Mathematical Representation Of The Marine Current Rotor Efficiency

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Abstract

Oceans represent enormous source of energy. The most important type of its energy is energy of the tidal current. The conversion of tidal energy into electrical energy represents one of the most interesting renewable energy resources. The tidal current power, mathematically, can be represented as well as wind energy. In that expression the certain coefficient, known as rotor efficiency, is characteristic. That coefficient depends on the rotor blade geometry, and of water speed. In this paper is made mathematical description of that coefficient for concrete tidal turbine. Using a least square method, the best interpolating functions which described that coefficient are obtained. The obtained result can be very useful for tidal power analysis, prediction and simply description.

1. Introduction

Interest in renewable energy in the world has increased continually over the past decade [1]. This interest is due primarily to security of supply issues and the effects of climate change. Access to energy sources is essential for economic development [2-5]. Ocean energy has an advantage over other renewable energy sources as it is predictable over long time scales [6-7].

Oceans cover more than 70% of Earth's surface. He is an enormous source of energy. It is estimated that 0.1% of the energy in ocean waves could be capable of supplying the entire world's energy requirements five times over. Currently, a number of technologies aimed at harnessing this potential have been investigated and are at different stages of development including tidal and marine energy, wave energy, and difference of temperature and salinity energy [8].

Ocean energy (Marine energy) or ocean power (marine power) refers to the energy carried by ocean waves, tides, salinity, and ocean temperature differences. The conversion of tidal energy into electricity has been widely investigated and can be compared to the technology used in hydroelectric power plants. In fact, electricity is generated by water flowing into and out of gates and turbines installed along a dam or barrage built across a tidal bay or estuary [9].

Marine current energy is one of the most interesting renewable and clean energy resources that have been less exploited respect to wind energy. The first large-scale tidal power plant (the Rance Tidal Power Station - France) started operation in 1966. Only in Europe this type of energy is available for 75 millions of kW and in terms of exploitable energy the amount is about 50 milliards of kWh. In the last years, the realization of horizontal axis turbine for the exploitation of the tidal currents is having, to world-wide level, a considerable increment [8-15].

The marine power can be represented using a simple equation, in which is dominant and characteristics a certain coefficient, known as rotor efficiency. This coefficient depends on rotor blade geometry, and in many cases in manufacturer datasheet is given as a function of the ratio of the downstream to the upstream water speeds. In this paper will be done mathematical description of that coefficient, what is very important for any marine power analysis. In the first part of this work, the marine current description will be given. After that, the main equations which describe marine power will be presented, as well as the description of power efficiency.

2. Generation of tidal energy [9-10]

Tidal power is the only form of energy which derives directly from the relative motions of the Earth–Moon system, and to a lesser extent from the Earth–Sun system. Tidal forces produced by the Moon and Sun, in combination with Earth's rotation, are responsible for the generation of the tides.

Tidal energy is extracted from the relative motion of large bodies of water. Periodic changes of water levels, and associated tidal currents, are due to the gravitational attraction of the Sun and Moon. Magnitude of the tide at a location is the result of the changing positions of the Moon and Sun relative to the Earth.

The tidal cycle can be approximated by a double sinusoid; one with a period of 12.4 h representing the diurnal tidal ebb and flow cycle, and the other with a period of 353 h representing the fortnightly spring neap period. The following equation provides a reasonable model for predicting the velocity V of a tidal current:

$$V = \left[K_0 + K_1 \cdot \cos\left(\frac{2\pi t}{T_1} \right) \right] \cdot \cos\left(\frac{2\pi t}{T_0} \right), \qquad (1)$$

where K_0 and K_1 are constants determined from the mean spring peak and the ratio between the mean spring peak and the mean neap peak currents, T_1 is the spring neap period (353 h) and T_0 is the diurnal tidal period (12.4 h), and t is time [6-7]. The tidal current speed vs. time, for K_0=2 and K_1=1, is presented on the Fig. 1.

