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Mathematical Philosophy Of Time In Minkowskian Space

Sudhakaran Gopalan

(7/M.Sc/Physics/1964,IIT Madras)
Author of gr-qc/0106029 in xxx.lanl.gov
Former Professor Of Physics, Sree Narayana College, Kollam,Kerala,India Pin:691001

Abstract

Time is a monotonic strictly increasing single valued real parameter that exists in spacetime. Here we consider an observer in his rest frame belonging to the Minkowskian spacetime. The order of the sequence of events on his World line is strictly preserved in the sense that the order of the sequence of events remains invariant under Lorentz transformations in Minkowskian spacetime: because the world line of the observer is always time-like.

Keywords - *Algebra on event field, proper time, real number field, time loop, proper world line, proper observer.*

1. Introduction

Time is awake when all things sleep.
Time stands straight when all things fall.
Time shuts in all and will not be shut.
Is,was,and shall be are Time`s children.
O Reasoning, be witness, be stable. [1]
VYASA,the Mahabharata [ca.A.D 400]

This universe has basic temporal structure. The fundamental nature of TIME in relation to human consciousness is evident as soon as we think that our judgements related to time and events in time appear themselves to be IN TIME.

Our analysis concerning SPACE do not appear in any obvious sense to be IN SPACE. But SPACE seems to be appeared to us all of a piece, whereas TIME comes to us only BIT by BIT. The Past exists only in our memory and the Future is hidden from us. Only the Present is the physical reality experienced by us. Thus TIME is always a ONE-WAY membrane. We cannot go from Present to the Past; while one can perform backward and forward motion in SPACE.

The free mobility in SPACE leads to the idea of transportable rigid rods. The absence of free mobility in TIME leads to the concept ONE-WAY membrane

TIME is a monotonic strictly increasing single valued real parameter corresponding to a non - spatial dimension represented by a straight line in Minkowskian spacetime and the SPACE is three dimensional. Minkowski unified space and time to a single entity called spacetime which is absolute. Einstein used the concept of spacetime for constructing spacetime geometry so that physics becomes part and parcel of geometry in Minkowskian spacetime. Einstein introduced the concept square of the distance between twoevents **$ds^2 = -dx^2-dy^2-dz^2+dT^2$ [2]**

here ds is distance between two events P(x,y,z,T) and Q(x+dx,y+dy,z+dz,T+dT).If ds^2 is greater than zero the separation between events is called time-like;if $ds^2= 0$, the separation between events is called null-like leading to the concept of Light Cone Structure in Special Theory of Relativity([3] &[4]) and if ds^2 is less than zero, the separation between events is called space-like. Time-like events are causally connected and also null-like events are causally connected; there is no causal connection between events separated by space-like interval. All real particles trace curves in spacetime. These curves are called time-like curves. Light rays travel along null curve in spacetime.Here we are concerned only with time-like curve so that the order of sequence of occurrence of events shall

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be the same for every observer under admissible co-ordinates transformations.

The world view proposed by Minkowski is often termed as Minkowskian spacetime [5] or M-space. It is said to have a (3+1) description of spacetime. Here "3" represents the Three Dimensional Euclidean space and "1" the One Dimensional time.

We introduce spacetime co-ordinates to order events. In M-space, the co-ordinates of an event can be represented by an ordered set of four real numbers, $<x^{1*},x^{2*},x^{3*},x^{4*}>$. Here the numbers x^{1*}, x^{2*}, x^{3*} and x^{4*} are taken to be PURE real numbers. 1,2,3,4 are superscripts used to specify the co-ordinates. It is always convenient to consider a Lorentz frame with orthonormal basis vectors $\mathbf{e_1},\mathbf{e_2},\mathbf{e_3}$,**and** $\mathbf{e_4}$ [1]. Relative to the origin of this frame the time-like worldline of a particle with real non-zero rest mass has a co-ordinates description

$$\mathbf{S}=x^{1*}(\tau)\,\mathbf{e_1}+x^{2*}(\tau)\mathbf{e_2}+x^{3*}(\tau)\,\mathbf{e_3}+x^{4*}(\tau)\mathbf{e_4} \quad\ldots\ldots(1)$$

Here **S** is the position 4-vector of the particle on its time-like wordline and τ is the proper time parameter defined along the time-like worldline of the particle.

