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Mathematical Modeling of Storage and Inventory Control Policies for Cost Minimization in Deteriorating Item Warehouse System

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Abstract - Inventory control is an important part of supply chain management, especially when products are stored in different warehouses and lose value over time. Many items such as food, medicines, and electronic products deteriorate due to spoilage or become outdated. Because of this, companies must choose proper storage and ordering policies to reduce total cost and avoid shortages.

This research develops a simple mathematical model for managing deteriorating items in a multi-warehouse system. The system includes one owned warehouse and two rented warehouses with different costs and deterioration rates. One rented warehouse has low rent but high deterioration, while the other has high rent but low deterioration. This situation is commonly seen in real business environments where firms use both cheap and high-quality storage facilities.

The main aim of the model is to find the best order quantity and the best way to distribute inventory among the warehouses so that the total average cost is minimized. The total cost includes holding cost, deterioration cost, and shortage cost. Demand is assumed to be known and changes according to location, such as village, city, and micro-city areas. Replenishment is instant, shortages are allowed and fully backlogged, and warehouse capacities are assumed to be unlimited.

The total cost function is developed and optimized using basic mathematical methods. A numerical example is used to compare two-warehouse and three-warehouse systems. The results show that the three-warehouse system gives a lower total cost.

The study concludes that using an additional low-deterioration warehouse, even with higher rent, helps reduce overall cost and improves inventory management

Keywords: ***Inventory control, Deteriorating items, Multi-warehouse system, Cost minimization, EOQ model, Supply chain management.***

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1.1. Introduction:

Inventory control constitution one of the most significant and practical challenges in contemporary logistics and supply chain management, particularly where goods must be preserved and managed across distant and diverse location. Effective inventory management demands the careful formulation of appropriate storage policies for warehouse, ensuring optimal preservation availability of goods. Model based solution to inventory problems generally address two primary objectives, first, the continuous availability of off seasonal goods over a specified time horizon; and second, the substantial retention of product quality over time.

Warehouse are typically classified into two categories:

a) Low rent warehouse characterized by high deterioration rates.
b) High rent warehouse characterized by low deterioration rate.

For example, rented warehouse may be either airconditioned (high rent with low deterioration) or non airconditioned (low rent with high deterioration). A business may strategically hire a combination of two type of warehouses according to its operational requirement. Typically, two operational model includes their warehouses; One owned warehouse (denoted as ow) and two rented warehouses (denoted as RW1 and RW2).

In this configuration, RW2 is associated with high rental costs but exhibits low deterioration rates, whereas RW1 incurs low rental costs but experiences higher rates of deterioration.

The primary objective of this model is to investigate two optional inventory policies in terms of Economic order quantity (EOQ) and End of life (EOL) management within such an environment, particularly when multiple locations incorporate a mixture of these to warehouse categories. Each warehouse is assumed to operate independently, determining its optimal policies as though functioning as an autonomous entity. This modality approach mirrors practical scenarios encountered in real world logistics system.

Organization with multiple warehouses often manage them centrally, particularly when it comes to inventory ordering. While centralization simplifies decision making, it also introduces complexity in optimizing inventory policies for each warehouse location.

Deterioration items are products that lose value over time due to physical decay or market obsolescence. They can be classified into two categories;

Physical Deterioration: Items like meat, vegetables and medicine, spoil decay or get damaged.

Value obsolescence: Items like electronics or fashion goods that lose value due to new technological developments or changing consumer preferences.

Despite its importance deterioration is often overlooked in warehouse planning and modeling.

Warehouse are classified based on location using factors such as population density, resource availability, and business activity. The classification is as follows:

Location type	Index (i)	Population Range	Notation
Village Level	i =1	0 % to u%	$0 \leq S \leq u$
City Level	i = 2	u% to v%	$u \leq S \leq v$
Micro City Level	i = 3	v% to 100%	$v \leq S \leq 100$

Constant u and v are determined based on official population records of the practical experience of supply chain experts and planners.

