Mathematical Modelling and Design Software for Cryogenic Regenerator

Debashis Panda, Manoj Kumar
Ph.D. Scholar, Cryogenics Engineering Laboratory
Department of Mechanical Engineering
National Institute of Technology,
Rourkela ,Odisha, India, 769008

A. K. Satapathy, R. K. Sahoo
Professor, Cryogenics Engineering Laboratory,
Department of Mechanical Engineering
National Institute of Technology,
Rourkela, Odisha, India,769008

Sunil K. Sarangi
Emeritus, Process Equipment and Design Laboratory,
Cryogenic Engineering Centre Indian Institute of Technology,
Kharagpur Professorial fellow,
IIT Bhubaneswar

***Abstract*—Closed cycle regenerative cryocooler provides a wide variety of technical merits, among them are ease of design and construction, highly reliable operation, minimal maintenance requirements and moderate cost. Stirling and pulse tube cryocoolers offer the simplicity of operation and vibration free operation. It also covers a wide range of cold end temperature and cooling capacity. The space required by Stirling and pulse tube cryocoolers are very small, so they are mostly used in space-based applications. In all regenerative cryocoolers, the regenerator is the chief component of these machines. Over past three decades, the mechanism of refrigeration has been understood and it is obvious that the performance of these machines directly depends upon the effectiveness of the regenerator. However, the design of regenerator may be complicated and tedious task to the engineers and scientists, pursing in this field due to the complex fluid flow and heat transfer process occurring inside it. To overcome these difficulties, design tools and software becomes very much essential to design regenerators to achieve required output. This paper is directed towards the creation of a general purpose simulation software to design regenerators. Both primary and secondary losses are taken into consideration. The effect of longitudinal conduction and wall conduction are also taken into consideration. The software is written in MATLAB, which has a decent graphical user interface. It assists the designer, not only in feeding input information but also in generating reports, those would support the user in future. The design is implemented through a process of successive simulation. The results are compared with published results and a good agreement is observed. A regenerator is simulated by using this software and various thermo-hydrodynamic phenomena are validated with published experimental results. The effect of various operating parameters on the performance of a typical cryocooler is also presented with interactive charts.**

Keywords—Regenerator,Softwrare, MATLAB, IR Sensors

I. INTRODUCTION

Regenerator is the crucial component of all regenerative type machines including Stirling, GM, VM, pulse tube cryocoolers and Stirling engines of alpha, beta and gamma configurations. Stirling and pulse tube cryocooler are used in IR sensors because of its compact size. The design of such miniature cryocoolers is a challenging problem to the scientists and engineers working in this field. The performance of cryocoolers and heat engines is an active function of the effectiveness of the regenerator. Regenerator is thermal energy storage device, consists of a hollow cylinder filled with meshes (woven meshes, random fiber meshes etc.) or lead balls, which stores heat from the fluid during heating period to cool the working fluid. To provide the required refrigerating effect, it releases heat to the fluid during the cooling period. Because of periodic and unsteady flow of fluid inside the matrix of the regenerator, its design may be a little bit complicated. The use of regenerator in hot air engine was reported by Stirling [1]. However, early mathematical modelling of regenerator was found in a German publication [2], in which Nusselt did the mathematical analysis of regenerator assuming infinite matrix heat capacity. The comparison of the regenerative heat exchanger with recuperative heat exchanger was carried out by Hausen [2]. IIife [3] performed the first order numerical analysis considering a transient variation of matrix temperature.

A lot of numerical and experimental work on regenerator was carried out by London et al. [4] for gas turbine plants. Wilmot [5-7] expanded the reduced length and reduced period method proposed by Hausen et al. by considering three-dimensional effects to examine multidimensional effects of a regenerator. Radebaugh [8, 9] gave an approximate design procedure for designing of the regenerator. REGEN series software developed by Radebaugh et al. is used for designing of regenerators. Several investigators used REGEN for carrying out parametric studies of regenerator by assuming various materials [10-15]. Kapitza [2] first reported the application of regenerators in cryogenic refrigerators. Regenerators are classified into two different types, depending upon the fact that regenerator matrix would be stationary or rotating. In stationary regenerator, the matrix is rest whereas in rotating type the matrix is rotating. In cryocoolers and heat engines, mostly stationary regenerators are used. While the REGEN software is mainly used for fixed matrix regenerators, whereas, the present software can be used for the design of fixed matrix regenerator of various meshes (woven mesh, random fiber mesh, spherical balls) and also rotary regenerators. In the present article, modelling procedure and results for fixed matrix regenerator are presented.