M-Space = $M=\{<x^{1*},x^{2*},x^{3*},x^{4*}> \mid x^{i*}$ belongs to real number field$\}$

Here we shall have inner product of $\mathbf{e_1},\mathbf{e_2}$, $\mathbf{e_3}$, and $\mathbf{e_4}$as follows:

$\mathbf{e_1}.\mathbf{e_1}=\mathbf{e_2}.\mathbf{e_2}=\mathbf{e_3}.\mathbf{e_3}=-1$ and $\mathbf{e_4}.\mathbf{e_4}=+1$. $\mathbf{e_i}.e_j=0$ if $i\neq j$

$\mathbf{e_i}.e_j$are the covariant components of the metric tensor of spacetime. This tensor determines the nature of geometry of spacetime.Here the spacetime is taken to have signature -2.

Now taking the inner product of **S** in equation (1) we write

$\mathbf{S.S}=S^2=-1\{[x^{1*}]^2+[x^{2*}]^2+[x^{3*}]^2\}+[x^{4*}]^2=-D^2+[x^{4*}]^2$

D is the spatial separation between the two events and $[x^{4*}]$ is temporal separation between the same two events. The minus sign associated with D^2 and the plus sign associated with $(X^{4*})^2$ show that the space and time are distinct physical entities even though they are interdependent to form SPACETIME which is absolute. For illustration let D=2 lakhs km and time (t) separation between the two events be 2 sec.Then in conventional units we may put $[x^{4*}]=ct$. Here c is the speed of light in free space.c=3 lakhs km/sec.Then

$S^2=-4+c^2t^2=-4+(9x4)=32$;

so $S=(\sqrt{32})$ lakhs km which is the distance between two events. If S^2 is +ve, the spatio-temporal separation of events is time-like. If S^2 is equal to zero, the separation is null-like and if S^2 is less than zero, the separation between events is space-like. Since we consider only (3+1) description of space time co-ordinates in M-space we assume that (x^{1*},x^{2*},x^{3*}) represents the rectangular Cartesian co-ordinate system to specify the spatial position vector of an event in M-space and x^{4*} represents the temporal position of the event. Thus the spatio-temporal position is represented by
$<x^{1*}, x^{2*}, x^{3*},x^{4*}>$.
We write the event $=<x^{1*}, x^{2*}, x^{3*},x^{4*}>$ and we may put $x^{1*}=x,x^{2*}=y, x^{3*}=z,x^{4*}=T$
The totality of events $M=\{<x^{1*}, x^{2*}, x^{3*},x^{4*}>\}$ constitutes the Minkowskian spacetime (M-space).
The three dimensional Euclidean space(contained in Minkowskian spacetime) is homogeneous and isotropic.

2. Homogeneity of Space

Because of the homogeneity of space a physical phenomenon is always the same, if physical conditions are the same,wherever it is observed: the same physical experiment performed in California gives the same result if performed in New Delhi under the same physical conditions. The physical equivalence of different points of space- the homogeneity of space- demands certain restrictions on physical phenomena leading to the expression of law of conservation of linear momentum.

3. Isotropy of Space

Isotropy of space says that the physical properties of space at any particular point of space are the same in every direction. Because of this property of space, we do have what is called law of conservation of angular momentum.

4. Homogeneity of Time

Because of time homogeneity a physical phenomenon is always the same if physical conditions are the same, whenever it is observed. This physical equivalence of different instants of time- the homogeneity of time-imposes restrictions on physical phenomena. It leads to the expression of law of conservation of energy (Energy can neither be created nor destroyed).**But the time is not isotropic; because we cannot go from present to the past.**

These are two of the basic fundamental symmetric properties of space and time. [6]

Three dimensional Euclidean space is taken to be a space where Euclidean geometrical notions are true. In this space distinct straight lines parallel to one another do meet only at infinity, sum of the angles of a triangle is always 180 degree etc. Often this space is called a flat space.

Any physical body does have a spatio-temporal existence, in the sense that it has both spatial extension and temporal existence. And it can be represented by world band in spacetime.

But energy only has temporal existence. Energy is purely temporal being. It does not have any spatial extension. In that sense photon does not have any spatial dimension and we may think that photon is quantized action per unit time.

