Mathematical Modeling of Vibration in Steering Wheel Assembly of Commerical Vehicles

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***Abstract* – Quality index of any commercial vehicle with respect to Noise and vibration is primarily governed by vibration levels at its tactile points such as steering wheel, gear lever, pedal, and seat. Steering wheel vibration is an important element which driver uses to express about the vehicle vibration quality. The work aims to study the vibration the steering assembly of a commercial vehicle and to establish a simplified mathematical model to represent the harmonics involved in the vibrating system. The work attempts to study the correlation of steering assembly vibrations between an existing commercial vehicle and optimized steering assembly. This work tries to present a basic mathematical model to which will help in understanding the various design and material conditions/constraints involved in designing of a steering wheel assembly and establishes a worst case criteria for developing the mathematical model. In general design of steering system is driven by ergonomics, packaging, durability, safety, vibration & ride and handling requirements.**

Keywords—Steering wheel, commercial vehicle, Mathematical Modeling

I. INTRODUCTION

Automobile drivers especially in commercial vehicles spend most of their lives inside the vehicle. So as design engineers it is one of our primary goals to deliver a vehicle which are comfortable to drive over long periods of time. The comfort of any commercial vehicle is governed by its noise, vibrations, harshness and ride and handling characteristics. The drivers fatigue will be drastically reduced if the driver is not continuously exposed to vibration. The vehicle vibrations reach the drivers primarily through the floor panel, the pedals, the gearshift lever, the seat and the steering wheel. The steering wheel is the most important of all these vibrations because of two reasons. Firstly it is the component with which the driver is in contact for most of the time and secondly due to the sensitivity of the skin tactile receptors of the hand as there is no intermediate medium such as shoes or clothing or seat cushioning which can attenuate vibration. Hence the work focuses on Steering vibrations. A Typical steering assembly consists of three sub-assemblies namely steering wheel assembly, steering column assembly and attachment assembly with the body structure, which can be direct attachment to firewall through brackets or cross beam to A-pillar.

II. INTRODUCTION TO MATHEMATICAL MODELING

Modeling is the part of solution of engineering problems that aims towards producing its mathematical description. This mathematical description can be obtained by applying known laws of physics with necessary assumptions to simplify the engineering problems to an extent that the physic laws may be applied to a real time problem. This part of modeling is called creation of the physical model. Application of the physics law to the physical model yields the wanted mathematical description that is called mathematical model. Process of solving of the mathematical model is called analysis and yields solution to the problem considered. One of the most frequently encountered engineering problems is vibration. There are primarily two approaches to mathematical Modeling of vibrating system,

1) Distributed - parameter approach
2) Lumped - parameter approach

In Distributed-parameter approach, the distributed mass and elasticity of some very simple components such as uniform shafts and plates are represented by partial differential equations. It is not generally possible to represent typical engineering systems (which tend to be more complicated) in this way. In Lumped - parameter approach, a set of discrete mass, elastic and damping elements are represented as spring mass system, resulting in one or more ordinary differential equations. The masses are concentrated at discrete points and are connected together by massless elastic and damping elements. The number of elements used dictates the accuracy of the model. It is suggested to have just sufficient elements for natural vibration modes and frequencies to avoiding unnecessary computing effort. In this work the Lumped parameter approach is used for modeling the steering wheel assembly as vibrating system.

III. CREATION OF PHYSICAL MODEL

Any system for creating a mathematical model creation and study of its physical model is important. The physical model for a multi body system shows the exact linkages in the model and how each body is connected to another and how forces/vibrations are getting transferred to the next system. The vibration path transmissibility of the system can be derived only from the physical model and reveals the exact system parameters to be used in the mathematical model. Any typical commercial vehicle is represented can be show as in figure 1. The chassis holds all the aggregates of the vehicle including the cab and represents the skeleton the vehicle.

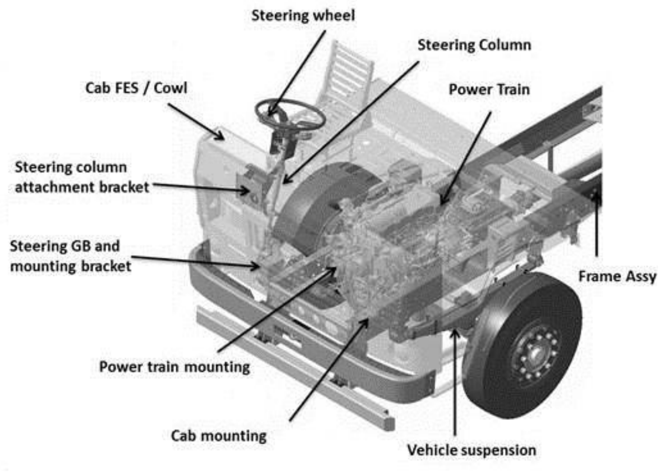

Fig. 1. Parts involved in steering vibration transfer path

The frame holds the powertrain which is mounted with the help of the engine or power train mounts. The cab is also mounted on the chassis through the cab suspension and cab mounting. The cab in turn holds the steering the column mounting bracket, the steering column and finally the steering wheel.