The significant losses that affect the performance of regenerator involve ineffective loss, conduction loss of matrix, temperature swing loss and wall friction loss and pressure drop loss. This article is directed towards the solution of such complex equations to develop general purpose software with an advanced interface that will help the designer's not only to design regenerators but also to predict its performance from thermal aspects. The results of this software are tested with the results available in open literature. A good agreement is obtained between them.

II. MATHEMATICAL MODELLING

The mathematical model presented here is based upon numerical solutions of thermal energy equations of working fluid and matrix of regenerator. Detailed description of this can be found elsewhere [2, 16]. The mathematical model is based on the assumption that considers one-dimensional longitudinal heat flow.

Ideal Model

Thermal energy equation of matrix

$$(\rho c_p \delta V)_m \frac{\partial T_m}{\partial t} = h\delta A_s (T - T_m) \quad (1)$$

Thermal energy equation of working fluid

$$(\rho c_p \delta V)_f \frac{\partial T}{\partial t} = h\delta A_s (T_m - T) \quad (2)$$

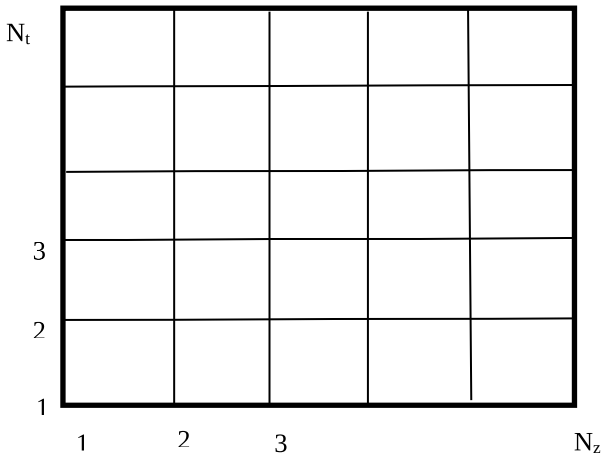

FIG: 1 Discretization of regenerator geometry section.

where i is number of spatial nodes and j is number of time step given as follows

$$i = 1, 2, 3, \ldots\ldots\ldots\ldots N_z$$

$$j = 1, 2, 3, \ldots\ldots\ldots\ldots N_t$$

Where, N_z the total number of spatial nodes and N_t is the total number of time step. The term on the left-hand side of Eq. (1) represents the unsteady term or accumulation of heat in regenerator matrix, whereas the term on the right-hand side signifies the convection heat transfer between fluid and matrix. Similarly, in Eq. (2) left-hand side represent the accumulation of heat in fluid, and right-hand side represents convection heat transfer between fluid and matrix.

Longitudinal conduction effect

In regenerator the matrix absorbs heat alternatively from hot fluid and cold fluid so there is conduction loss along the matrix of regenerator. Matrix thermal equation is

$$\frac{\partial}{\partial z}\left(K_m \frac{\partial T_m}{\partial z}\right)\delta V_m + h\delta A_s\left(T - T_m\right) = \left(\rho c_p \delta V\right)_m \frac{\partial T_m}{\partial t} \quad (3)$$

Fluid thermal equation is

$$h\delta A_s (T - T_m) = -(\rho c_p \delta V)_f \frac{\partial T}{\partial t} \quad (4)$$

Longitudinal conduction and wall effects.