5. Event Field in Minkowskian Spacetime (M-Space)

We define the event field in M-space

$F=[\{<x^1,x^2,x^3,x^4>\},\oplus,\blacktriangle]$

$=[\{P_0,P_1,P_2,\ldots\ldots P_\alpha\ldots\}=\{P\},\oplus,\blacktriangle]$ .... (2)

Here, $\{P\}=(M-\{Q_0 \mid Q_0 \neq P_0\})$

Q_0 is the event that does not have ▲multiplicative inverse.

$P_0 = <0, 0, 0,0>$ is the additive identity.

$P_\alpha=<x_\alpha^1, x_\alpha^2, x_\alpha^3, x_\alpha^4>$ is the α^{th} event .... (3)

Now introduce two binary operations ⊕&▲in event field of M- space.

$\oplus : \{P\} \times \{P\} \longrightarrow \{P\}$

$\blacktriangle : (\{P\} - \{P_0\}) \times (\{P\} - \{P_0\}) \longrightarrow (\{P\} - \{P_0\})$

⊕is the event Addition operator and▲is the event Multiplication operator. We consider only the event field in M-space which obeys the following algebra.

<u>Event field Algebra</u>

6. Postulates A

(A_1) **F** is an abelian group under **⊕, P_0 =<**0,0,0,0**>**is the additive identity ⊕ is called the addition of events in M-space

$P_\alpha\oplus P_\beta= <x_\alpha^1 , x_\alpha^2, x_\alpha^3, x_\alpha^4>\oplus <x_\beta^1 , x_\beta^2, x_\beta^3, x_\beta^4>= <x^1_\alpha+x^1_\beta, x^2_\alpha+x^2_\beta, x^3_\alpha+x^3_\beta, x^4_\alpha+x^4_\beta>$ belongs to M-space .... (4)

Now $P_\alpha\oplus P_\beta= P_\beta\oplus P_\alpha$ (thus commutative property and closure property under ⊕ are obeyed)

(A_2)$P_\alpha\oplus (P_\beta\oplus P\mu)= (P_\alpha\oplus P_\beta)\oplus P\mu$ (Associative Law under ⊕ holds good)

(A_3) $P\oplus(-P)=(-P)\oplus P=P_0$ i.e. .... (5)

$<x^1,x^2,x^3,x^4>\oplus<-x^1,-x^2,-x^3,-x^4>=<0,0,0 \mid 0>= P_0$ which is the additive identity and event (-P) is the ⊕ additive inverse of the event P.

(A_4)$P\oplus P_0 = P_0\oplus P=P$ .... (6)

7. Postulates B

B_1) $P_\alpha\blacktriangle P_\beta= P_\beta\blacktriangle P_\alpha=<x_\alpha^1, x_\alpha^2, x_\alpha^3, x_\alpha^4>\blacktriangle <x_\beta^1 , x_\beta^2, x_\beta^3, x_\beta^4>=<x_\alpha^1 x_\beta^1 ,x_\alpha^2 x_\beta^2,x_\alpha^3 x_\beta^3, x_\alpha^4 x_\beta^4>$, ... (7)

(Thus Closure Property under ▲Multiplication is obeyed)

B_2) $P_\alpha\blacktriangle (P_\beta\blacktriangle P\mu) = (P_\alpha\blacktriangle P_\beta)\blacktriangle P\mu$
(Associative Property under ▲Multiplication is obeyed) …………...(8)

B_3) $P_\alpha\blacktriangle P_I = P_I\blacktriangle P_\alpha= P_\alpha$here $P_I=<1,1,1,1>$ is ▲Multiplicative Identity………………………........(9)

B_4) $P\blacktriangle P^{-1}= P^{-1}\blacktriangle P=< x^1,x^2, x^3 ,x^4>\blacktriangle < (x^1)^{-1} ,(x^2)^{-1},(x^3)^{-1},(x^4)^{-1}>=<1,1,1,1> = P_I=P^{-1}\blacktriangle P$............. (10)

Here P is not equal to P_0: that is▲inverse does not exist for P_0.Event (P^{-1}) is the ▲ multiplicative inverse of the event P.

