Mathematical Modelling of Non-Isolated Bi-Directional DC-DC Converter on Transients and Steady State Response

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***Abstract*—Energy management strategy is gaining much popularity because now the load is taking energy from different non-conventional/conventional sources as well as from energy storage element to provide uninterruptable power supply to the load. The bi-directional converter placed in between a DC voltage source and a battery to allow energy transfer. The converter connected with high voltage DC-bus and also the converter backed up with battery. Using the bi-directional dc-dc converter operation modes (Buck and Boost) this battery will be charged and discharged as per the suitable condition for uninterrupted power supply to the load. In this paper, a non-isolated half-bridge bi-directional dc-dc converter is studied for hybrid vehicle technology. The state space formulation of the bi-directional dc-dc converter in ideal case as well as with parasitics in different modes of operations is derived. The averaging and linearization technique is applied to get the small signal model of the converter.. To verify the methodology, converter model is developed in MATLAB/SIMULINK environment.**

Keywords-Bi-directional DC-DC Converter, Energy management system,State space modelling, Small-Signal Analysis Hybrid electric vehicle etc.

I. INTRODUCTION

Using a bi-directional dc–dc converter along with low-voltage energy storage for the high-voltage dc bus has been a prominent option for hybrid electric storage technology. Its have huge applications on hybrid electric vehicles and with non-conventional energy sources like fuel cell or photovoltaic cell etc. On this kind of hybrid storage technology, there is need to create an energy management strategy because to maintain continuous operation which provides uninterruptable power supply to the load with a backup planning [1].This topology improves the performance of the system; also it reduces the size and the cost of the system.

Asinglebi-directional dc-dc converter can replace two uni-directional converters. A single bidirectional dc-dc converter is capable to flow the power in opposite directions and provides the functionality of two uni-directional converters in a single converter unit. The converter is required to draw power from the high voltage dc-bus side to charge up the battery, and when the condition arrives it will to draw the power from battery to boost up the bus [2-4].

Here, a brief review and simulation of developed non-isolated bi-directional dc-dc power converter for hybrid storage technology application is presented. The bi-directional dc-dc power stage model is derived with the state-space averaging method. This derived model is validated by comparing between control-to-inductor current transfer function from the simulation results and the derived mathematical model. This power stage model can be used under different operating modes of the bidirectional converter.

A. *Circuit topology and its Power Stage Modelling*

The objective of dc-dc conversion is to convert a source voltage to a near-constant output voltage under disturbances at the source voltage and load. A dc-dc converter must provide a regulated dc output voltage under such condition like, input Voltage conditions, varying load, as well as converter component values. A bi-directional dc-dc converter topology is a combination of buck and boost converter. A bi-directional dc-dc converter consists with some basic functional blocks like, the power stage (plant), the modulator, and the controller. Here a proposed modelling method is used based on modelling of each component individually, and then combining them to a complete model. The power stage was modelled using state-space averaging. After that the controllers aredesigned. The combined small-signal model generates all the transfer functions required for design purposes.

As discussed, a non-isolated bi-directional dc-dc converter technology is to combine a buck and a boost converter in a half-bridge configuration. When charging the battery, this converter working as a buck converter, it operates in voltage step-down mode during the battery discharging its working as a boost converter; it operates in voltage step-up mode. Fig. 1 shows a non-isolated half bridge bi-directional dc-dc power converter circuit topology. The bi-directional dc-dc converter is placed in between high-voltage and low-voltage sources to allow energy transfer. This kind of power converters use in many applications like in hybrid vehicles, in aerospace etc.

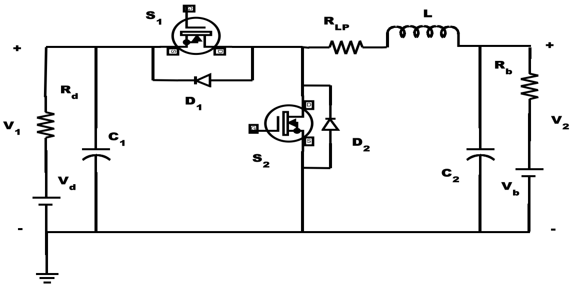

Fig. 1 Basic circuit of the proposed bi-directional dc-dc converter

II. STATE SPACE FORMULATION

The mathematical models for the non-isolated bi-directional dc-dc converter have been developed for both the step-down and step-up mode operation in the continuous current conduction mode. State-space formulation method is employed for the modelling of the bi-directional dc-dc converter with the following assumptions [11].