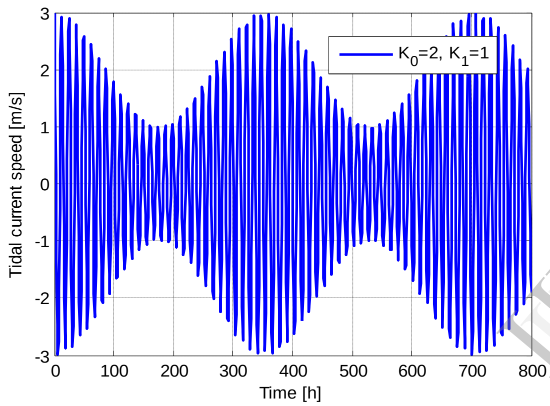

Figure 1. Tidal current speed vs. time

2.1 Generating methods

Tidal power can be classified into three generating methods:

- ✓ Tidal stream generator,
- ✓ Tidal barrage,
- ✓ Dynamic tidal power.

A tidal stream generator is a machine that extracts energy from moving masses of water, or tides. These machines function very much like underwater wind turbines, and are sometimes referred to as tidal turbines. Tidal stream generators are the cheapest and the least ecologically damaging among the three main forms of tidal power generation.

A Tidal barrage is a dam-like structure used to capture the energy from masses of water moving in and out of a bay or river due to tidal forces. Instead of damming water on one side like a conventional dam, a tidal barrage first allows water to flow into the bay or river during high tide, and releasing the water back during low tide. This is done by measuring the tidal flow and controlling the sluice gates at key times of the tidal cycle. Turbines are then placed at these sluices to capture the energy as the water flows in and out.

Dynamic tidal power or DTP is a new and untested method of tidal power generation. It would involve creating large dam like structure extending from the coast straight to the ocean, with a perpendicular barrier at the far end, forming a large 'T' shape. This long T-dam would interfere with coast-parallel oscillating tidal waves which run along the coasts of continental shelves, containing powerful hydraulic currents. The detailed descriptions about tidal currents generating methods are presented in [9].

2.2 The expression for tidal current power

The kinetic energy (E_k [J]) in water of mass "m" moving with speed V [m/s] is given by the following in SI units:

$$E_k = \frac{1}{2} m V^2 \qquad (2)$$

The power (P [W]) in moving air is the flow rate of kinetic energy per second. Therefore:

$$P = \frac{1}{2} \{ mass\_flow\_per\_second \} V^2 \qquad (3)$$

where ρ is water density [kg/m^3], A is area swept by the rotor blades [m^2]. The actual power extracted by the rotor blades is the difference between the upstream and the downstream tidal powers. Therefore,

$$P = \frac{1}{2} \{ mass\_flow\_per\_second \} \left(V^2 - V_0^2 \right) \qquad (4)$$

where V is upstream water velocity at the entrance of the rotor blades, V_0 is downstream water velocity at the exit of the rotor blades. The water velocity is discontinuous from V to V_0 at the "plane" of the rotor blades in the macroscopic sense. The mass flow rate of water through the rotating blades is, therefore, derived by multiplying the density with the average velocity. That is:

$$mass\_flow\_per\_second = \rho \cdot A \cdot \frac{V + V_0}{2} \qquad (5)$$

The mechanical power extracted by the rotor, which is driving the electrical generator, is therefore:

$$P_0 = \frac{1}{2}\left[\rho \cdot A \cdot \frac{V+V_0}{2}\right]\left(V^2 - V_0^2\right), \quad (6)$$

or,

$$P_0 = \frac{1}{2}\cdot\rho\cdot A\cdot V^3 \frac{\left(1+{V_0}/{V}\right)\left(1-\left({V_0}/{V}\right)^2\right)}{2} \quad (7)$$

The power extracted by the blades is customarily expressed as a fraction of the upstream wind power as follows:

$$P_0 = \frac{1}{2}\cdot\rho\cdot A\cdot V^3\cdot C_p$$

$$C_p = \frac{\left(1+{V_0}/{V}\right)\left(1-\left({V_0}/{V}\right)^2\right)}{2} \quad (8)$$

The C_p is the fraction of the upstream wind power, which is captured by the rotor blades. The factor C_p is called the power coefficient of the rotor or the rotor efficiency [1, 15]. For a given upstream wind speed, the value of C_p depends on the ratio of the downstream to the upstream water speeds, that is (V_0/V).

