Mathematical Modeling Applications in Water Assessment and Management

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***Abstract*—An assessment of the available water resources is a pre-requisite to undertake an analysis of the stress on the water resources and to subsequently adopt appropriate management strategies to avoid adverse environmental effects and reconcile conflicts between users.**

During the last five decades there has been a sharp increase in water consumption owing to the population explosion, unprecedented rise in standard of living, and enormous economic development. The situation has become even more difficult because of the increasing pollution of water resources. This in turn has caused serious problems impeding sustained economic and social development in many regions, even those not located in arid zones. These problems are caused not only by natural factors, such as uneven precipitation in space and time, but also by mismanagement and the lack of knowledge about existing water resources.

Research on the development and application of water balance models at different spatial and temporal scales has been carried out since later part of the 19th century. As a result, a great deal of experience on various models and methods has been gained. A large number of models has been proposed in the hydrology field. The various methods for modeling includes both traditional long-term water balance methods and the new generation distributed models for assessing available water resources under stationary and changing climatic conditions at different spatial and temporal scales. Modern models make use of GIS and remote sensing to provide adequate information. Although these models have progressed fairly through all these years, still they face a lot of challenges.

I. INTRODUCTION

Growing population faces a severe threat of stress due to water scarcity. Problems are accentuated by excess use of the limited water resources and the deteriorating quality of the available water. This combined with our lack of knowledge regarding the knowledge of exactly how much water is available poses difficulties for effective regional, national, and international water resources management (Hultcrantz,1997) . Moreover, the situation becomes even more complicated by the looming climate change which, in the longer period has the potential to decrease the availability of natural water resources in many areas of the world due to probable changes in the rainfall distribution and the increase in temperature. Due to the complexities, there arises the need to understand the technical aspects and the broader managerial and social aspects. These aspects are very well studied using the various mathematical models proposed all along the 19th century.

This paper reviews the existing methods/models for assessing regional water resources under stationary and changing climate conditions at different spatial and temporal scales, identify the progress and challenges that remain and discuss the possible further developments in the field. It discusses the representative methods/models and the different categories of these models instead of the individual models.

The models can be classified as models under stationary climate conditions and under constant climate change. The methods for simulating water resources under stationary climate conditions includes the long term water balance methods, conceptual lumped-parameter models and spatial hydrologic GIS supported models.

Methodologies for assessing hydrological responses to global climate change include five methods which include the use of direct GCM-derived hydrological output, the method of coupling GCMs and macro scale hydrologic models, the use of dynamic downscaling, the use of statistical downscaling and the use of hypothetical scenarios as input to hydrological models.

II. WATER RESOURCE MODELING UNDER STATIONARY CLIMATE

These models include the models which are applicable during constant climate. There are a large number of popular mathematical models of respectively large and small watershed hydrology. Watershed modeling includes data acquisition , model components, model construction, calibration and verification, analysis of risk and reliability (Singh a,b,2002) .

A. Long Term Water Balance Method

It is a traditional way for water resources assessment which is based on the long-term average water balance equation (say one year or many years) over a basin as

$$P = AE + Q \qquad .....(1)$$

where P, AE and Q are the long-term average annual precipitation, actual evapotranspiration and stream flow, respectively.

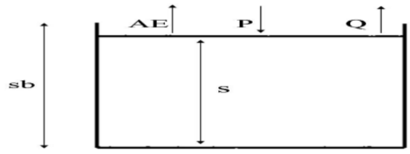

Fig1.Lumped Bucket Model

The concept represented in Eq(1) can be represented by a simple lumped bucket model. In order to solve the Eq(1) and calculate the available water resources, Q, two terms P and AE, must be known. The areal precipitation P is usually computed from point measurements. The key element in the long-term water balance of a catchment or a region is the value of the actual long-term evapotranspiration, AE. In order to calculate this value, we can relate the changes in actual evapotranspiration to the changes in precipitation and potential evapotranspiration, PE.

There arises two extreme conditions :-

a) Water limited system

b) Energy limited system

In the first case where precipitation is much less than the potential evapotranspiration, on the annual or longer time scale, all incoming precipitation is evaporated back. The actual evapotranspiration rate is controlled by the amount of rainfall, regardless of how high PE is. Thus this is a "water limited" system. The first limiting condition occurs when

$$AE = P \text{ and } Q = 0 \qquad .....(2)$$

In the second case precipitation is much greater than the potential evapotranspiration, we would have the second limiting condition

$$AE = PE \text{ and } Q = P - AE \qquad .....(3)$$

There are variations present in the value of precipitation and potential evaporation each year. This results in different soil moisture content during different time periods. The value of AE thus remains between these two limiting cases. The ratio AE/P is represented as a function of PE/P. Various such relationships have been proposed according to the data collected around the world. These were proposed by