So Based on the physical model the Engine firing provides the input forces for the system, which is transferred to frame through the engine power train mounts. The vibration from frame is transferred to steering column through 2 paths. One is through the steering gearbox assembly and other through cab and cab mounting assembly and finally from steering column to steering wheel. So the steering vibration transfer path is as shown in figure 2.

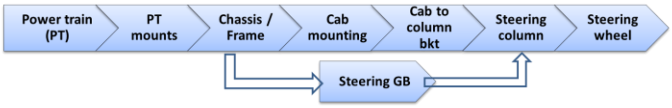

Fig. 2. steering vibration transfer path

IV. IDENTIFICATION OF WORST CRITERIA FOR MODELING

Vibrations of a steering wheel are variable depending on the position of the steering column. By Experiments and by theory the maximum vibrating position and minimum vibrating positions are identified as:

1) Max position = position closest to dashboard (12 o Clock position)

2) Min position = position closest to driver seat (6 o Clock position)

So by taking measurement at the maximum position at various conditions we can establish the worst case criteria for any vehicle. Instrumentation at 12 o clock position is as shown in figure 3.

Fig. 3. Steering system worst case measurement location

A Sample Intermediate commercial vehicle (ICV) of 12T capacity is taken for testing and measuring the vibration accelerations at various conditions. The Table 1 shows the engine specification of the vehicle. Based on the engine rpm and no of cylinders we can calculate the frequency of the engine.

TABLE 1 ENGINE SPECIFICATION

Attributes	ICV
Engine idle R.P.M	600
Fly-up r.p.m	2800
No. of cyl.	4
C.R.D.I / D.I	DI
Idle rpm Frequency – Hz	20
Fly-up rpm Frequency – Hz	93.3

For Example,

Engine speed Idle rpm is 600 i.e. 10cyles per second.

For a 4 Cylinder engine we need to take the second order vibrations so engine’s frequency at idle rpm is calculated as 10X2 = 20Hz

The steering wheel vibration accelerations at various conditions for the selected ICV are shown in Table 2.

Table 2 Steering wheel vibration at various conditions

S.No	Direction	RMS value in m/s^2		
		Idle Speed	@ 40 kmph	@ 60 kmph
1	X axis	2.26	1.13	1.83
2	Y axis	1.51	0.67	1.78
3	Z axis	4.92	3.44	3.8

From the values we can conclude that idle rpm condition is the worst case for steering wheel vibrations. So this work focuses on mathematical modeling of steering wheel vibrations at idle rpm. The testing data of the vehicle also validates the literature review findings that the idle rpm vibrations are the worst case criteria for refinement of the steering wheel vibrations (M. Ajovalasit et al [11] (2008)).

V. MATHEMATICAL MODELING AND ITS SIMPLIFICATION

The Lumped-parameter approach of mathematical modeling is used to represent the system. This is the simplest and easiest method of representing such complex system. Two scenarios of the system have been identified for modeling. The Natural Free – Free condition and Idling Engine condition which is the worst case for steering wheel vibrations have been modeled in the figures 4 & 5 respectively.

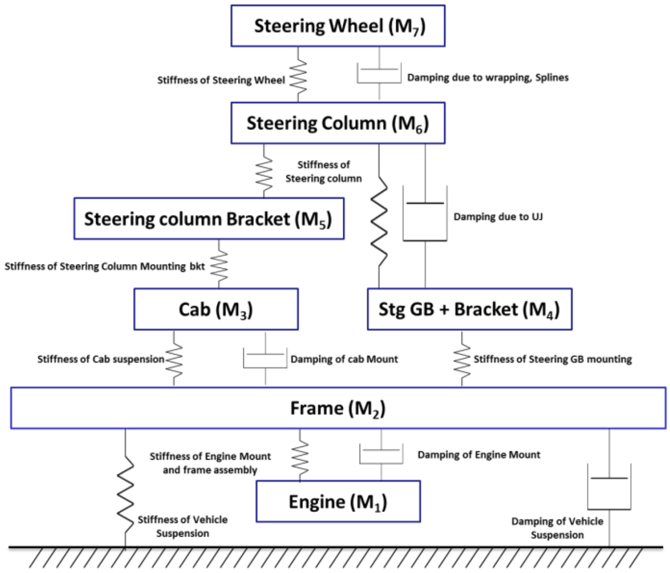

Fig. 4. Mathematical Model for Free-Free condition

The above model replicates the mechanical model and is based on the following assumptions:

1. The elements in the model have only single directional stiffness at any point of time. This stiffness is either directional stiffness or the RMS stiffness
2. Mass will be a lumped mass that will act at its connection point.
3. The input from engine during idle state modeling will be in terms time versus acceleration data and only the real part of the data is used for ease of calculation.
4. Modal damping ratio (ε) is assumed to be 3% based on industry practice and same values is used for Universal joints and splines of steering column.

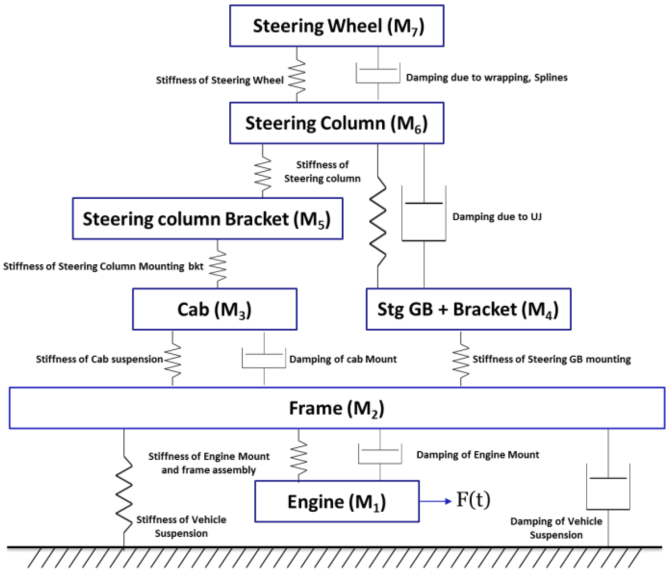

Fig. 5. Mathematical Model for Idling condition

For further simplification of the model effect of cross car beam (CCB) to steering column mounting bracket and Steering gear box assembly on steering wheel vibration is found out by physical testing. The below shown testing set up in figure 6 is used to understand this effect. The testing was done on different vehicles to check for its validity. The schematic shows the locations of accelerometers for the component isolation study.

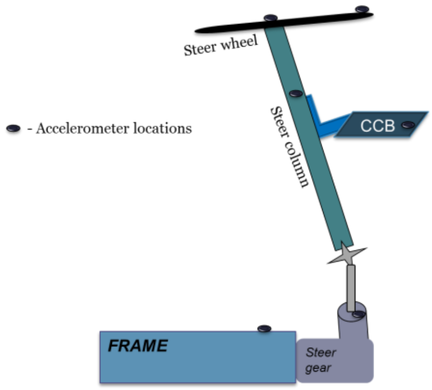

Fig. 6. Testing for Sensitivity of steering gear box and CCB

Based on the below test results shown in table 3 we can conclude that Steering GB contributes only marginally to steering wheel vibrations and hence can be neglected for ease of calculations for ease of calculations. Based on these results we can modify our mathematical model to make it into a simple multi body system.

TABLE 3 SENSITIVITY OF STEERING GB AND CCB

Condition	Steering wheel R.M.S values		
	X	Y	Z
Baseline	2.26	1.51	4.92
S.Gear fixed / CCB removed	0.42	0.65	0.98
S.Gear removed / CCB fixed	0.83	1.11	4.22
Both removed	0.16	0.11	0.26

The Mathematical model can be simplified to below system as shown in figure 7.