A. Step-down mode

In step-down mode operation battery is charging. During his step-down mode converter switch S1 remain ON and the switch S2 OFF. There are three energy storage components high side capacitor voltage, low side capacitor voltage, inductor current.

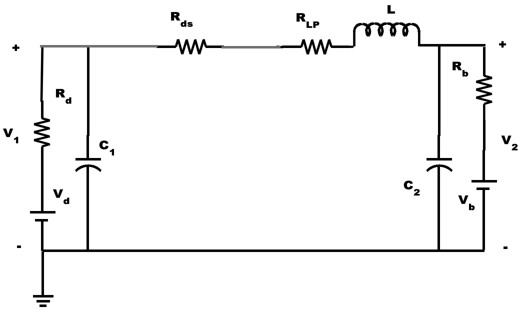

Fig. 2 Converter circuit in step-down mode

The converter equivalent circuit represent in Fig. 2. According to the above circuit when the switch S2 is only on, the following equations are derived by using KVL and KCL formula.

$$\frac{di_L}{dt} = -\frac{R_P i_L}{L} - \frac{V_2}{L}$$

$$\frac{dV_1}{dt} = -\frac{V_1}{R_d C_1} + \frac{V_d}{R_d C_1}$$

$$\frac{dV_2}{dt} = \frac{i_L}{C_L} + \frac{V_b}{R_b C_2} - \frac{V_2}{R_b C_2}$$

From the above equations

$$\begin{bmatrix} I_L \\ V_1 \\ V_2 \end{bmatrix} = \begin{vmatrix} \frac{-R_p}{L} & \frac{1}{L} & \frac{-1}{L} \\ \frac{-1}{C_1} & \frac{-1}{R_d C_1} & 0 \\ \frac{1}{C_2} & 0 & \frac{-1}{R_b C_2} \end{vmatrix} \begin{vmatrix} I_L \\ V_1 \\ V_2 \end{vmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{R_d C_1} \\ \frac{1}{R_b C_2} & 0 \end{bmatrix} \begin{bmatrix} V_b \\ V_d \end{bmatrix}$$

B. Step-up mode

This operation mode applies when battery discharges the power to the load to the connected DC bus. Converter operates in voltage step-up mode. Switch S2 remain ON and the switch S1 OFF. On during step-up mode bi-directional converter equivalent circuit shown in Fig. 3.

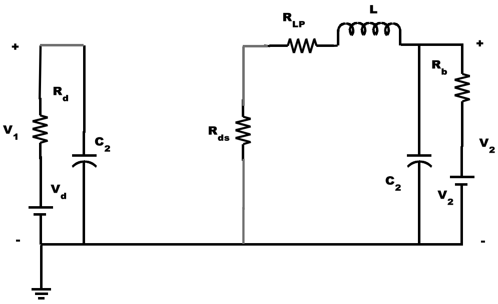

Fig. 3 Converter circuit in step-up mode

According to the above circuit when the switch S2 is only on, the following equations are derived by using KVL and KCL formula.

$$\frac{di_L}{dt} = -\frac{R_P i_L}{L} - \frac{V_2}{L}$$

$$\frac{dV_1}{dt} = -\frac{V_1}{R_d C_1} + \frac{V_d}{R_d C_1}$$

$$\frac{dV_2}{dt} = \frac{i_L}{C_L} + \frac{V_b}{R_b C_2} - \frac{V_2}{R_b C_2}$$

From the above equations

$$\begin{bmatrix} I_L \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{-R_p}{L} & 0 & \frac{-1}{L} \\ 0 & \frac{-1}{R_d C_1} & 0 \\ \frac{1}{C_2} & 0 & \frac{-1}{R_b C_2} \end{bmatrix} \begin{bmatrix} I_L \\ V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{R_d C_1} \\ \frac{1}{R_b C_2} & 0 \end{bmatrix} \begin{bmatrix} V_b \\ V_d \end{bmatrix}$$

