error based on slope of line

Area = area1 ±actual error

D].Mathematics (fig.1)

a) When only one slope is known initially then point of intersection is given directly as follows:

In above procedure, if angle bisector is act as median over straight line joining two data points or over third side of triangle ABC.
Where in triangle ABC sides are,

$AB=b_1=[(x_1^1-x_0)^2+(y_1^1-y_0)^2]^{(1/2)}$

$AC=b_2=[(x_1^1-x_1)^2+(y_1^1-y_1)^2]^{(1/2)}$

$BC=c=[(x_1-x_0)^2+(y_1-y_0)^2]^{(1/2)}$

and $c_1=c_2=BD=DC=(c/2)$
Then by applying cosine rule and using principle for angle between two tangent, finalize equation is as follows

$\cos\tan^{-1}[(m0-m)/(1+m0\times m)] = [(7/4)(x_0^2+y_0^2)+(3/4)(x_1^2+y_1^2)-(5/2)[(x_1*x_0)+(y_1*y_0)]-[(x_1-x_0)*(1+m_0m)*x_1^1]]/[(x_1-x_0)^2*(1+m_0^2)*(1+m^2)*(x_1^1-x_0)^2]$

By solving this equation we will get value of x_1^1 and then we will easily get value of y_1^1; hence new point can be plot easily.

b) When both the slopes are known initially then point of intersection of tangent is given directly as follows

In above procedure, if angle bisector act as median then finalize formula for getting point of intersection is as follows

$\cos[\Theta 2-\Theta 1)]= [x_1-x_1^1)^2*(1+m_1^2)+(1+m_0^2)*(x_1^1-x_0)^2 - [(x_1-x_0)^2+(y_1-y_0)^2]]/[2*(x_1-x_1^1)^2*(1+m_0^2)*(1+m_1^2)*(x_1^1-x_0)^2]$

By solving this equation we will get value of x_1^1 and then we will easily get value of y_1^1; hence new point can be plot easily

E].Numerical

E.g. Problem of numerical integration:

X	1	2	3	4	5
Y	1	4	9	16	25

Solution: by tangent method

Step1: find degree of equation by Newton forward difference table

X	Y	Δ y	Δ ^{2}y	Δ ^{3}y
1	1	3	2	0
2	4	5	2	0
3	9	7	2	-
4	16	9	-	-
5	25	-	-	-

As Δ ^{2}y column of table is same; hence above data points should follows second degree polynomial equation.
Step 2: find equation of second degree polynomial

$y=a_0+a_1x+a_2x^2$

Hence for finding three constant, number of equation should be equal to three and obtain these equations from data points as follows

$y=a_0+a_1x+a_2x^2$
$1=a_0+a_1*1+a_2*1$
$y=a_0+a_1x+a_2x^2$
$4=a_0+a_1*2+a_2*4$
$y=a_0+a_1x+a_2x^2$
$25=a_0+a_1*5+a_2*25$

By solving these three equations by crammers rule or by gauss elimination method, we get constant as follows

$0=a_0$
$0=a_1$
$1=a_2$

Then equation of curve which will fit for data points is as follows

$y=x^2$

step3: find derivative or slope equation of curve

$m=(dy/dx)_{at\ x,y}$.
As m=2*x

Iteration 1:
Step4: draw tangent 1 and tangent 2 and find their point of intersection as follows
Take initial points as follows A(1,1)and B(5,25)
Draw tangents, their slopes are $m_0=2*1=2$ and $m_1=2*5=10$

$y_1^1-y_0=m_0(x_1^1-x_0)$
$y_1-y_1^1=m_1(x_1-x_1^1)$

And equations are

$y_1^1-1=2(x_1^1-1)$
$25-y_1^1=10(5-x_1^1)$

Solving these two equations we will get point of their intersection as $(x_1^1,y_1^1)=(3,5)$
Then draw angle bisector
Where angle of first tangent
$\Theta_1=\tan^{-1}(m0)=63.434$
Angle of second tangent
$\Theta_2=\tan^{-1}(m1)=84.2894$

$\Theta^1 = (\Theta_1 + \Theta_2)/2 \; =73.8621$, $m^1 = \tan \Theta 1 = 3.456$

13) Hence equation for angle bisector should be as follows

$y-y^1=m^1(x-x^1)$
$y-5=3.456(x-3)$

And point of intersection of curve with angle bisector can be given by putting value of y in equation of curve $y=x^2$
On solving we get
$(x_2,y_2)=(1.728,2.985984)$

