Mathematical Modeling of Power Generation by Solar and Wind

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***Abstract*— Today's demand of electricity goes on increasing day by day, but to meet such demand we have limited energy resources. So, we need to find or search for alternatives and finally we turn towards renewable or non-conventional energy to fulfill our electricity demand. In this paper, we present the mathematical models of power generation using solar and wind energies.**

***Keywords*— Solar power; wind power; mathematical model**

I. INTRODUCTION

Need of energy plays an important role in human life. Energy in nature is in many forms like heat, light, kinetic, potential and electrical. In spite of all these we are mainly concerned about electrical energy. Demand of electrical power is increasing day by day. If we use coal for generation, then it is non-renewable and it is also costly. The same applies for nuclear power. So, we choose an alternative source among the renewable resources such as solar, wind, tidal, geothermal etc. for the production of electricity. From all these renewable sources, solar and wind is available everywhere and so the power generation using them is better than other resources.

In this paper mathematical models of power generation using solar and wind are presented. The organization of the paper is as follows. The next section describes the mathematical modelling of power generation using wind energy. The mathematical model of power generation using solar energy is presented in the third section and finally the conclusions are presented in the fourth section.

II. MATHEMATICAL MODELLING OF WIND ENERGY

The factors on which production of electricity through wind is dependent are:-

a) Output curve of power

b) Velocity of wind

c) Height of hub

The most suitable model for wind turbine power is:

$$P_{wind} = P_{RE}*(V_w - V_{wci}) / (V_{WR} - V_{wci}) \text{ if } V_{wci} < V_w < V_{WR}$$

$$P_{wind} = P_{RE} \text{ if } V_{WR} < V_w, < V_{WEF}$$

$$P_{wind} = 0 \text{ if } V_W < V_{WEF} \ \& \ V_w > V_{WEF}$$

Where P_{RE} = rated electrical power

V_{wci} = cut-in wind velocity or speed
V_{WR} = rated wind speed
V_{WEF} = cut – off wind speed

Cut in wind speed is relatively small for small scale wind turbines. So, even when wind speed is not very high the turbine will operate.

Speed of wind changes with the height. We have two laws for determining the wind speed at some vertical height. They are:

A) Log law
B) Power law

Here, we use power law for determining the vertical height which is as-

$$V_h/V_{rh} = (Z_h/Z_{rh})^{\gamma}$$

V_h=wind speed at vertical height

V_{rh}=wind speed at reference height

V_{rh}= vertical height of tower

Z^{rh}=reference height
γ=power law exponent generally it is taken as 1/7 when there are no specific site data