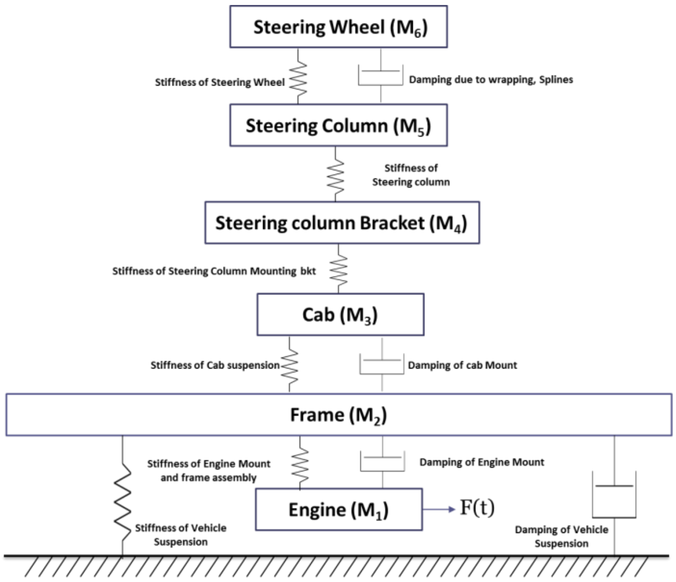

Fig. 7. Simplified mathematical model for idling condition

$$M_1\dot{X_1} = K_1\ [X_1] + C_1[\dot{X_1}] - M_1 g\ - f(t)$$

$$M_2\dot{X_2} = K_2\ [X_2] + C_2[\dot{X_2}] - M_2 g\ - K_3[X_3 - X_2] \\ - C_3[\dot{X_3} - \dot{X_2}]$$

$$M_3\dot{X_3} = K_3[\ X_3 - X_2] + C_3[\dot{X_3} - \dot{X_2}] - M_3 g \\ - K_4\ [X_4 - X_3 - X_2]$$

$$M_4\dot{X_4} = K_4[X_4 - X_3 - X_2] - M_4 g \\ - K_5[\ X_5 - X_4 - X_3 - X_2]$$

$$M_5\dot{X_5} = K_5[X_5 - X_4 - X_3 -\ X_2] - M_5 g \\ - K_6[X_6 - X_5 - X_4 - X_3 - X_2] \\ - C_6[\dot{X_6} - \dot{X_5} - \dot{X_4} - \dot{X_3} -\ \dot{X_2}]$$

$$M_6 X_6 = K_6\ [X_6 - X_5 - X_4 - X_3 - X_2] \\ + C_6[\dot{X_6} - \dot{X_5} - \dot{X_4} - \dot{X_3} - \dot{X_2}] - M_6 g$$

The above equations are called the coupling equations of the steering wheel system. The f (t) values represent the excitation forces of the engine which need to be physically measured. The same data will act as input to CAE analysis also. The other mass, stiffness and damping values will be the design intended values which will be given as input to CAE analysis also. So the final correlation should be compared with the CAE results since the same inputs will be used for validating the model.

VI. STEERING ASSEMBLY DESIGN MODIFICATION OPTIONS

With reference to the vibration transmissibility path described earlier the various options available for modifications in the system to refine the steering wheel vibrations are listed below.

A. Steering Wheel

The design parameters which play a very vital role in steering wheel are Wheel Mass, Diameter, Rim tube thickness and its diameter, Encapsulation damping, Ribbing pattern on wheel and No of Spokes, its spacing and its cross section, Boss plate stiffness. These parameters mainly affect the rim bending mode (fore-aft) and Wheel bending mode (lateral) of vibration.

B. Steering column

The design parameters of steering column which affect the steering wheel vibrations are Vertical Overhang of the column, Horizontal overhang column, Diameter of column, Thickness of column and No of universal joints in the column. These parameters affect Lateral bending Mode and Vertical bending mode of steering column and in turn the steering wheel itself.

C. Steering column mounting bracket and CCB

The only design parameter of steering column mounting bracket and Cross Car Beam (CCB) structure that affects the steering wheel vibrations is Column mounting bracket thickness and section which gives refers to the stiffness of the structure itself. The position of mounting bracket determines the steering column's vertical and horizontal overhang. This again affects the Lateral bending Mode and Vertical bending mode of steering column and in turn the steering wheel itself.

D. Power train and Power train mounts

The power train parameters which affect the steering wheel vibrations are Idle firing frequency, No. of cylinders and power train mount isolation characteristics. Since the power train is the source of the exciting force plays a critical role in overall steering wheel vibrations at all the conditions.

VII. ANALYSIS METHOD AND RESULTS

The mathematical model is a mathematical description of the physical problem. To obtain the results or solution to the problem, the mathematical model needs to be solved to obtain the required results. In our case we need the accelerations of steering wheel due to excitation of the engine at idle rpm. To obtain the acceleration values the following steps needs to be followed:

1. To above coupling equations, Laplace transform has to be applied in order to convert the equations from time domain to Laplace domain.
2. Solve the Laplace domain equations to obtain the displacements at of all the parts of the steering system.
3. Laplace domain has to be converted into frequency domain by taking (s=i* ω).
4. Inverse fast Fourier transform is applied to convert steering wheel displacements to time domain displacement values.
5. Numerical differentiation is applied to get the acceleration values in the time domain.
6. The test result values & the mathematical values will be compared to check the Correlation of the model.