Matrix thermal equation is

$$\left[\frac{aK_m}{T_m}\left(\frac{\partial T_m}{\partial z}\right)^2 + K_m \frac{\partial^2 T_m}{\partial z^2}\right] A_m \Delta z + \left(hA_s\right)_{\Delta z} (T - T_m) = [(Mc_p)_m]_{\Delta z} \frac{\partial T_m}{\partial t} \quad (5)$$

Wall thermal equation is

$$\left[\frac{bK_{wl}}{T_{wl}}\left(\frac{\partial T_{wl}}{\partial z}\right)^2 + K_{wl} \frac{\partial^2 T_{wl}}{\partial z^2}\right] A_{wl} \Delta z + \left(hA_{wl}\right)_{\Delta z} (T - T_{wl}) = [(Mc_p)_{wl}]_{\Delta z} \frac{\partial T_{wl}}{\partial t} \quad (6)$$

Fluid thermal equation is

$$\left(hA_s\right)_{\Delta z} (T - T_m) + \left(hA_{wl}\right)_{\Delta z} (T - T_{wl}) = -(\dot{m} c_p)_f \frac{\partial T}{\partial t} \quad (7)$$

The governing equations are converted to algebraic equations according to the procedure proposed by Ackermann et al. [2]. Then, the algebraic equations are solved by means of an iterative method. The temperature on the left hand side is assumed to be *T*h and cold end is *T*c. Linear variation of temperature is assumed as initial condition. The effect of various losses are affect the performance of regenerator are presented by Panda et al. [16]. Pressure drop has been taken into consideration as reported by Kays et al. [4]. All other parameters (e.g. inefficiency) are calculated as reported by Ackermann et al. [2].

III. CRESP-REGEN SOFTWARE OVERVIEW

This section explains the detailed design and explanation of the Graphical User Interface of CRESP-REGEN package, which has been written in MATLAB [17]. Fig. 2 shows the input interface of woven matrix configuration that accepts various input parameters from the user including geometrical parameters and operating parameters. Also, it allows the user to choose the solid and fluid materials. After entering the values of parameters, the user has to choose the model "ideal", the model including "longitudinal effect" and "longitudinal effect and wall effect", then hitting "run" it will solve the equations and produce results, which will be displayed after user will click "results". The detailed description of various "menu" and "toolbar" icons as well as other configurations for e.g. Random fiber meshes, spherical balls, parallel plates and rotary regenerators are beyond the scope of this paper. The GUI is converted into Microsoft Visual Studio 2012 to develop the executable package that runs in Windows operating system without MATLAB [18]. In an "ideal" model, least time is required to get results, on the other hand non-ideal case (Longitudinal effect and longitudinal conduction) more computational time is needed.

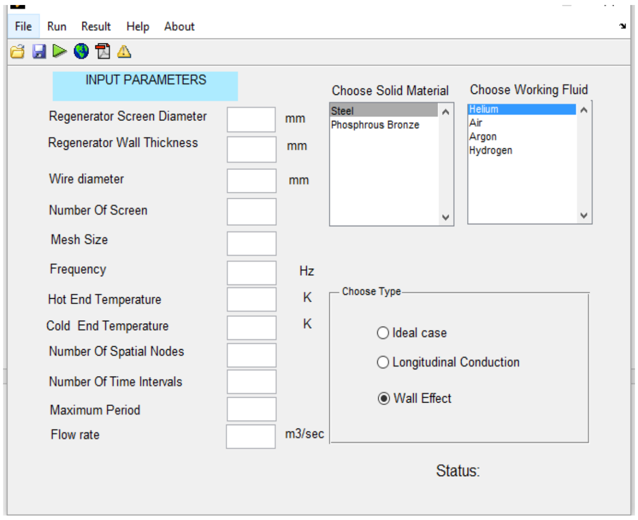

FIG: 2 input GUI of CRESP-REGEN woven matrix configuration.

IV. VALIDATION OF PRESENT SOFTWARE

First, the numerical programme is validated with the previously published results by Ackerman et al. [2]. The input parameters for validation are shown in Table 1. Since the solution procedure is based upon discretization approach, the results will vary with the change in time intervals and space coordinates. If the number of space coordinates and time intervals are small, then accuracy will be better but the computational time will be more. Hence, there must be a comfortable balance between accuracy and computational time. The results of present software with results published by Ackerman et al.[2] is shown in Table. 2.