B_5) $P_\alpha\blacktriangle (P\mu\oplus P_\beta) = (P_\alpha\blacktriangle P\mu)\oplus (P_\alpha\blacktriangle P_\beta)$
$(P\mu\oplus P_\beta)\blacktriangle P_\lambda = (P\mu\blacktriangle P_\lambda)\oplus (P_\beta\blacktriangle P_\lambda)$
(Distributive law holds good)..................................(11)

B_6) $P_\alpha\blacktriangle P_\beta= P_\beta\blacktriangle P_\alpha$(commutative) thus we have an event field which is commutative in M-space. ....(12)

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8. Proper Time, Proper World Line, Proper Observer in M-Space

In Minkowskianspacetime,**time** is homogenous but not isotropic. Even though space and time are interdependent entities in M-space,but not identical in all respects, we may denote event P in Minkowskian space time (when the observer is spatially at rest) $P=\langle x^1,x^2,x^3 | x^4\rangle$=**<spatial position of occurrence of P | temporal position of P>**

We consider an observer in his time-like world line. He is always in his own rest frame. That is he is always spatially at rest with respect to his own rest frame. This observer can be represented by $\langle \mathbf{x^1=c^1,x^2=c^2,x^3=c^3 | x^4=\tau}\rangle$; $\tau$is a real number.

where c^1,c^2,c^3areconstants. For sake of convenience we may put the spatial co-ordinates (c^1,c^2,c^3) $=(0,0,0)=\mathbf{0}$=spatial zero vector (zero spatial vector) which remains constant vector throughout the history of the observer in his rest frame. It is his HERE. We may put $\mathbf{0}(\tau)$ to show the HERE of the observer on his world line at the proper timeτ. We may call this observer, the proper observer possessing the proper worldline

$W=\{\langle Here(\tau) | \tau\rangle : \tau\epsilon\mathbb{R}\}$.

$=\{\langle O(\tau) | \tau\rangle\}$;$\tau$is the proper time measured by the observer.

Consider the ordered sequence of events on his proper worldline W along which only τvaries. The set of events on his worldline is represented by

$\{P_0, P_1 \ldots\ldots\ldots\}$. P_0 is the$\oplus$ identity = $\langle \mathbf{0(0)} | 0\rangle$.

$P_1 = \langle \mathbf{0(1)} | 1\rangle$is the$\blacktriangle$multiplicative identity.

Event(-P) = $\langle \mathbf{0}(-\tau) | -\tau\rangle$is the$\oplus$ additive inverse of the event P.

Event (P^{-1}) = $\langle \mathbf{0}(\tau^{-1}) | \tau^{-1}\rangle$is the$\blacktriangle$multiplicative inverse of event P.

a1) $P_1 \oplus P_2 = P_2 \oplus P_1$ .... (13)

$\{\langle o(\tau_1) | \tau_1\rangle\}\oplus\{\langle o(\tau_2) | \tau_2\rangle\}(\langle o(\tau_1+\tau_2) | \tau_1+\tau_2\rangle : \oplus$ closureproperty obeyed

a2) $(P_1\oplus P_2)\oplus P_3 = P_1\oplus(P_2\oplus P_3)$ associative law obeyed under $\oplus$.. (14)

a3)$\langle O(0) | o\rangle\oplus\langle O(\tau) | \tau\rangle=\langle O(\tau) | \tau\rangle$; $\oplus$ identity .... (15)

a4) $\{\langle O(\tau) | \tau\rangle\oplus\langle O(-\tau) | -\tau\rangle=\langle O(o) | o\rangle$ Thus $\oplus$ inverse.. (16)

hence we have an abelian group under event addition $\oplus$.

b1) $\langle O(\tau_1) | \tau_1\rangle\blacktriangle\langle O(\tau_2) | \tau_2\rangle=O(\tau_1\tau_2) | \tau_1\tau_2\rangle$......(17)

$\blacktriangle$ closure property obeyed

b2)$[P_1\blacktriangle P_2]\blacktriangle P_3= P_1\blacktriangle[P_2\blacktriangle P_3]$ ....(18)

$\blacktriangle$ associative property obeyed

b3) $\langle O(\tau) | \tau\rangle\blacktriangle\langle O(1) | 1\rangle=\langle O(\tau) | \tau\rangle$.................. (19)

$\langle O(1) | 1\rangle$ is $\blacktriangle$ multiplicative identity

b4 $\langle O(\tau) | \tau\rangle\blacktriangle\langle O(\tau^{-1}) | \tau^{-1}\rangle=\langle O(1) | 1\rangle$............(20)

Thus $\blacktriangle$inverse exists except for $\langle O(0) | 0\rangle$

b5)