Again draw third tangent at that point, which has equation as follows

$m_2=2*x=2*1.728=3.456$
Then equation is
$y=3.456x-2.985984$

a)then point of intersection of tangent 1 and tangent 3 is at point (x_2^1,y_2^1) and obtain by solving equation of tangent 1and3,which is
Point $(x_2^1,y_2^1)=(1.364,1.728)$
b)then point of intersection of tangent 2 and tangent 3 is at point (x_3^1,y_3^1) and obtain by solving equation of tangent 2 and 3,which is
Point $(x_3^1,y_3^1)=(3.364,8.64)$
Hence in this way, first iteration is completed and avail data points are as follows

point (x_0,y_0)	(1,1)
point (x_2^1,y_2^1)	(1.364,1.728)
point (x_3^1,y_3^1)	(3.364,8.64)
point (x_1,y_1)	(5,25)

Hence area under curve is given by putting value in equation for area of trapezoid as follows

$AREA=[(x_2^1-x_0)/2][y_0+y_2^1]+ \; [(x_3^1-x_2^1)/2][y_2^1+y_3^1]+ \; [(x_1-x_3^1)/2][y_3^1+y_1]$

After putting value in above equation, we will get area under curve after first iteration is equal to
AREA=38.382016
ITERATION 2:
Repeat same procedure as step 4 from points (x_3^1,y_3^1) and (x_2^1,y_2^1)
Points after second iteration, we get point of intersection of tangent 4 with tangent 1 and 3 and tangent 5 with tangent 2and 3 are as follows

point (x_0,y_0)	(1,1)
point (x_4^1,y_4^1)	(1.13957,1.279)
point (x_5^1,y_5^1)	(1.5034,2.2099)
point (x_6^1,y_6^1)	(2.2761,5.0819)
point $(x71,y_7^1)$	(3.79526,12.9526)
point (x_1,y_1)	(5,25)

Area after second iteration, by formula of trapezoid and summation we will get as
AREA=40.17106336
In this way as iteration proceed; we will get closer to actual answer, where actual answer for above problem by integration for confirmation is equal to 41.33.

In this way, very high degree polynomial with unknown equation or equation which is difficult to integrate can be solve by iterative tangent method.
It can be used for very high degree of polynomial, where other method only suitable up to certain degree of polynomials. E.g. Simpsons (1/3) rule only use for second degree polynomial with great accuracy.
**Data points which are obtain after iteration 2 can be use for curve plotting by just join them by straight line and accuracy can be increased with increasing number of iterations.

F]. Scope
1. As in this method, we are finding approximate equation of curve and then by tangent driven approach, we can find area of curve more accurately.
2.We can get area of curve of high degree of polynomial with less number of iteration (3 tangent iteration) as compare to number of steps by trapezoidal or Simpsons method, where gauss Legendre method can also handle only curve up to five degree.
3.Area under curve for low degree polynomial can be find more accurately and with less error by incorporating three tangent (one iteration) between two points when solve by regular stepping.
4. There are some mathematical functions which cannot easily solve by integration and hence for solving such equation with great accuracy tangent method can be used. Also for minimizing gap between straight line and curve obtain from data points; we can use iterative tangent method.
5. In this way, this is generalizing procedure of integration or to find area under any curve with great accuracy, with good convergence for any nature of curve or for any function of mathematics.
6) Area under any curve can be finding without knowing equation of curve by graphical tangent method approach.
7) As we are drawing tangent or straight line which is first degree polynomial at data point to consider nature of curve and in the same way higher degree polynomial curve can be draw at data points and continue further by initial procedure accuracy obtain will be to good.

G].Limitations
generally, limitation of this method will be only divergence otherwise tangent property will always execute nature of curve and by using angle bisector approach we are always try to stabilize convergence of procedure, hence limitation for this process are as follows:
1.if accurate point of cut for angle bisector with curve is not choose correctly, then procedure accuracy may get damaged.
Reason: if point of cut of angle bisector with the curve is not first and it is second or third point of cut of angle bisector with curve then definitely it will not converge process as from next choose point we will draw third tangent to the curve and if that next point is not in between two points from which tangents 1 and 2 are drawn ,then definitely third tangent will out of that two points; hence third tangent is not able to minimize gap between tangents and curve which we want to minimize iteration by iteration

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and approximate whole curve with tangents to get accuracy .

Solution: we can choose first point of cut of angle bisector with curve only by use of distance formula over the angle bisector, we have to choose point out of all points at which angle bisector cut the curve with distance formula such that distance of that point of cut over the curve should be minimum from point of intersection of two tangents.