So, from the above equation the state space average dc model become

$$0 = \begin{bmatrix} -\frac{R_p}{L} & \frac{D}{L} & -\frac{1}{L} \\ -\frac{D}{C_1} & -\frac{1}{R_d C_1} & 0 \\ \frac{1}{C_2} & 0 & -\frac{1}{R_b C_2} \end{bmatrix} \begin{bmatrix} I_L \\ V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{R_d C_1} \\ \frac{1}{R_b C_2} & 0 \end{bmatrix} \begin{bmatrix} V_b \\ V_d \end{bmatrix}$$

The state-space averaged ac model become

$$\frac{d}{dt} \begin{bmatrix} \hat{i}_L \\ \hat{V}_1 \\ \hat{V}_2 \end{bmatrix} = \begin{bmatrix} \frac{-R_p}{L} & \frac{D}{L} & \frac{-1}{L} \\ \frac{-D}{C_1} & \frac{-1}{R_d C_1} & 0 \\ \frac{1}{C_2} & 0 & \frac{-1}{R_b C_2} \end{bmatrix} \begin{bmatrix} \hat{i}_L \\ \hat{V}_1 \\ \hat{V}_2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{L} & 0 \\ \frac{-1}{C_1} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_L \\ V_1 \\ V_2 \end{bmatrix} \hat{d}$$

III. SMALL-SIGNAL ANALYSIS OF THE SYSTEM

Signal ac analysis for the different modes of the bi-directional converter operation under current mode control and also derives the transfer functions which describing the converter characteristics.

The state variables of the above system are the capacitor voltages and the inductor current. Therefore by considering ideal switching, the following two sets of state-space equations can be derived for each circuit state:

When switch S1 on during d(t) period

$$\dot{x} = A_1 x(t) + B_1 u(t)$$

When switch S2 on during (1-d)(t) period

$$\dot{x} = A_2 x(t) + B_2 u(t)$$

From the above equation, system, where “t” is the switching period,

$$\dot{x} = [A_1 x(t) + B_1 u(t)]dt + [A_2 x(t) + B_2 u(t)][1 - d(t)]$$

$$\text{Where} \quad d(t) = (D + \hat{d})$$

$$\text{And}\, [1 - d(t)] = (D' - \hat{d})$$

Now by substituting the perturbations terms the equations becomes

$$\dot{X} + \dot{\hat{x}} = [A_1(X + \hat{x}) + B_1(U + \hat{u})](D + \hat{d}) + [A_2(X + \hat{x}) + B_2(U + \hat{u})](D' + \hat{d})$$

The perturbed state-space description in above equation becomes nonlinear due to the presence of x^ and d^. The duty cycle is the control input, not being an element in the input vector u.

A. *Linearization*

The perturbed state-space averaged model is nonlinear. By Taylor series expansion and under the assumption of small-signal operation, linearization is done around the points (X, D, u), and nonlinear terms of higher orders are cancelled, i.e., departures from the steady-state values are negligible compared to the steady-state values themselves [14],

So it can be say

$$\hat{x}.\hat{d} \approx 0$$

$$\hat{u}.\hat{d} \approx 0$$

$$\hat{x}.\hat{d} \approx 0$$

$$\hat{u}.\hat{d} \approx 0$$

Now in linear approximation of the state space equations representing the averaged state space model

$$\dot{\hat{x}} = (A_1 D + A_2 D')\hat{x} + (B_1 D + B_2 D')\hat{u} + [(A_1 X - A_2 X) + (B_1 U - B_2 U)]\hat{d}$$

B. *Small-signal transfer function*

Taking the Laplace transform of equation with zero initial condition, we getting the following equation

$$\frac{x(s)}{u(s)} = \frac{[B_1 D + B_2 D']}{\mathrm{inv}[sI - (A_1 D + A_2 D')]}$$

The above expression is basically used to analyze the bidirectional converter’s dynamic behavior. Equation need to put in to a standard state space form.