Parameter	Value
Screen diameter	19.05 mm
Wire diameter	0.069 mm
Regenerator thickness	0.152 mm
Number of screens	640,480,320,220
Mesh size	#150 phosphorous bronze
Flow rate	0.00661m^3/s
Cold end temperature	80 K
Hot end temperature	300 K

Table 1. INPUT PARAMETERS FOR VALIDATION.

Parameter		Ackerman et al.	Present result	Relative error (%)
Length(cm)		10.16	10.14	-0.19
Heat transfer area(m2)		0.44	0.48	8.33
Reynolds number		53	53.77	1.43
NTU		192	210	8.57
Stanton number		-	0.16	-
Mass of regenerator		0.1	0.14	28.5
Ineffectiveness (%)	Ideal	0.62	0.68	8.82
	Longitudinal conduction	.71	0.79	10.12
	Longitudinal conduction + wall effect	0.99	1.02	2.94

Table 2 VALIDATION OF PRESENT CODE WITH PUBLISHED RESULTS

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V. PARAMETRIC STUDIES

A parametric study is conducted to examine the effect of various geometrical and operating parameters upon the performance of a regenerator for cryocooler based applications. The input parameters chosen for parametric investigation is given in Table 3.

Parameter	Value
Regenerator Screen Diameter(mm)	10
Thickness of wall(mm)	0.5
Wire diameter(mm)	0.04
Mesh size	# 250SS
Number of Mesh	670
Cold end temperature(K)	80
Hot End temperature(K)	300
Frequency(Hz)	28
Number of spatial Nodes	580
Number of time interval	85
Maximum period	120

Table 3. INPUT PARAMETERS PARAMETRIC STUDY.

Mesh size

Mesh sizes are available in various ranges in markets (50×50 to 635×635) for both academic and industrial-based applications. 50×50 mesh means 50 numbers of openings per inches. As the mesh size increases, then number of mesh opening per inch increases, results in decreasing porosity. Also, thermal penetration depth increases that lead to an increase in heat transfer area. It is clearly observed from Fig. 3, with an increase in mesh size inefficiency decreases. Also, the pressure drop increases with increase in mesh size.

Wire diameter

Wire diameter is also another significant geometrical parameter related to the screen mesh that affects the performance of regenerator. Increase in wire diameter decreases the porosity of mesh. Since porosity is inversely proportional to the mass of regenerator, a decrease in porosity increases the mass of regenerator and also heat transfer area. From Fig. 4, it is evident that there is an inefficiency of regenerator with increase in wire diameter of regenerator.

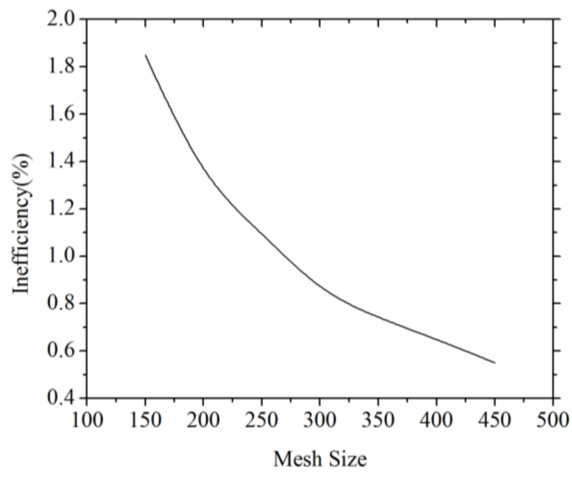

FIG: 3 Eeffect of mesh size on regenerator inefficiency

Wall thickness

Figure 5 indicates that, with increase in regenerator wall thickness, inefficiency increases due to the increase in conduction heat loss through the wall. However, this has negligible effect as compared to mesh size and wire diameter. In case of low temperature applications (e.g. 4 K) the conduction loss is a significant parameter to decrease its performance.