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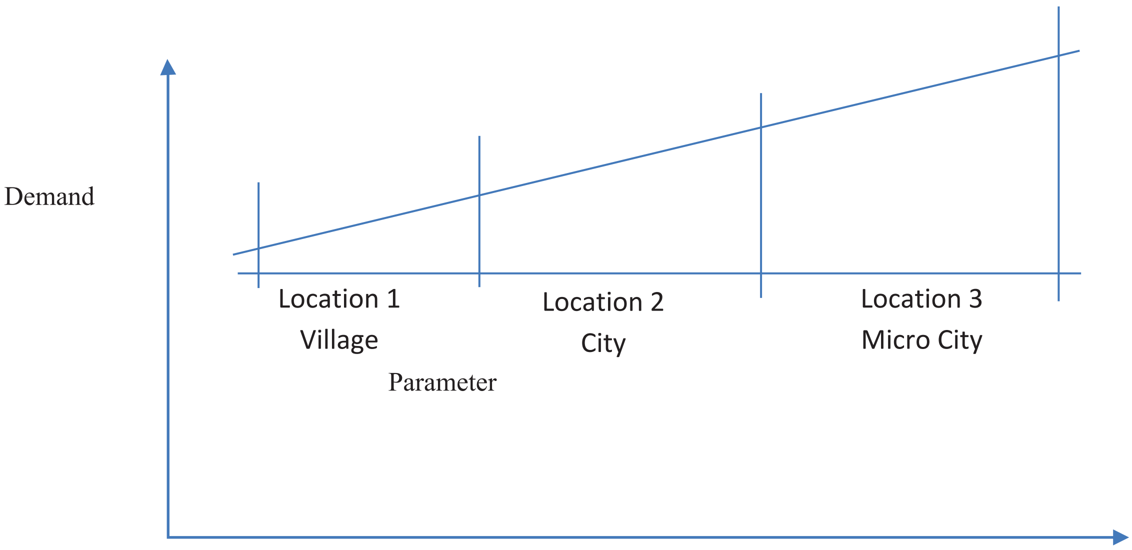

Fig.-i: Warehouse Classification based on location

1.2. A Review:

Chen and Kang (2009-a) [1] formulated the integrated inventory models under the two-level trade credit policy with price sensitive demand and negotiation scheme. A review article on trade credit can be reviewed by Cheng et al (2009) [2] incorporated the concept of vendor and buyer integration and order size –dependent trade credit. Different other researchers [7-10] have developed inventory models assuming demand as constant, time dependent, stock dependent or price dependent. X. Wang, Tang, and Zhao [6] worked on fuzzy economic order quantity inventory model without backordering.

Teng and Chang (2009) [5] gave the models discussing Vendor-buyer inventory models with trade credit financing under both non-cooperative and integrated environments. Jagi (1995)

discussed the effect of deterioration of units when delay in payments is permissible. Teng et al (2005) [3] developed retailer's optimal ordering and pricing policy for the assumption of deterministic and constant demand.

1.3. Symbols and Assumptions:

The proposed model is based on the following assumption:

Item demand is deterministic, and the demand rate varies by location and over the given time horizon. The demand rate at the i^{th} location is given by $R_i = Q_i/T$, where Q_i is the order quantity and T is the total schedule time.

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- The schedule time T is constant.
- Replenishment occurs instantaneously (infinite replenishment rate)
- The order quantity for the i^{th} location is Qi. The variable a_i, b_i and c_i represent the quantities stored in warehouse OW_i , RW_{1i}, RW_{2i}, respectively. Here a_i, is the primary storage preference, followed by b_i and finally c_i.
- The deterioration rates of items stored in OW_i, RW_{1i}, RW_{2i}at the i^{th} location are denoted by x_i, y_i, and z_i respectively.
- The holding costs per unit for items stored in OW_i, RW_{1i}, RW_{2i} at the i^{th} location are represented by B_{1i}, B_{2i}, and B_{3i} respectively.
- Thus, the order quantity is $Q_i = a_i + b_i + c_i$ and the deterioration adjusted value is

$$\frac{a_i}{x_i} + \frac{b_i}{y_i} + \frac{c_i}{z_i}$$

- Although the warehouse capacities OW_i, RW_{1i}, and RW_{2i} are considered unlimited, the optional stock level for each is determined by comparing their holding cost and deterioration rates to minimize overage cost.
- D_i represents the shortage cost per unit time.