2.if curve have more than one maxima or sudden peaks in its nature then procedure may take less iteration and accuracy will be good too but it make procedure too complex and sometime it may diverge process too.

Solution: to avoid problem of complexity and divergence of procedure we should follow stepping over the curve in direction of axis which represents the independent quantity over it or either follows the iterative tangent method with stepping for such problems.

3. from point of intersection of tangents, we are able to draw two angle bisector at two different angles and we may choose any one of them if second angle bisector is not touching the curve or not in between extreme points and if we choose wrong angle bisector then we may diverge out of that points from which two tangents are drawn and process get diverge.

Solution: we should draw angle bisector such that it has point of cut over the curve in between only two points from which tangents 1 and 2 are drawn, we should follow above rule if both the angle bisector are cutting the curve and we have to choose either of them.

H].Conditions

1. This process for any branch of iteration continues till two tangents coincide with each other or till having ill condition or we can apply accuracy criterion to slopes to increased speed and accuracy of process and at two points for any iteration if two tangents are parallel, then process should be continued by drawing tangent at midpoints of these two points.

2. If angle bisector cuts high degree curve at more than one point then for procedure first point of cut should be consider for further iteration and point should be find by minimum distance formula from point of intersection to that point of cut with the curve.

3. Number of points after all iteration is obtain from relation as

Number of points= $(2)^{\text{number of iteration}}+2$

E.g. for 5 iteration, points should be 34 in number

Number of points $=(2)^5+2=34$

4. Number of tangent after all iteration is given by relation as follows

Number of tangents= $(2)^{\text{number of iteration}}+1$

Number of tangents=number of points-1

5. For low degree polynomial curve general iterative tangent method preferred but for high degree curve or for spline curve we must preferred iterative tangent method with stepping.

6. When curve intersect at x axis then we should introduce point (x,0) in between two data points.

7.when angle bisector is not touching the curve anywhere or in between limit of integration or when root of equation are imaginary which lead to divergence and to avoid that we should further proceed by drawing tangent at their midpoint or we must preferred stepping along x axis.

8. When tangent are parallel to each other, then mid-point in x or y data point has to choose and in x direction it is obtain as follow

$x_m=(x_1+x_2)/2$

$y_m=f(x_m)$

Then draw tangent at point (x_m,y_m).

9. When we have to find area under close curve above the axis line then for avoiding interference of tangent we should shift curve along respective axis depend on symmetricity of curve otherwise shift it along y axis.

$y= f(x)$

$(\frac{dy}{dx}) = f^1(x) =0$

$(\frac{d^2y}{dx^2}) = f^2(x)$ = negative =maximum=y (max)

If positive =minimum=y (min)

$(h^1)=(y(max)+y(min))/2$

Shift whole system by

$(h)= y-h^1$

Then apply tangent method by drawing tangent along the curve and choose point of intersection of angle bisector with only positive y or negative y as per side about x axis.

Then add those two area together to get area enclose by that curve.

10. Important conditions regarding angle bisector: As when slope of two tangents is obtain, let it have any combination like both slope positive or both slope negative or one slope is positive and other one is negative

Then for any single combination of slopes, angle made by angle bisector will always have two options , where first is angle bisector lie in same two quadrants in which tangents lie and its angle is given as follows

$\Theta^1= (\Theta_1 +\Theta_2)/2$, $m^1=\tan \Theta 1$

And second option will arise when angle bisector diverges away from curve or angle bisector do not intersect the curve or if root of equations are imaginary then angle made by angle bisector will lie in another two quadrant and its angle is obtain by

$\Theta^{11}= \Theta^1+90$, $m^1=\tan \Theta 1 1$

11. From any point of intersection of two tangents always arises two possibilities of angle bisector at angle Θ^1 and at angle Θ^{11} ,then during the process both the angle bisector are touches the curve then choose only those angle bisector which will converge the process by cutting curve in between two points from which tangents 1 and 2 are drawn and also we can proceed further in process by drawing tangent at a time at both the possible angles ,if both of them are covering different enclosed region in outermost extreme points and whichever not cover before.