$\langle O(\tau_1) | \tau_1\rangle\blacktriangle[\langle O(\tau_2) | \tau_2\rangle\oplus O(\tau_3) | \tau_3\rangle]=\langle O(\tau_1\tau_2) | \tau_1\tau_2\rangle\oplus\langle O(\tau_1\tau_3) | \tau_1\tau_3\rangle$

$=\langle O(\tau_1\tau_2+\tau_1\tau_3) | \tau_1\tau_2+\tau_1\tau_3\rangle$

Similarly

$[P_2\oplus P_3]\blacktriangle P_1=\langle O\langle\tau_2\tau_1+\tau_1\tau_3) | \tau_2\tau_1+\tau_1\tau_3\rangle$........(21)

Distributive law holds

b6) $P_1\blacktriangle P_2 = P_2\blacktriangle P_1$ .... (22)

Thus we have an event field being commutative on proper world line of the observer in M-space.

9. Total Order of Event Field constituting the world line of a proper observer [7] & [8]

The event field of proper observer on his world line,

F=[{<O(τ) | τ>

such thatτbelongs to real number field**},⊕,▲]**together with a **TOTAL ORDER** "≤" on the event field if the order "≤" satisfies the following properties. Here "≤" stands for <u>precedes</u>

This relation "≤" satisfies the following properties.

1. P "≤" P for all P belonging to W – the proper worldline of the observer (reflexivity).
2. If P "≤" Q and Q "≤" P, then P=Q (antisymmetry)
3. If P "≤" Q and Q "≤" R, then P "≤" R (transitivity)
4. Either P "≤" Q or Q "≤" P (totality)

Here P,Q and R are events on the WORLDLINE "W" of the observer.

Also we may have the following properties.

- If event P "≤" event Q then

$(P\oplus R)$ "≤" $(Q\oplus R)$ here $P = \langle O(\tau_p)|\tau_p$ …… (23)
$Q = \langle O(\tau_q)|\tau_q\rangle$ ………................................ (24)
$R = \langle O(\tau_r)|\tau_r\rangle$…………..............................(25)

If P_0 "≤" P and P_0 "≤" Q then P_0 "≤" (P▲ Q) where P_0 is $\langle O(o)|o\rangle$ which is the additive identity of event field in M-space
i.e., $\langle O(o)|o\rangle$"≤"$\langle O(\tau_p)|\tau_p\rangle\langle O(o)|o\rangle$"≤"$\langle O(\tau_q)|\tau_q\rangle$ then
$\langle O(o)|o\rangle$"≤"$\langle O(\tau_p)|\tau_p\rangle$▲$\langle O(\tau_q)|\tau_q\rangle$
i.e., $\langle O(o)|o\rangle$"≤"$\langle O(\tau_p\tau_q)|\tau_p\tau_q\rangle$iff proper time of occurrence of P and that of Q are NOT LESS THAN ZERO.
Here $\tau_{p,}\tau_q$ and $\tau_r$are proper times of occurrence of events P,Q and R respectively.
The world line
$W = \{\langle O(\tau)|\tau\rangle;\tau$ belongs to $\mathbb{R}\}$.

Every event belonging to W does have a number associated with that. This number is the proper time associated with the event. And every number representing proper time τ is associated with an event on W.There does exists one to one correspondence

between W and $\{\tau|\tau$ belongs to$\mathbb{R}\}$

Thus the totality of events on world line W of the proper observer is a real mathematical field. Consider the inequality $\tau_i \leq \tau_j$.

Here $\tau_i<\tau_j$ must imply that the event P_i occurs prior to the event P_j on the world line W of the observer and if $\tau_i= \tau_j$, P_i becomes identical to $P_{j:}$ thus the order is also preserved for the event field W in M-space. Since this field of events is ISOMORPHIC to real number field$\mathbb{R}$ we have shown that W is an ordered mathematical field. Here W does have one to one correspondence with $\{\tau\}$ and that $\{\tau\}$ is also an ordered mathematical field being ISOMORPHIC to$\mathbb{R}$ .So closed **time loop** cannot exist in Minkowskianspacetime, for $\mathbb{R}$ can never be a closed curve.

Thus we do have the theorem
Theorem :
If proper time is an ordered mathematical field, no closed time loop can exist in Minkowskianspacetime.

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