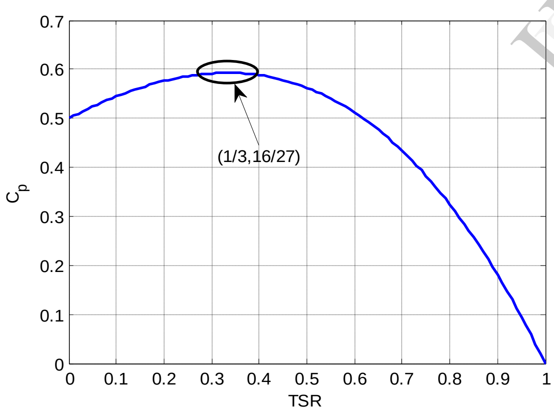

Figure 2. C_p vs. TSR

The ration of the downstream to the upstream water speeds is known as a tip-speed ration (TSR).

$$TSR = \frac{V_0}{V} \quad (9)$$

This equation can be represented as a

$$TSR = \frac{V_0}{V} = \frac{R\cdot\omega}{V}, \quad (10)$$

where R and ω are the rotor radius and the angular speed, respectively. For a given wind speed, the rotor efficiency C_p varies with TSR

The plot of power coefficient versus (V_0/V) shows that Cp is a single, maximum-value function (Fig. 2). It has the maximum value of 0.59 when the (V_0/V) is one-third. The theoretical maximum value of C_p is 0.59. In practical designs, the maximum achievable C_p is below or less than 0.5.

3. Mathematical representation of the marine current power efficiency

In this paper, the Cp vs TSR relation for concrete marine turbine (uniform flow=3.08 m/s for diameter 20), are taken form [15]. This dependence is presented in Table I, and on a Fig. 3. As it can be seen, this dependence doesn't have the form of the ideal Cp characteristics.

The mathematical description of this dependency is very important for marine power analysis, its better understanding and simple calculation.

Table I: C_p vs TSR (appears in [15]), for tidal turbine with diameter of 20m, and with average speed of 3.08m/s

C_p	TSR	C_p	TSR
0.035	0.65	0.39	3.88
0.135	1.3	0.38	4.2
0.2	1.62	0.364	4.51
0.275	1.93	0.31	5.15
0.355	2.59	0.27	5.51
0.374	2.91	0.16	6.15
0.4	3.6	0.08	6.5

Using the least square method [16], we are proposed different interpolation function using which we are make fitting of the experimental result from Table I. The summarized result of this interpolation procedure is presented in the Table II. For example, in the Fig. 4 the graphical presentation of the curve C_{p1}-C_{p4} are presented.

As it can be seen from Table II, the best fitting function, for this experimental results are curve C_{p2}, C_{p8}, C_{p15}, C_{p19}, (also presented in the Figs. 5-7) for different type of the used interpolation function. So, for any proposed interpolating function can be found the best fitting curve of the experimental results.

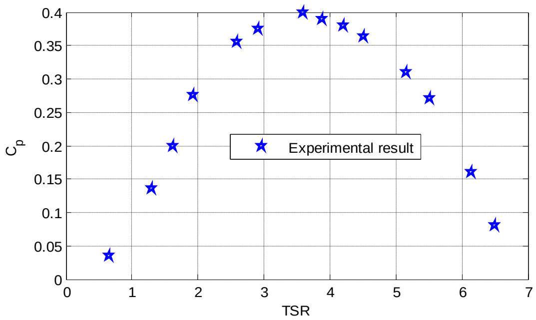

Fig. 3. C_p vs. TSR for marine turbine from [15]