$$\frac{\hat{i}_L}{\hat{d}} = \left[\begin{vmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & S \end{vmatrix} - \begin{vmatrix} -\frac{R_p}{L} & \frac{D}{L} & \frac{-1}{L} \\ -\frac{D}{C_1} & -\frac{1}{R_d C_1} & 0 \\ \frac{1}{C_2} & 0 & \frac{1}{R_b C_1} \end{vmatrix}\right]^{-1} . \begin{vmatrix} 0 & \frac{1}{L} & 0 \\ -\frac{1}{C_2} & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

R_d	R_b	L (µH)	C_1 (µF)	C_2 (µF)	F_{SW} (KHZ)	R_{dson} (miliohom)	V_1 (volt)	V_2 (volt)	R_{lp} (miliohom)
15	8	7	20	20	70	36	42	14	36

So the duty cycle-to-inductor current transfer function become

$$\frac{\hat{i}_L}{\hat{d}} = \frac{S^2.a.bV_1 + S(a.V_1 + b.V_1 - a.D.I_L.R_d) + V_1 - D.I_L.R_d}{S^3.a.b.L + S^2(a.L + b.L + a.b.R_p) + S(L + a.R_p + b.R_p + a.D^2.R_d + b.R_b) + R_b + R_p + D^2.R_d}$$

The control to high-side voltage transfer function become

$$\frac{\hat{V}_1}{\hat{d}} = \frac{-(\frac{I_L}{C_1} + \frac{D}{C_1}.\frac{\hat{i}_L}{\hat{d}})}{S + \frac{1}{C_1 R_d}}$$

The control to low-side voltage transfer function become

$$\frac{\hat{V}_2}{\hat{d}} = \frac{\frac{1}{C_2}}{S + \frac{1}{C_2 R_b}} . \frac{\hat{i}_L}{\hat{d}}$$

When the low side voltage Vb is zero, Rb is treated as a resistive load and Rd is negligible, the model derived in duty cycle-to-inductor current transferfunction behaves like a standard second-order buck converter model. The Buck Mode with Resistive Load Converter Mode state space equation become,

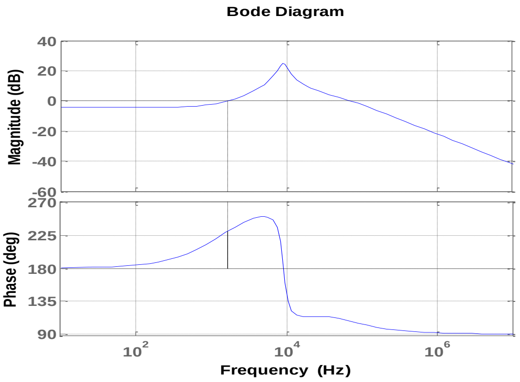

Fig. 4 Bode plot of the control to high-side voltage transfer function

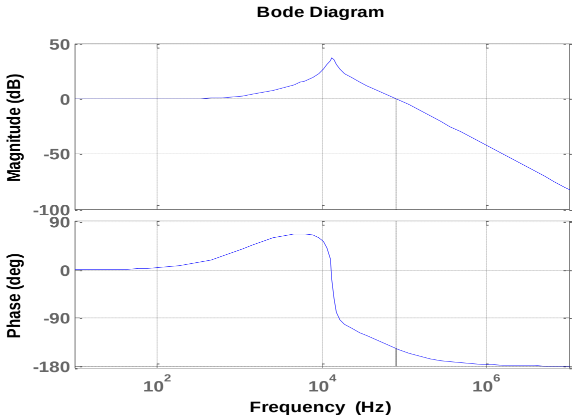

Fig. 5 Bode plot of the control to low-side voltage transfer function

$$G(s)=\frac{V_1(C_2R_bs+\mathbf{1})}{R_bC_2s(R_p+Ls)+R_p+Ls+R_b}$$

When the high-side voltage $V_d$is zero, R_d is treated as a resistive load and R_b is negligible, duty cycle-to-inductor current transfer function is simplified into a standard second-order boost converter model. The State-space equation become,

$$G(s)=\frac{(s+\frac{1}{C_1R_b})\frac{V_1}{L}-\frac{I_LD}{LC_1}}{(s+\frac{1}{C_1R_b})(s+\frac{R_p}{L})+\frac{D^2}{LC_1}}$$

IV. RESULTS AND DISCUSSIONS

The transfer function which is required to form the dynamic model of the converter for control purposes is the duty cycle-to-inductor current transfer function. Taking the inductor current as the output variable, the transfer function to the duty cycle has been derived.