Schreiber :-

$$AE/PE= (P/PE)[A-\exp(-PE/P)] \qquad .....(4)$$

Ol'dekop :-

$$AE/PE= \tanh(P/PE) \qquad .....(5)$$

Turc-Pike

$$AE/PE=P/PE/\text{sqrt}(1+(P/PE)\text{^}2) \qquad .....(6)$$

This method uses meteorological data for estimating the water resources. It is one of the most simplest methods used. However it faces the disadvantage that in arid and semi arid regions the river run off is very small and close to the error of determination of evaporation and precipitation. Also it is very difficult to obtain estimates of water resources.

B. Lumped Conceptual Water Balance Models

Various conceptual lumped rainfall models are being developed and used nowadays. The principal concept behind these models is the catchment water balance. The distinguished feature of these models is soil moisture accounting. Such models relate the rates of water storage change within the control volume to flows of water across the control surface. The equation for such models can be summarized as

$$S(t + 1) = S(t) + P(t) - AE(t) - Q(t) \qquad .....(7)$$

Here $S(t)$ is the amount of soil moisture stored at the beginning of the time interval t, $S(t + 1)$ the storage at the end of that interval, and the flow across the control surface during the interval consists of precipitation $P(t)$, actual evapotranspiration, $AE(t)$, and soil moisture surplus, $Q(t)$, which supplies stream flow and groundwater recharge.

The actual evapotranspiration is estimated using a soil moisture extraction fraction or coefficient of evapotranspiration. This coefficient is used to express the relation between the actual rate of evapotranspiration AE and the potential rate of evapotranspiration PE. This is based on the function of current soil moisture content and moisture retention properties of the soil.

A basic form of such functions can be represented as

$$AE= PE.f(SMT/SMC) \qquad .....(8)$$

Here SMT is the actual moisture storage and SMC is the soil moisture storage at field capacity. The moisture is extracted from the soil at the potential rate until some critical moisture content is reached when evapotranspiration is no longer controlled by meteorological conditions. Below this critical moisture content, there is a linear decline in soil moisture extraction until the wilting point is reached.

In the lumped models, the catchment is considered as a single entity which transforms excess rainfall into outflow hydrograph. The soil moisture content is considered as a resident in some storage area or reservoir. The different storage concepts (Fleming, 1975) are:-

1) Linear reservoir :- Q = K .S

2) Logarithmic reservoir :- Q = K . exp (S)

3) Non-Linear reservoir :- Q = K . S^(1/m)

Here Q is reservoir outflow, S is the storage K is the storage constant and m is the exponent. These models are usually used for a short period of time like for a day. Models for time period spanning over a month have also been developed now.

The major drawback with this model is regarding the function of current soil moisture content and moisture retention properties of the soil. This function vary with the leaf area index i.e. the vegetation amount and pattern of the soil. Also determining the water holding capacity is quite a difficult task. To improve the results, it is desirable to correlate the function with the satellite derived indices of vegetation cover of the area.

C. Spatial Hydrology Models

These models make use of GIS data structures to stimulate the water flow and transport in a specified region (Maidment, 1996). These models are used for a variety of operational and planning purposes. It provides spatial resolution finer than the one that can be provided by observed data. It also gives great amount of information regarding effects of land use and climate variability and help in estimating point and non-point sources of pollution leading to the streams.

The current models uses the factors which determine the formation of runoff which include the topography of the water reservoir, negative exponential law relating transmissivity of the soil with the vertical distance from the ground level. In these models the total flow is the sum of surface runoff and the flow in the saturated zone. The surface runoff is also a sum of the water on the ground surface entering the soil and that generated by saturation excess mechanism.

For these models the catchment area can be divided either into hydrological response units which are similar with respect to the selected characteristics and which are modeled separately (Arnold 2000, Becker and Braun 1999) or into equally spaced square grid elements (Arnold 1999, Yao and hashino 2001). In the first approach the problem that arises is that the characteristics should be related to hydrological process. If there are a large number of characteristics, the partitioning will be very detailed and if we consider few characteristics, we are neglecting the fact that all the water units are different in some aspect. In the second approach the problem is that the different units will have completely different characteristics and hence there will be large differences in the modeling of these units

III. WATER ASSESMENT UNDER CLIMATE CHANGE

The regional water resources are greatly affected by the climatic changes happening in that region. These water resources affect our day to day life, the changing eater resources have vast social economical effects. Thus to account for these climatic changes climate models which simulate climatic effects of the increasing concentration of the greenhouse gases, hydrological models which simulate hydrological effects of the climatic changes and downscaling techniques which acts as a link between these two models has been developed.