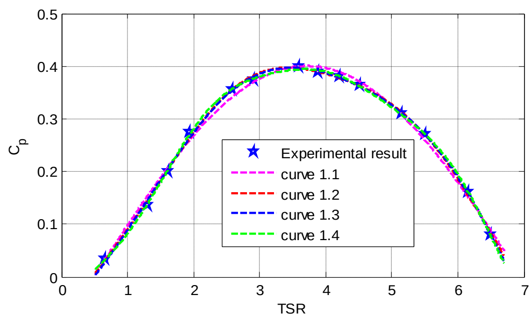

Figure 4. C_p vs. TSR for curve C_{p1}, C_{p2}, C_{p3} and C_{p4}

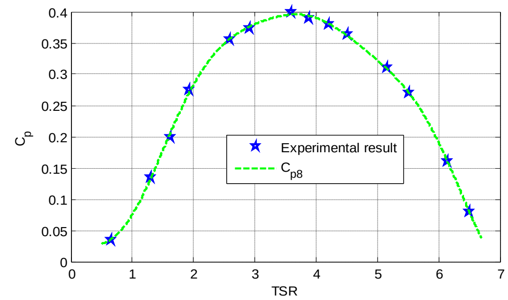

Figure 5. C_p vs. TSR for curve C_{p8}

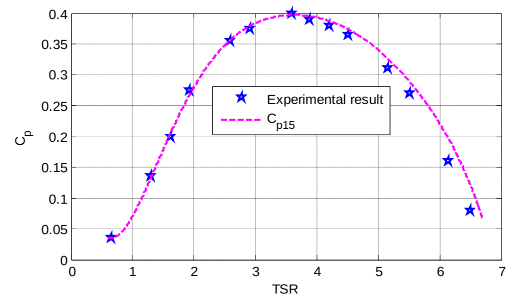

Figure 6. C_p vs. TSR for curve C_{p15}

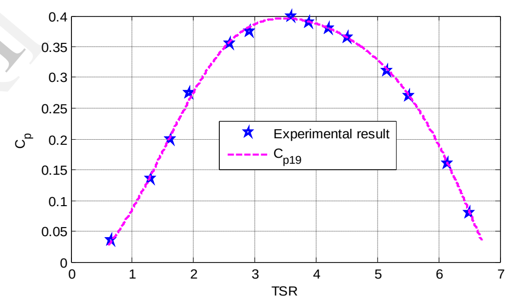

Figure 7. C_p vs. TSR for curve C_{p19}

Table II: The obtained result for different interpolation function (the coefficient λ represent TSR)

$C_{p1}=a_1\cdot\sin(b_1\cdot\lambda+c_1)$	a_1=0,4005;b_1=0.4862; c_1=−0.2399	0.01138
$C_{p2}=a_1\cdot\sin(b_1\cdot\lambda+c_1)+a_2\cdot\sin(b_2\cdot\lambda+c_2)$	a_1=0.4015;b_1=0.4854; c_1=-0.2295;a_2=0.01203;b_2=2.117;c_2=−4.062	0.007707
$C_{p3}=a_1\cdot\sin(b_1\cdot\lambda+c_1)+a_2\cdot\sin(b_2\cdot\lambda+c_2)+a_3\cdot\sin(b_3\cdot\lambda+c_3)$	a_1=0.4381; b_1=0.5127; c_1=−0.2102; a_2=0.05787; b_2=0.7719; c_2=2.828; a_3=0.01723; b_3=1.768; c_3=3.315	0.009565
$C_{p4}=a_1\cdot\sin(b_1\cdot\lambda+c_1)+a_2\cdot\sin(b_2\cdot\lambda+c_2)+a_3\cdot\sin(b_3\cdot\lambda+c_3)+a_4\cdot\sin(b_4\cdot\lambda+c_4)$	a_1=0.4266;b_1=0.4994; c_1=−0.2347; a_2=0.03047; b_2=0.7125; c_2=2.63; a_3=0.01697; b_3=2.194; c_3=1.928;a_4=0.009704; b_4=3.029; c_4=1.782;	0.009148
$C_{p5}=a_0+a_1\cdot\cos(\lambda\cdot\omega)+b_1\cdot\sin(\lambda\cdot\omega)$	a_0=−0.1419; a_1=0.02693; b_1=0.5399; ω=0.4086;	0.01168