1.4. Proposed Inventory Model:

The total lot size $Q_i + S_i$ enter the system where S_i represent any leftover shortages from the previous cycle and Q_i is the inventory for the current period T. Out of Q_i the inventory is distributed as follow: a_i units in OW_i b_i unit in RW_{1i} and c_i units in RW_{2i} all stored simultaneously.

The consumption process being with goods Z_i from RW_{2i} which are consumed and cleared by time t_2. Then from time t_2 to t_1 ,y_i from RW_{1i} is consumed. Finally, from time t_1 to t_0, consumption of a from OW_i starts. Deteriorated items are discarded once that inventory is depleted.

Due to deterioration, shortage occur and are associated with cost coefficients x_i, y_i, z_i for OW_i, RW_{1i}, RW_{2i} respectively. The total shortage cost over the period T is given by $x_i a_i + y_i b_i + z_i c_i$. The objective is to determine the optimal inventory level a_i^0 for OW_i and b_i^0 for RW_{1i} such that the total average cost is minimized across different location.

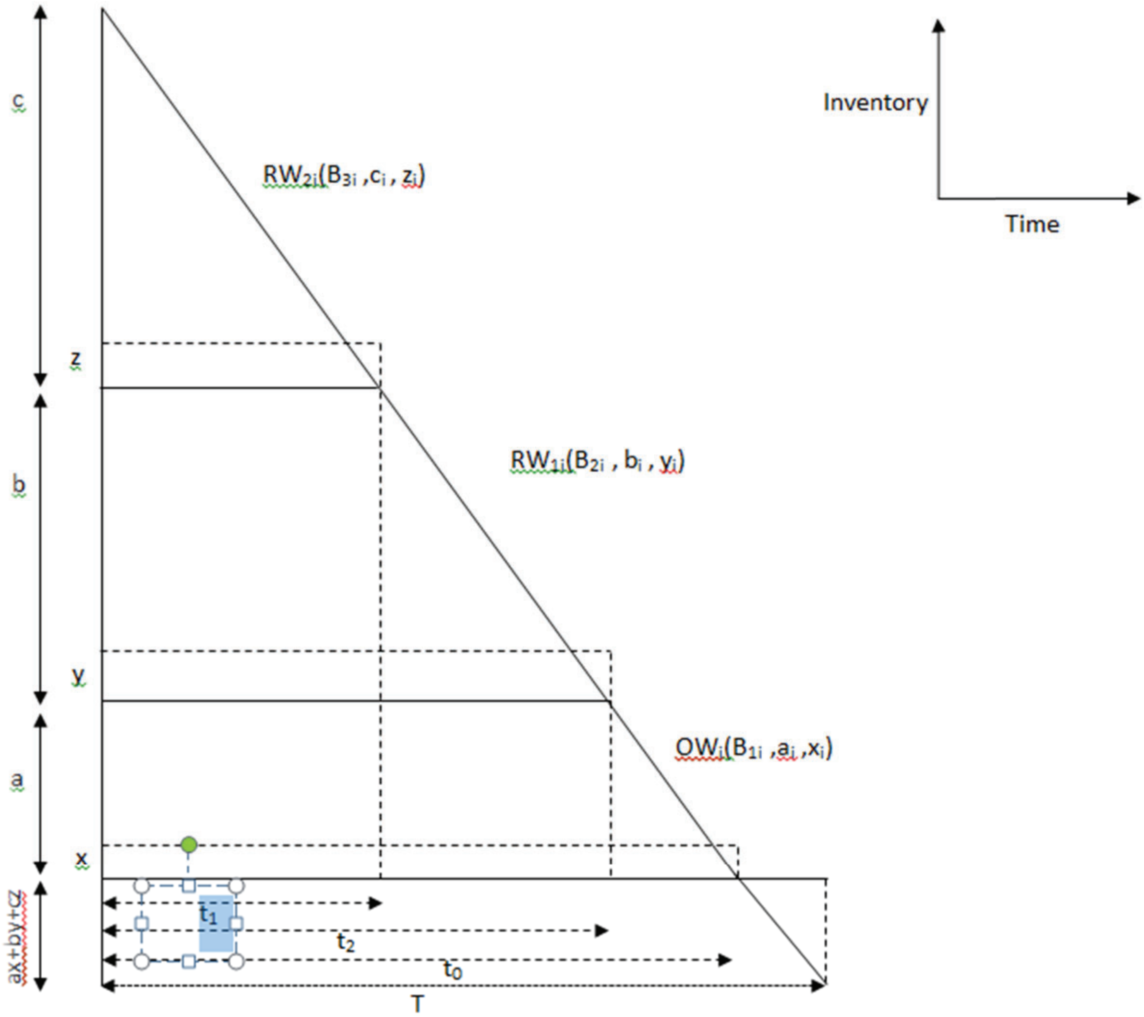

Fig.-ii: Three-warehouse set-up

1.5. Mathematical Formulation:

As per assumption for demand

$$\frac{(1-z_i)c_i}{t_2} = \frac{(1-y_i)b_i}{t_1-t_2} = \frac{(1-x_i)a_i}{t_0-t_1} = \frac{x_i a_i + y_i b_i + z_i c_i}{T-t_0} = \frac{Q_i}{T} = R_i \text{------ (1)}$$