12. If curve contain more than one maxima or complexity in the nature then it may possible that procedure may diverge by drawing tangent at wrong position or may take

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more iteration to complete ;hence above problem can be solved by iterative tangent method with stepping.
13. If tangents drawn over the curve are not able to execute whole nature of curve then we must follow iterative tangent method with stepping or increase number of steps in iterative tangent method with stepping instead of having just general iterative tangents method.
14. Accuracy of this process can be check at each instant by finding distance between point of intersection of tangent and point at which angle bisector cut curve or length of angle bisector, for each iteration as follows
Where length of angle bisector is as follows

h=((y$_2$-y$_1^1$)^2+(x$_2$-x$_1^1$)^2)^(1/2)

Accuracy criterion
h ≤ accuracy
15. In this way accuracy of integration process can be obtain with iterative tangent method with regular stepping and also in this case half iteration to number of iteration can be performed and can make closer approach to accurate answer.
16. "Iterative triangle tree" for iterations should be as follows

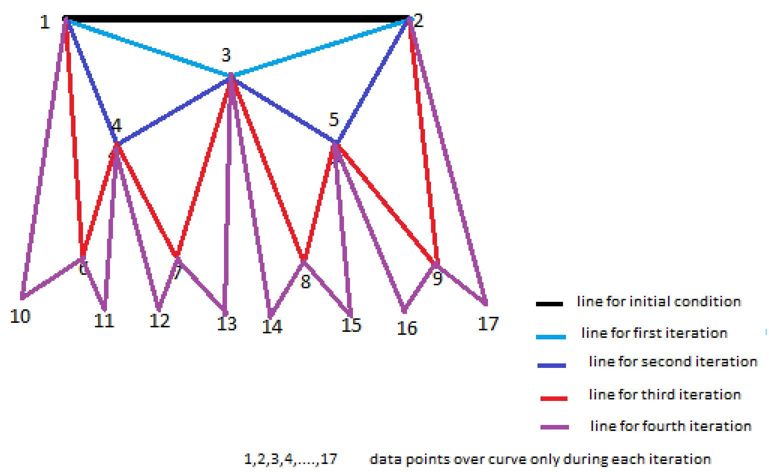

Fig.4 Iterative triangle tree

For first iteration data points over curve (x_0,y_0) ,(x_1,y_1) & (x_2,y_2) are point 1,2 and 3 in above diagram respectively and point 1 and 2 will give point of intersection of tangent 1 and tangent 3 and point 2 and 3 will give point of intersection of tangent 2 and tangent 3 respectively.
For second iteration process will pass through point 1, 4,3,5,2 and give point of intersection of further tangents and get respective point to obtain area under curve by basic principle of tangent method.

4. CONCLUSIONS

1. Conclusive steps of tangent method as follows
A. finds equation of curve or start with equation of curve.
B. find first derivative of equation of curve.
C. find slope of curve at various points on curve.
D. draw tangents at all points over curve by any of any tangent method as mentioned above.
E. finds point of intersection of all tangents as per process flow along independent axis of curve.
F. draw angle bisector over curve with appropriate conditions follow to avoid divergence of process as mention in procedure point (12) and find intersection with curve and then draw next tangent over curve at that point for iterative approach within two tangents domain and continue iteration further.
G. applies basic formula for area of trapezoid to that all data point which will approximate nature of curve.
H. sums all area together to get area under the curve.

2. In these way, by tangent method while finding area we can consider points on high degree polynomial as compare to over straight line or first degree polynomial and also try to join points on curve by straight line using tangent by iterative approach; hence we are taking care of degree of curve that data points has followed, which improve accuracy of method. As we are tracking each nature of curve by approximating curve properly with tangents; hence we are able to find area under or enclosed by any curve with suitable changes in procedure of tangents method.
3. This method will give very high accuracy with more degree of convergence; we can use computer software to avoid cumbersome procedure and iterations for problem solving.
4. There are some mathematical functions which cannot easily solve by integration and hence for solving such equation with great accuracy tangent method can be used. Also for minimizing gap between straight line and curve obtain from data points; we can use iterative tangent method.

5. As tangent is property of all kind of curve; hence tangents over the curve are able to approximate nature of any curve of any degree, similarly multiple tangents over the curve execute nature of curve and minimize area gap between curve and tangents over the curve, also difficult mathematical equation can be integrate easily; hence

iterative approach of integration by using tangent method provide simple generalize method for integration and also try to remove all drawbacks of previous methods of integration as mention above.

6. Iterative integration by using tangent method serves to be a generalize method for integration with acceptable accuracy, good convergence and capacity to solve all problems by various approaches with simple procedure for any nature of curve or for any function of mathematics.

7) Any degree of curve can be accurately plotted with this method and accurate curve can be obtained by increasing number of iterations by iterative tangent method.

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