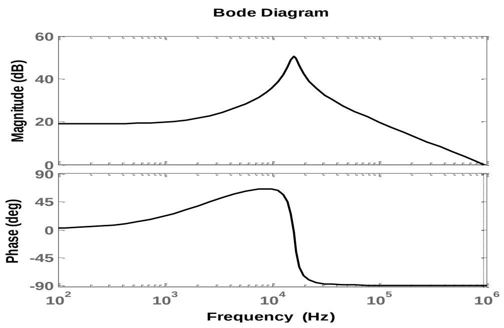

Fig. 6 Bode plot for duty cycle-to-inductor current transfer function

Using MATLAB, the Bode plot has been done for this duty cycle-to-inductor current transfer function to analyze the stability of the system. The Bode plot for is shown in Figure 6. Note the RHP zero does not appear in this transfer function, and it is inherently stable. For boost resistive load, Rd indicates a resistive load, where Vd does not exist. Battery internal resistance Rb is as small. Here Rb and C2 are negligible. At low frequency of less than kHz, the equivalent circuit is simplified. Taking the inductor current as the output variable at theboost mode, the transfer function tothe duty cycle has been derived and the Bode plot is shown in Figure 7.

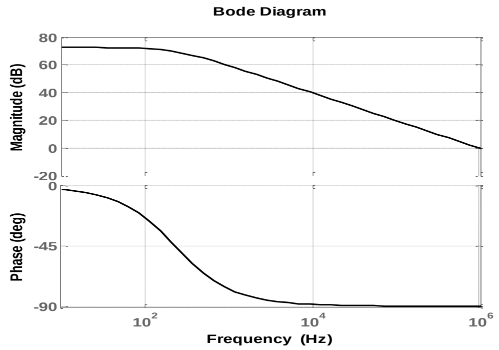

Fig 7 Bode plot for duty cycle-to-inductor currenttransfer function in boost mode condition.

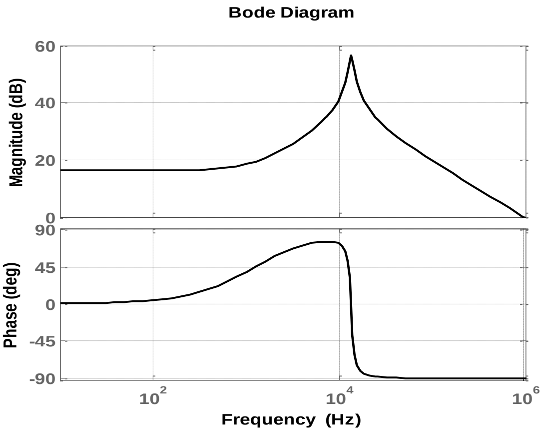

Fig. 8 Bode plot for duty cycle-to-inductor current transfer function in buck mode condition.

In Buck Mode with Resistive Load Converter Model, the low side voltage Vd is zero, Rb is treated as a resistive load and the high side resistance Rd is negligible. That behaves like a standard second-order buck converter model; the stability analysis is done through Bode plot, shown in Figure 8.

V. CONCLUSION AND FUTURE WORK

The analysis, state space formulation, controller design and the simulation of the non-isolated bi-directional DC-DC converter were examined. The converter topology is analyzed with state space formulation in different modes of operations and then by using averaging and linearization process the small-signal models of the converter derived. So as a conclusion, the project objective is to mathematical modelling of a non-isolated bidirectional converter on transients and steady state responsefor energy management system is done and system stability has been analysis through the MATLAB/SIMULATION. The Future work will be designing the controller circuit both the buck and boost mode for the bi-directional dc-dc converter and to maintains the voltage label at a standard value on the different operations mode.

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