Average holding cost of warehouse RW_{2i}

$$= \frac{1}{T}\{c_i t_2 z_i + \frac{1-y_i}{2} c_i t_2\} A_{3i}$$

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Average holding cost of warehouse RW_{1i} is

$$=\frac{1}{T}\left\{b_i t_2 + \frac{1-y_i}{2}(t_1 - t_2)b_i + y b_i(t_1 - t_2)\right\}A_{2i}$$

Average holding cost of warehouse $OW_i$is

$$=\frac{1}{T}\left\{a_i\ t_1 + \frac{1-x_i}{2}(t_0 - t_1)a_i\ + x a_i\ (t_0 - t_1)\right\}A_{1i}$$

Average shortage cost over time T

$$=\frac{1}{2T}(x_i a_i\ + y_i b_i\ + z_i)(T\ to\ D_i)$$

So, the total average cost TC_i is: -

$$Tc_i = \frac{1}{T}\left[\left\{c_i t_2 z_i + \frac{1-z_i}{2}c_i t_2\right\}A_{3i} + \left\{b_i t_2 + \frac{1-y_i}{2}b_i(t_1 - t_2) + y_i b_i(t_1 - t_2)\right\}A_{2i}\right]$$

$$+ \frac{1}{T}\left[\left\{a_i t_1 + \frac{1-x_i}{2}(t_0 - t_1)a_i + a_i x_i(t_0 - t_1)\right\}A_{1i}\right]$$

$$+ \frac{1}{2T}[(x_i a_i\ + y_i b_i\ + z_i c_i\ )(T - t_0)]D_i \text{---------------------}(2)$$

$$Tc_i = \frac{1}{T}\left[\left\{\frac{1+z_i}{2}c_i t_2\right\}A_{3i} + \left\{b_i t_2 + \frac{1-y_i}{2}b_i(t_1 - t_2)\right\}A_{2i}\right]$$

$$+ \frac{1}{T}\left[\left\{a_i t_2 + a_i(t_1 - t_2) + \frac{1+x_i}{2}a_i(t_0 - t_1)\right\}A_{1i}\right]$$

$$+ \frac{1}{T}\left[\frac{1}{2}(x_i a_i\ + y_i b_i\ + z_i c_i\ )(T - t_0)D_i\right] \text{--------------------------}(3)$$