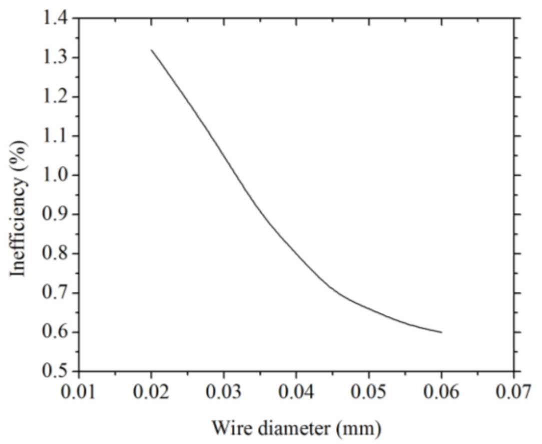

FIG: 4 Effect of wire diameter on regenerator inefficiency.

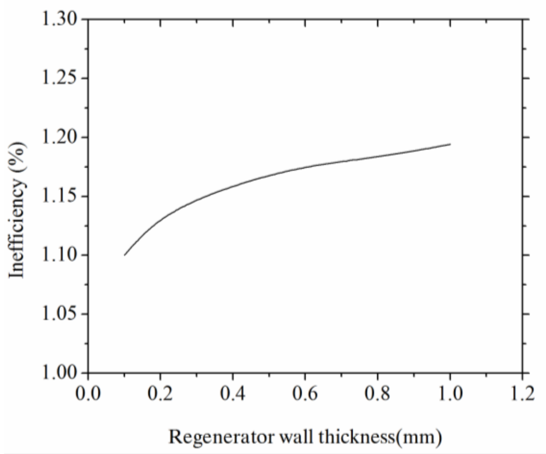

FIG: 5 Effect of wall thickness on regenerator inefficiency.

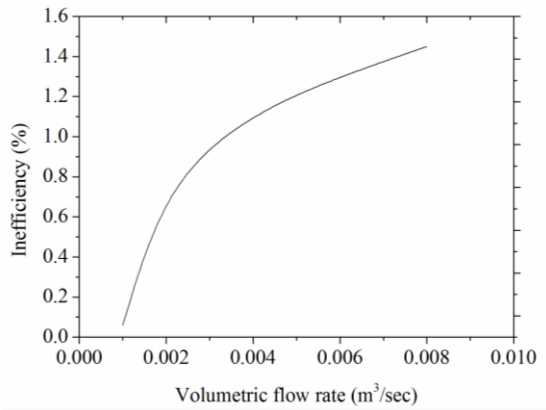

FIG: 6 Effect of volumetric flow rate on regenerator inefficiency.

Volumetric flow rate variation

As volumetric flow rate increases, the effectiveness of regenerator decreases due to increase in inefficiency and reduction in NTU as indicated in Fig 6.

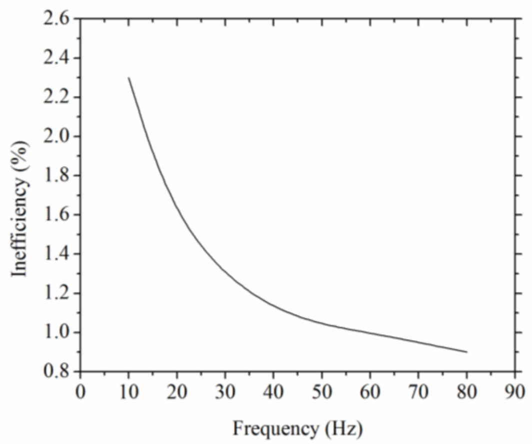

FIG: 7 Effect of operating frequency on Regenerator inefficiency.