$C_{p6}=a_0+a_1\cdot\cos(\lambda\cdot\omega)+b_1\cdot\sin(\lambda\cdot\omega)+a_2\cdot\cos(2\cdot\lambda\cdot\omega)+b_2\cdot\sin(2\cdot\lambda\cdot\omega)$	$a_0=0.2312$; $a_1=-0.1923$; $b_1=-0.02158$; $a_2=-0.03047$; $b_2=-0.01475$; $\omega=0.8735$;	0.007298
$C_{p7}=a_0+a_1\cdot\cos(\lambda\cdot\omega)+b_1\cdot\sin(\lambda\cdot\omega)+a_2\cdot\cos(2\cdot\lambda\cdot\omega)+b_2\cdot\sin(2\cdot\lambda\cdot\omega)+a_3\cdot\cos(3\cdot\lambda\cdot\omega)+b_3\cdot\sin(3\cdot\lambda\cdot\omega)$	$a_0=1.089\cdot10^7$;$a_1=-1.624\cdot10^7$; $b_1=-1.881\cdot10^6$; $a_2=6.373\cdot10^6$; $b_2=1.497\cdot10^6$; $a_3=-1.028\cdot10^6$; $b_3=-3.708\cdot10^5$; $\omega=0.02793$;	0.005729
$C_{p8}=a_0+a_1\cdot\cos(\lambda\cdot\omega)+b_1\cdot\sin(\lambda\cdot\omega)+a_2\cdot\cos(2\cdot\lambda\cdot\omega)+b_2\cdot\sin(2\cdot\lambda\cdot\omega)+a_3\cdot\cos(3\cdot\lambda\cdot\omega)+b_3\cdot\sin(3\cdot\lambda\cdot\omega)+a_4\cdot\cos(4\cdot\lambda\cdot\omega)+b_4\cdot\sin(4\cdot\lambda\cdot\omega)$	$a_0=0.1895$; $a_1=-0.1876$; $b_1=0.1039$; $a_2=0.007384$; $b_2=-0.002735$; $a_3=0.0192$; $b_3=-0.02429$; $a_4=0.00449$; $b_4=-0.01412$; $\omega=0.714$;	0.004944
$C_{p9}=a_0+a_1\cdot\cos(\lambda\cdot\omega)+b_1\cdot\sin(\lambda\cdot\omega)+a_2\cdot\cos(2\cdot\lambda\cdot\omega)+b_2\cdot\sin(2\cdot\lambda\cdot\omega)+a_3\cdot\cos(3\cdot\lambda\cdot\omega)+b_3\cdot\sin(3\cdot\lambda\cdot\omega)+a_4\cdot\cos(4\cdot\lambda\cdot\omega)+b_4\cdot\sin(4\cdot\lambda\cdot\omega)+a_5\cdot\cos(5\cdot\lambda\cdot\omega)+b_5\cdot\sin(5\cdot\lambda\cdot\omega)$	$a_0=-1.129\cdot10^5$; $a_1=1.359\cdot10^5$; $b_1=1.328\cdot10^5$; $a_2=-2549$;$b_2=-1.117\cdot10^5$; $a_3=-3.005\cdot10^4$;$b_3=3.213\cdot10^4$; $a_4=1.045\cdot10^4$; $b_4=-452.7$; $a_5=-849$; $b_5=-765.2$; $\omega=0.2076$;	0.004989
$C_{p10}=p_1\cdot\lambda^2+p_2\cdot\lambda+p_3$	$p_1=-0.04011$; $p_2=0.2983$;$p_3=-0.1605$	0.01229
$C_{p11}=p_1\cdot\lambda^3+p_2\cdot\lambda^2+p_3\cdot\lambda+p_4$	$p_1=-0.0003988$;$p_2=-0.03578$; $p_3=0.2849$;$p_4=-0.15$;	0.01271
$C_{p12}=p_1\cdot\lambda^4+p_2\cdot\lambda^3+p_3\cdot\lambda^2+p_4\cdot\lambda+p_5$	$p_1=0.0006163$; $p_2=-0.009129$; $p_3=0.005473$; $p_4=0.2114$;$p_5=-0.1113$;	0.01226
$C_{p13}=p_1\cdot\lambda^5+p_2\cdot\lambda^4+p_3\cdot\lambda^3+p_4\cdot\lambda^2+p_5\cdot\lambda+p_6$	$p_1=-0.0008622$; $p_2=0.01597$; $p_3=-0.1093$; $p_4=0.2961$; $p_5=-0.1482$; $p_6=0.03156$;	0.006666