$$Tc_i = \frac{1}{T}\left[\left\{\frac{1+z_i}{2}c_i t_2\right\}A_{3i} + \left\{b_i t_2 + \frac{1+y_i}{2}b_i(t_1 - t_2)\right\}A_{2i}\right]$$

$$+ \frac{1}{T}\left[\left\{a_i t_2 + a_i(t_1 - t_2) + \frac{1+x_i}{2}a_i(t_0 - t_1)\right\}A_{1i}\right]$$

$$+ \frac{1}{T}\left[\frac{1}{2}(x_i a_i\ + y_i b_i\ + z_i c_i\ )(T - t_0)D_i\right] \text{--------------------------}(4)$$

Also , from the fig we have $Z_i = Q_i - a_i - b_i$ and after replacing the proportional value of

$t_2, (t_1 - t_2), (t_0 - t_1)and(T - t_0)$ from equation (1) we get

$$Tc_i = \frac{1}{Q_i}[(Q - a_i - b_i)^2\frac{1-z_i^2}{2}A_{3i} + \{(Q_i - a_i - b_i)b_i(1 - z_i) + \frac{1-y_i^2}{2}y_i^2\}A_{2i}]$$

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$$+\frac{1}{Q_i}\left[\left\{a_i b_i(1-y_i)+(Q-\ a_i-\ b_i)a_i(1-z_i)+\frac{a_i^2}{2}(1-x_i^2)\right\}A_{1i}\right]$$

$$+\frac{1}{Q_i}\left[\frac{1}{2}\{a_i x_i+b_i y_i+z_i(Q_i-\ a_i-\ b_i)\}^2 D_i\right] \text{----------- (5)}$$

On differentiating Tc_i w.r. to a_i and b_i we have

$$\frac{\partial Tc_i}{\partial a_i}=\frac{1}{Q_i}[-(Q_i-\ a_i-\ b_i)(1-z_i^2)A_{3i}-b_i(1-z_i)A_{2i}]$$

$$+\frac{1}{Q_i}[\{b_i(1-y_i)+(1-z_i)(Q_i-\ 2a_i-\ b_i)+a_i(1-x_i^2)\}A_{1i}]$$

$$+\frac{1}{Q_i}[\{a_i x_i+b_i y_i+z_i(Q_i-\ a_i-\ b_i)\}(x_i-\ z_i)D_i]=0 \text{-----------(6)}$$

And, $\frac{\partial Tc_i}{\partial b_i}=\frac{1}{Q_i}[-(Q_i-\ a_i-\ b_i)(1-z_i^2)A_{3i}+\{(Q_i-a_i-2bi)(1-z_i)+b_i(1-y_i^2)\}A_{2i}]+$ $\frac{1}{Q}[\{a_i(1-y_i)-a_i(1-z_i)\}A_{1i}+\{a_i x_i+b_i y_i+z_i(Q_i-\ a_i-\ b_i)\}(y_i-\ z_i)D_i]=0$

------------------(7)

Now above two equations can be written as –

$$F_{1i}a_i+\ F_{2i}b_i=F_{3i} \text{-----------------------------(8)}$$

And

$$F_{2i}a_i+\ F_{4i}b_i=F_{5i} \text{------------------------------(9)}$$

Where,

$$F_{1i}=[\ (1-z_i^2)A_{3i}-(1-2z_i+\ x_i^2)A_{1i}+(x_i-z_i)^2 D_i]$$

$$F_{2i}=[\ (1-z_i^2)A_{3i}-(1-z_i)A_{2i}+(z_i-y_i)A_{1i}+(y_i-z_i)(x_i-z_i)D_i]$$

$$F_{3i}=[\ z_i Q_i(z_i-x_i)D_i+\ Q_i(1-z_i^2)A_{3i}-\ Q_i(1-z_i)A_{1i}]$$

$$F_{4i}=[\ (1-z_i^2)A_{3i}-\ (1-2z_i+\ y_i^2)A_{2i}+\ (y_i-z_i)^2 D_i]$$

$$F_{5i}=[\ z_i Q_i(z_i-\ y_i)D_i+\ \ Q_i(1-z_i^2)A_{3i}-\ Q_i(1-\ z_i)\ A_{2i}]$$