To project the climate changes, global climate models are used. These models are the type of general mathematical climatic models of the general circulation of the planetary atmosphere or ocean based on equations on rotating sphere with thermodynamic terms for the energy sources. Atmospheric and oceanic models are the key components of the global climatic models along with sea ice and land surface components. In these models, scientists divide the planet into a 3-dimensional grid, apply the basic equations, and evaluate the results. Atmospheric models calculate winds, heat transfer, radiation, relative humidity, and surface hydrology within each grid and evaluate interactions with neighboring points.

Global Climate Models (GCMs) used for climate studies and climate projections are run at coarse spatial resolution and are unable to resolve important sub-grid scale features such as clouds and topography. As a result GCM output cannot be used for local impact studies. To overcome this problem downscaling methods are developed to obtain local-scale surface weather from regional-scale atmospheric variables that are provided by GCMs. Two main forms of downscaling technique exist. One form is dynamical downscaling, where output from the GCM is used to drive a regional, numerical model in higher spatial resolution, which therefore is able to simulate local conditions in greater detail. The other form is statistical downscaling, where a statistical relationship is established from observations between large scale variables, like atmospheric surface pressure, and a local variable, like the wind speed at a particular site. The relationship is then subsequently used on the GCM data to obtain the local variables from the GCM output.

All these methods consists of these basic steps (Wilby 1994) :-

a) The global climate models are used to derive the parameters which are used for prediction of future climatic scenarios.

b) These climatic scenarios are further downscaled into local or regional climatic scenarios with various statistical tools.

c) These are then used to run previously tested hydrological models.

These models don't necessarily predict the future climate changes, they only show how much hydrological sensitive the system is to the possible climatic changes in that region.

IV. CALIBRATION AND VALIDATION OF THE MODELS

After the formation of models, next important step is to calibrate and validate these models. There have been several methods and tests developed to check these models and make sure desirable results are achieved. The tests developed follows a hierarchical framework because the modeling tasks are ordered according to the increasing complexities (Klemes 1986). In the stationary conditions simple tests are used while in transient conditions same tests are used along with differential approach.

In simple split sample test, measured time series data is split into two sets and each of them is used for testing and calibration. The results from both samples are then compared with each other. For differential test the data is divided according to some variable which is meant to show that the model has general validity so that it can predict the value of the output variables for conditions different from which it is calibrated. The remaining methods remains same.

In proxy basin test, the data is collected from two different basins. This test shows that model has greater general validity as the model is calibrated against the data of one basin and is validated against the data of another basin. In differential approach the data is separated into two sets based on some output variable like rainfall.

In both of the differential models, the models are calibrated against the data for one set of model and validated for another contrasting set of output variable. For example if the model is calibrated for a dry season then it should be validated for a wet season. For the distributed models, the comparison is done according to the data at the specific points which are at the external boundaries or on the intermediate locations in the grid.

V. PROGRESS MADE BY THESE MODELS

Modeling is perhaps the most accurate way to address the water resource problem and to access the quantity of the available water resources. These days more and more models are being developed and used continuously for water resources planning, development, assessment, and management.

1) Many hydrological analyses uses lumped conceptual models for estimating outflow

2) The use of these models is increasing day by day as the demand for forecasting of future hydrological conditions resulting from climatic changes is increasing.

3) With the development of technology, new computer simulations can be created to determine water quality, ecology, risk and uncertainty, environmental impact, etc in addition to normal hydrology assessments.

4) The readily available data regarding all the water resources and terrain has made modeling easy and practical.

VI. CHALLENGES FACED BY THESE MODELS

Despite all the progress these models have made, there are a large number of challenges these models face.

1) The integration of the models for ground water and surface water has not been done properly till date.

2) Due to complex nature of modeling of large lakes or rivers it is still done through traditional modeling techniques.

3) To use the climatic models at a smaller scale we require downscaling. Without downscaling the data from global climate models can only be used at macro level modeling.

4) These models cannot be directly applied to the water bodies whose data is not available with us. A large amount of uncertainty can occur if we use a model calibrated for a certain water region in some other water body. To use these models these uncertainties has to be removed continuously.

VII. CONCLUSION

In today's world with pressure on all the resources including the water resources, the modeling of the water bodies have become a necessity. The Mathematical models have provided a great basis for assessment of the water resources. Through these models, better water resource management has been possible. The modeling provides great amount of information regarding the fresh water availability with us along with helping us in future predictions regarding climatic sensitivity of these water resources. With the new technologies coming into picture, the modeling has becoming easier. Still these models face a certain amount of challenges and research should be continued in this area.

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