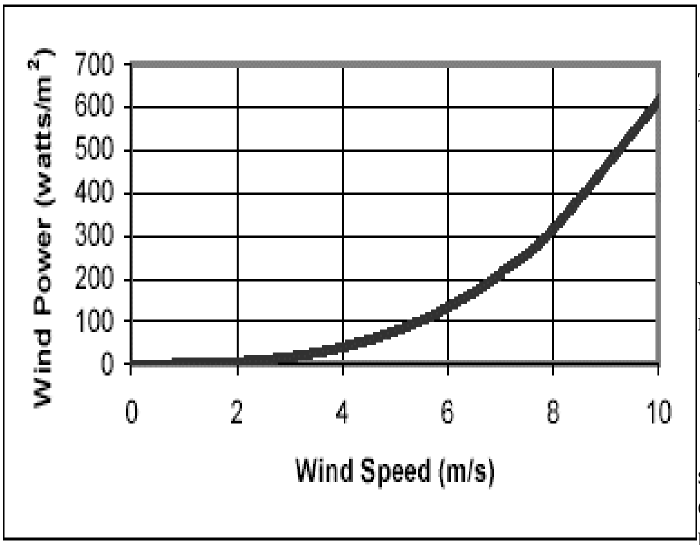

Fig. 1. Wind power vs. Wind speed

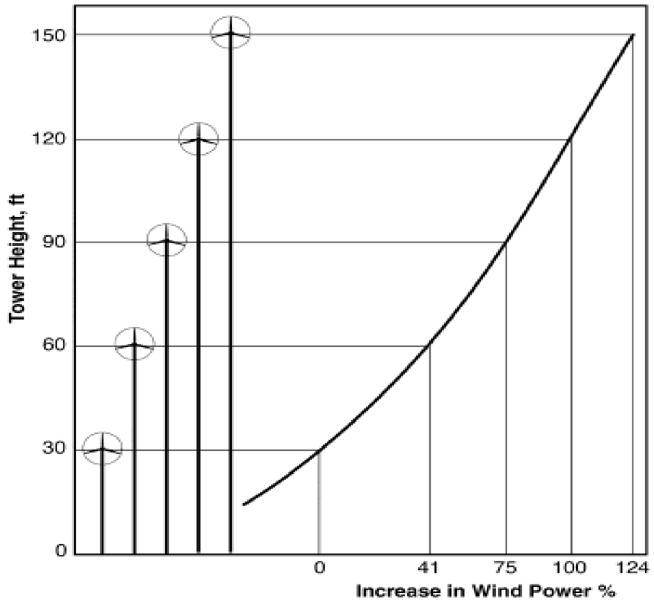

Fig. 2 Increase in Wind power vs. Tower Height

The fundamental equation governing the mechanical power capture of the wind turbine rotor blades, which drives an electrical generator is:

P_w=1/2*{air density(kg/m^3)}*{area swept of rotor(m^3)}^{3}*{power coefficient(e)}*efficiency of AC/DC converter

The maximum value of '*e*' is 0.59. Its value generally depends on rotor speed to wind speed ratio denoted by α

$$\alpha = w \times r/v$$

where w is the angular speed of the rotor and r is turbine radius.

III. MATHEMATICAL MODELLING OF SOLAR ENERGY CONVERSION SYSTEM

The three main parts that composed Photovoltaic (PV) system is:

a) PV modules
b) PV array
c) Solar radiation absorbed by PV modules

We describe each term one by one for mathematical modelling:-

a) PV modules

Performance of PV modules is a function of PV cell silicon, the temperature of solar cell and solar irradiances exposed on the solar cell. Regression parameter for maximum power output of PV module is

$$P= -(\alpha G +\beta)*(T +0.3375G) + \gamma G + ʒ$$

where
G = Total solar radiation absorbed by PV module in w/m^2

T = Temperature around PV module
α ,β, γ and ʒ are constant from result of PV modules

b) PV array

To meet the demand, a number of PV modules are connected in series and parallel connection. Series connection determines the DC output voltage and parallel connection determine the capacity of PV array output.

$$V_t = n \times v_p$$

$$P_t = m \times v_p \times i_p$$

Where V_t= Total voltage output

n= no. of PV's connected in series

v_p= single PV voltage

P_t= total capacity of PV system

m= no. of PV connected in parallel

i_p= single PV current

c) *Solar radiation absorbed by PV modules:*

Solar radiation absorbed by PV modules is dependent upon the tilted angles between solar panel and solar radiation. Perez model is utilized to determine the diffused solar radiation on any tilted angles. Modern Perez gives better result including all parameters like isotropic diffused radiation, horizon brightening etc.

We have to calculate sky clearance K_1 and sky brightness K_2:

$$K_1 = [(G_{1h} + G_{2h})/G_{1h} + 1.041\Theta^3]/[1 + 1.041\Theta^3]$$
$$K_2 = (G_{1h} \text{ x } m_a)/G = (G_{1h} \text{ x } m_a)/(G/\cos\Theta) = G_{1h}/G$$

Where m_a is mass of air
K_1 and K_2 are used for reduction of brightness co-efficient called Perez co-efficient.

K_1 and K_2 helps to calculate brightness co-efficient B_1 and B_2.
$$B_1 = B_{11}(k_1) + B_{12}(K_1)K_2 + B_{13}(K_1)\Theta$$
$$B_2 = B_{21}(K_1) + B_{22}(K_1)K_2 + B_{23}(K_1)\Theta$$

Now we calculate solar diffuse radiation on the tilted surface.
$$G_{1t} = G_{1h} \text{ x } \cos^2(\beta/2) \text{ x } (1-B_1) + G_{1h} \text{ x } B_1(a/c) + G_{1h} \text{ x } B_2 \text{ x } \sin\beta$$

Where (a/c) is used to determine angular location of circumsolar and is given by:

$$(a/c) = \max[0, \cos\phi]/\max[\cos 85, \cos\Theta]$$

IV. CONCLUSIONS

Power generation using renewable sources of energy is becoming increasingly important in the modern era. Mathematical models for power generation using these renewable sources would be of great importance for engineers. Two mathematical models, one for power generation using wind energy and another for power generation using solar panels was presented in this paper. The author intends to provide the mathematical models of other renewable sources in his future work.

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