Solving equation (8) and (9) we have optimum result a_i^0 and b_i^0 as

Also consider, $r_i=\frac{\partial^2 Tc_i}{\partial a_i^2}=\frac{1}{Q_i}[(1-z_i^2)A_{3i}-(1-2z_i+\ x_i^2)A_{1i}+(x_i-z_i)^2 D_i]$--------(10)

$$s_i=\frac{\partial^2 Tc_i}{\partial a_i \partial b_i}=\frac{1}{Q_i}[(1-z_i^2)A_{3i}-(1-\ z_i)A_{2i}+\ (z_i-y_i)A_{1i}+(y_i-z_i)(x_i-z_i)D_i]$$

---------------(11)

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$$t_i = \frac{\partial^2 Tc_i}{\partial b_i^2} = \frac{1}{Q_i}[(1 - z_i^2)A_{3i} - (1 - 2z_i + y_i^2)A_{2i} + (y_i - z_i)^2 D_i] \quad \text{------------(12)}$$

Here $A_{1i} < A_{2i} < A_{3i} < D_i$ and x_i, y_i, z_i are probabilities of deterioration lies between 0 to 1.

Now Hessian matrix H for $Tc_i\ (a_i, b_i)$is:

$$\mathbf{H} = \begin{bmatrix} \boldsymbol{r_i} & \boldsymbol{s_i} \\ \boldsymbol{s_i} & \boldsymbol{t_i} \end{bmatrix}$$

To confirm a minimum, the Hessian must be positive definite:

1) $\boldsymbol{r_i} > 0$,
2) **Determinant of it** $\boldsymbol{r_i t_i} - \boldsymbol{s_t^2} > 0$

Here $r_i = \frac{\partial^2 Tc_i}{\partial a_i^2}$---------------------(13)

$$= \frac{1}{Q_i}[(1 - z_i^2)A_{3i} - (1 - 2z_i + x_i^2)A_{1i} + (x_i - z_i)^2 D_i] > 0$$

And also since, $r_i t_i - s_t^2 > 0$------------------(14)

Clearly from (13) and (14) we observe that TC is minimum for a_i^0, b_i^0 and $c_i^0 = Q_i - a_i^0 - b_i^0$

1.6. Example and Comparison:

The monthly demand for the item remains uniform at 500 units. Order size is remained constant for a year and shortages are backlogged. The firm has storage facility they are own warehouse and two rented warehouses. Holding cost for the storage.

Capacities are rupee 10,12,15 per unit time respectively deteriorations are 30,20 and 10 on accordance and shortage is at the rate Rs.2 per unit time.

Given data: -

- Demand = 5000 unit / month
- Time Horizon (T) = 1 year
- Total Annual Demand (Q_0) = 5000 × 12 = 60000 unit
- Capacities of warehouse.

Own warehouse (a_1) = 10, Rented warehouse (a_2) =12, (a_3) =15

- Deterioration rate: 0.3, 0.2,0.1 respectively,
  Shortage Cost Rs.2 per unit time

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*** For two warehouses:**

a_1 = 42353 unit

b_1 = 17647 unit

Average cost (TC)=Rs.313324.6

*** For three warehouses:**

a_1 = 19917 unit, a_2 = 38265 unit, a_3 = 1818 unit

Average cost (TC) = Rs.308888.9

Clearly A_i =Rs. (313324.6 – 308888.9) =Rs. 4435.7 which is positive

1.7. Sensitivity analysis:

Sensitivity with Respect to Holding Cost

Table-1.Sensitivity Table: Holding cost: Holding costs are varied by **±10% and ±20%**.