$C_{p14}=p_1\cdot\lambda^6+p_2\cdot\lambda^5+p_3\cdot\lambda^4+p_4\cdot\lambda^3+p_5\cdot\lambda^2+p_6\cdot\lambda+p_7$	$p_1=0.000259$; $p_2=-0.006418$; $p_3=0.06234$; $p_4=-0.2989$; $p_5=0.6883$; $p_6=-0.5243$;$p_7=0.1562$;	0.005301
$C_{p15}=p_1\cdot\lambda^7+p_2\cdot\lambda^6+p_3\cdot\lambda^5+p_4\cdot\lambda^4+p_5\cdot\lambda^3+p_6\cdot\lambda^2+p_7\cdot\lambda+p_8$	$p_1=-9.396\cdot10^{-5}$; $p_2=0.002663$; $p_3=-0.03141$; $p_4=0.1978$; $p_5=-0.7065$; $p_6=1.357$; $p_7=-1.065$;$p_8=0.3159$;	0.005048
$C_{p16}=p_1\cdot\lambda^8+p_2\cdot\lambda^7+p_3\cdot\lambda^6+p_4\cdot\lambda^5+p_5\cdot\lambda^4+p_6\cdot\lambda^3+p_7\cdot\lambda^2+p_8\cdot\lambda+p_9$	$p_1=4.862\cdot10^{-5}$; $p_2=-0.001508$; $p_3=0.01983$; $p_4=-0.1444$; $p_5=0.6349$; $p_6=-1.714$; $p_7=2.69$;$p_8=-1.975$; $p_9=0.5549$;	0.005208
$C_{p17}=p_1\cdot\lambda^9+p_2\cdot\lambda^8+p_3\cdot\lambda^7+p_4\cdot\lambda^6+p_5\cdot\lambda^5+p_6\cdot\lambda^4+p_7\cdot\lambda^3+p_8\cdot\lambda^2+p_9\cdot\lambda+p_{10}$	$p_1=6.815\cdot10^{-5}$; $p_2=-0.002186$; $p_3=0.02982$; $p_4=-0.2255$; $p_5=1.03$; $p_6=-2.91$; $p_7=4.968$; $p_8=-4.816$; $p_9=2.525$;$p_{10}=-0.5201$;	0.005158
$C_{p18}= p_1\cdot\lambda+p_2/\lambda^4+q_1\cdot\lambda^3+q_2\cdot\lambda^2+q_3\cdot\lambda+q_4$	$p_1=1.638$; $p_2=2.073$; $q_1=-13.82$; $q_2=71.27$; $q_3=-158.9$; $q_4=145.8$	0.01651
$C_{p19}=p_1\cdot\lambda^2+p_2\cdot\lambda+p_3/\Lambda^4+q_1\cdot\lambda^3+q_2\cdot\lambda^2+q_3\cdot\lambda+q_4$	$p_1=-4.813$; $p_2=34.79$; $p_3=-12.33$; $q_1=-17.2$; $q_2=106.7$; $q_3=-283.2$; $q_4=398.8$	0.006354
$C_{p20}= p_1\cdot\lambda^4+p_2\cdot\lambda^3+p_3\cdot\lambda^2+p_4\cdot\lambda+p_5/\lambda^4+q_1\cdot\lambda^3+q_2\cdot\lambda^2+q_3\cdot\lambda+q_4$	$p_1=-73.71$; $p_2=604$; $p_3=-664$ $p_4=506.2$; $p_5=-135.7$; $q_1=283.1$; $q_2=82.11$; $q_3=1968$; $q_4=394.2$	0.01659
$C_{p21}= p_1\cdot\lambda^5+p_2\cdot\lambda^4+p_3\cdot\lambda^3+p_4\cdot\lambda^2+p_5\cdot\lambda+p_6/\lambda^5+q_1\cdot\lambda^4+q_2\cdot\lambda^3+q_3\cdot\lambda^2+q_4\cdot\lambda+q_5$	$p_1=-1.08$; $p_2=2.593$; $p_3=36.35$; $p_4=-5.82$; $p_5=-47.32$; $p_6=28.01$; $q_1=-14.86$; $q_2=103.8$; $q_3=-4.9$; $q_4=-8.826$; $q_5=7.423$	0.01084

9. Conclusion

In this paper is mathematically analyzed power efficiency of the concrete tidal turbine. For the purposes of that the least square method is used. Very good agreement between experimental and different fitting function is obtained. In the future work, the attention should be paid to the mathematical analyses of the power-TSR curve of the concrete tidal turbine.

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