Operating Frequency

The operating frequency of Stirling cryocooler and Stirling type pulse tube cryocooler is higher (near about 30-45 Hz), on the other hand, frequency of GM cryocooler and GM type pulse tube cryocooler is about 2-5 Hz. Thermoacoustic driven pulse tube cryocoolers operates at a frequency higher than 300

Hz to generate thermoacoustic waves. Also, the frequency of typical pulse tube cryocoolers can be increased to 120-150 Hz. With increase in operating frequency results in decrease in the size of the system, so that it will be suited for space based applications. With increase in operating frequency of the regenerator, heat flow loss starts decreasing, so the inefficiency decreases.These results are in aggrement with those of found in the literaature [14]. But in special types of Stirling type pulse tube refrigerators (e.g., Inertance type pulse tube refrigerartor), increase in operating frequency generates the inertance effect inside the inertance tube that causes an additional phase shift between pressure wave and mass flow rate, thus resulting in increase in cooling power and COP.

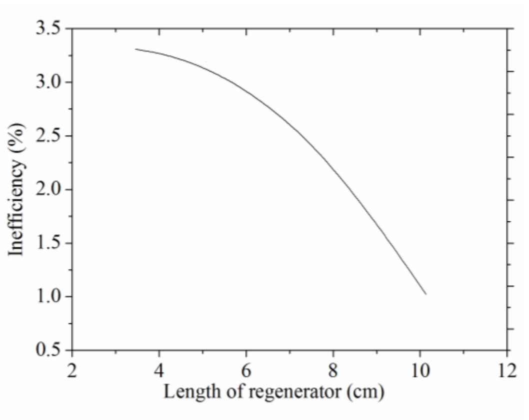

FIG: 8 Effect of length of regenerator on Regenerator inefficiency.

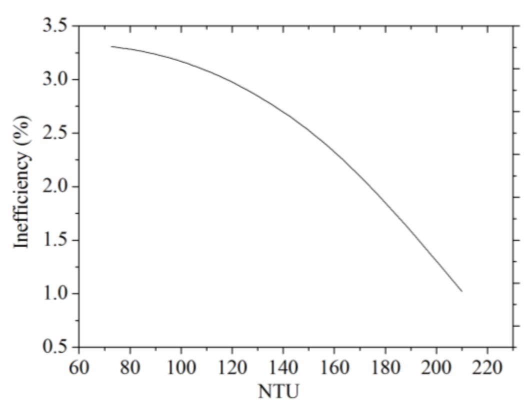

FIG: 9 Effect of NTU of regenerator on Regenerator inefficiency.

Length of regenerator

There is an increase in axial conduction loss and viscous loss with increase in length of regenerator, as a result of this inefficiency of regenerator decreases, as shown in Fig. 8.

NTU of regenerator

NTU (Number of transfer unit) is related to the size of regenerator. With increase in NTU of regenerator, its inefficiency decreases.

VI. CONCLUSIONS

A software has been developed by solving the governing equations of working fluid and matrix of the regenerator. Both ideal and non-ideal cases (effects including longitudinal conduction and both longitudinal conduction and wall effect) are taken into consideration. The present package is validated with the available results in the literature and a good agreement is observed between them. Parametric studies have been performed to examine the effect of important geometrical and operating parameters on the performance of a typical regenerator for cryogenic based applications. It is observed that parameters related to screen mesh of regenerator, such as mesh size and wire diameter, have a significant effect on the inefficiency of the regenerator, as compaired to thickness of the regenerator wall. Also, volumetric flow rate and operating frequency have a considerable impact on the performance over other operating parameters. The software will be extended in future by adding more materials and its temperature dependent properties. Also, the numerical model presented here will be extended in to higher order models considering the effect of additional losses. These works will be published in future papers.

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NOMENCLATURE

A	Area of crossections, m^2
a	Constant depends upon material property
C_p	Specific heat at constant pressure, J/kg-K
D	Hydraulic diameter, m
L	Length of the regenerator, m
K	Thermal conductivity of working fluid, W/m-K
h	Instantaneous local heat transfer coefficient, W/m^2-K
p	Static pressure, Pa
Re	Reynolds number
Nu	Average Nusselt number
x	Cartesian *x*-coordinate, m

Greek Symbol

ν	Kinematic viscosity, m^2/s
ρ	Density, kg/m^3
Δ	Length of control volume,m
μ	Dynamic viscosity, kg/m-s

Subscript

f	fluid
m	Matrix of regenerator
wl	Wall of regenerator