Change in Holding Cost	Two-Warehouse TC (RS.)	Three-Warehouse TC (RS)
−20%	301,120	296,540
−10%	307,215	302,610
Base	313,324.60	308,888.90
10%	320,540	316,120
20%	328,910	324,780

Total cost increases almost in a straight line as holding cost increases. The three-warehouse system always gives lower total cost, showing it works better. When holding cost is higher, firms keep less inventory and use shorter cycle times.

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Sensitivity with Respect to Deterioration Rate: Deterioration rates are increased and decreased by ±10% and ±20%.

Table-2: Sensitivity Table: Deterioration Rate

Change in Deterioration Rate	Two-Warehouse TC (RS)	Three-Warehouse TC (RS)
−20%	305,420	300,610
−10%	309,380.00	304,780.00
Base	313,325	308,889
10%	318,910	314,540
20%	325,840	321,720

Total cost changes a lot when the deterioration rate changes. The impact is stronger in the two-warehouse system because items deteriorate more there. The third warehouse, with low deterioration but higher rent, helps reduce losses by acting as a buffer.

Sensitivity with Respect to Shortage Cost: As, shortage cost is varied by ±10% and ±20%.

Table-3: Sensitivity Table: Shortage Cost:

Change in Shortage Cost	Two-Warehouse TC (RS)	Three-Warehouse TC (RS)
−20%	309,110.00	305,340.00
−10%	311,215	307,120
Base	313,325	308,889
10%	316,890	312,440
20%	325,840	321,720

When shortage cost increases, total cost increases but not too much. The three-warehouse system reduces shortages better because inventory is managed more efficiently.

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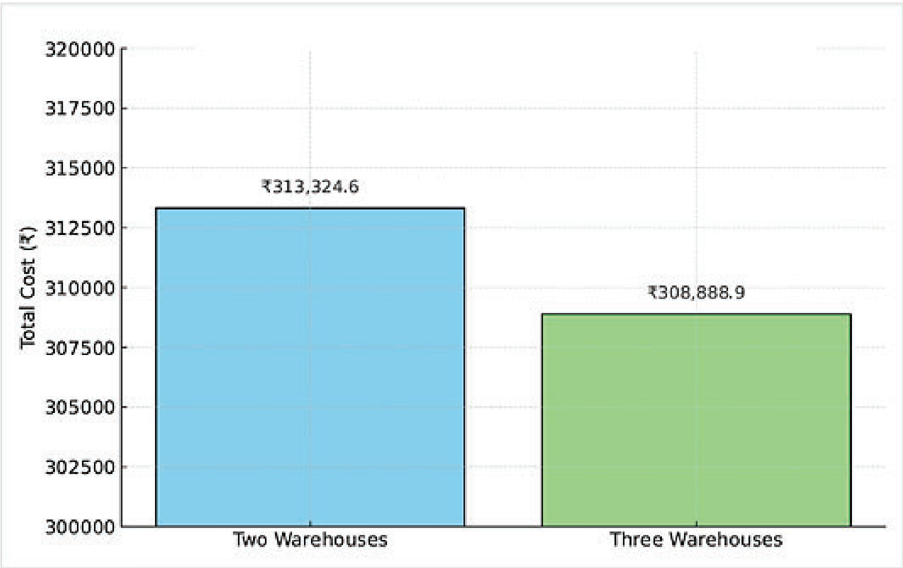
Table-4: Comparative Sensitivity Summary:

Parameter	Sensitivity Level	Key Insight
Holding cost	High	TC rises sharply; incremental storage is beneficial
Deterioration rate	Very High	Major driver of cost; justifies multi-warehouse policy
Shortage cost	Moderate	Less influential than holding and deterioration

Conclusion of Sensitivity analysis

The three-warehouse system stays the best choice even when conditions change, showing it is very reliable. Firms that handle fast-deteriorating items gain a lot from using warehouses with low deterioration, even if rent is high. The sensitivity results show that investing in better storage is more important than just choosing the cheapest rent.



Inventory Cost Comparison: Two vs Three Warehouses

1.8. Conclusion: - This indicates that the three-warehouse setup provides cost saving compared to the two-warehouse system.

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