The above method is validated with a real time vehicle and a correlation of about 45% was obtained with respect to CAE analysis results. The entire acceleration spectrum for the f (t) could not be obtained due to system capacity issues. Instead the correlation was obtained only at the corresponding peak acceleration points with help of preexisting CAE results and its corresponding time point. The results of the same are tabulated below in Table 4:

TABLE 4 CORRELATIONS OF CAE AND MATHEMATICAL MODEL

Steering wheel vibrations	@ IDLING CONDITION (In m/s^2) Z Direction RMS Value			
	CAE	Mat. Model	% Correlation	% Correlation for effect of modification
Base wheel design	2.96	1.27	43%	
Improved wheel design	1.64	0.66	40%	93%
Improved wheel design with improved Eng. mounts	1.14	0.44	38%	89%

The above results compare the steering wheel vibration of an existing steering wheel, an improved steering wheel and improved steering wheel with improved engine mounts. The correlation percentage is about 43% primarily because of the approximations and simplifications made in the model and because of neglecting the impact of other directional vibrations. The model is assumes that the vibration transfer is only in the Z direction. It is also observed that the effect of the modifications of steering wheel assembly have been captured with a correlation of about 89%. This means that the mathematical model accurately replicates the behavior of the system.

VIII. CONCLUSION AND FUTHER WORK PLANNED

A. *Conclusions*

A Typical Commercial vehicle steering system is studied and the vibration transmissibility path is understood from the physical model. Based on the physical testing of an intermediate commercial vehicle (ICV) Idling condition is established as the worst case for vibrations of a steering wheel. A simple mathematical model has been formulated using which the vibrations of steering wheel assembly can be estimated in terms of acceleration and displacement. The work throws light on various parameters which affect the steering wheel vibrations. A general methodology has been established for calculating the steering wheel displacements and accelerations and the same was tested using a test case of commercial vehicle and a decent level of correlation was achieved.

B. *Further work planned*

A Software or Macro development is planned which will help in performing the steps described for the analysis of the mathematical model to quickly study the effect of planned modifications. This will help in reducing design iteration time for refinement of the steering wheel vibrations. This will help in also understanding the various parameters which will affect the steering wheel vibrations.

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REFERENCES

[1] Zane Z.Yang, Srini V.Raman and Deren Ma (2005) “The virtual tests for facilitating steering wheel development” SAE Paper No. 2005-01-1072
[2] Jerome Rousseau., “Optimization techniques leading the development of steering wheel”, Jaguar cars limited, Altair technology conference, Bangalore, Feb 2004
[3] Vijay Antony John Britto, Sadasivam Sivasankaran, Ekambaram Loganathan and Saisankaranarayana Kupplili Saisankaranarayana (2013) “Commercial Vehicle NVH Refinement through Test-CAE Development Approach”, SAE Paper No.2013-01-1006.
[4] Vijay Antony John Britto, Ekambaram Loganathan, Sivasankaran Sadasivam, Kalyankumar Hatti, and Sai Sankaranarayana (2013) “Methodology of Steering Assembly Development for NVH for Medium and Heavy Commercial Vehicle”, SAE Paper No. 2013-01-2351
[5] Yasuhiko Fujiwara and Miki Nakayasu (1971) “Analysis of the vibrational models of vehicle steering mechanisms” SAE Paper No. 710627
[6] Hiroshi Sugita and Makoto Asai (1989) “Experimental Analysis for the steering wheel vibration using mechanical impedance method” SAE Paper No. 870971
[7] M.A.Demers (2001) “Steering wheel vibration diagnosis” SAE Paper No. 2001-01-1607
[8] Priede, T., “The Problems of Noise of Engines in Different Vehicle Groups” SAE Paper No. 750795, 1975.
[9] Drinan.T.H, Herring. R.A.,” The Correlation of a Steering Column F.E. and Experimental Modal Models”, SEM.org-IMC-XVIII-181403, 2000.
[10] Kim, K., Choi, I., and Kim, C., “A Study on the Advanced Technology Analysis Process of Steering System for Idle Performance”, SAE Paper No.2007-01-2339
[11] M. Ajovalasit and J. Giacomin (2008) “A Survey Study of Steering Wheel Vibration and Sound in Automobiles at Idle
[12] Michel A Bellmann (2002) “Perception of Whole Body Vibration: Effect of seat and steering wheel vibrations on passenger comfort inside the vehicle”.