

# Mathematical Modeling and Simulation of Pantograph Mechanism in MATLAB

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**Abstract**—The paper explains the mathematics behind simulation of Pantograph Mechanism in MATLAB. Pantograph mechanism is used for copying 2D objects like photographs or different shapes, and uniformly scaling them such that the copy is enlarged or shrieked by a scale factor.

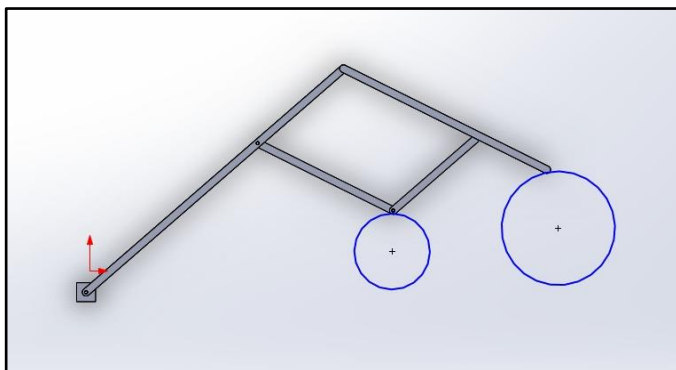
**Keywords**—Pantograph Mechanism, Mathematical Modeling, Simulation MATLAB.

## I. INTRODUCTION

Pantograph Mechanism is a 5-bar mechanical linkage in which the links are connected in a way such that if one point is tracing an image then the second point will produce identical, enlarged or miniaturized model of the same, because the links are coupled based on parallelogram principle. This mechanism has various applications in the areas such as sculpture, engraving, minting, milling, etc.[3]

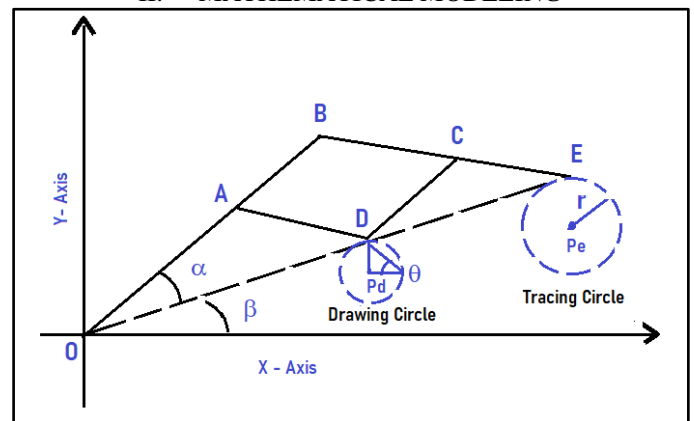
The pantograph was initially used for drafting i.e copying and scaling line drawings. However, contemporary use of it is seen in sculpture and minting, where 3 dimensional pantographs are used [1].

Before the advent of Computer Numeric Control (CNC) and Programmable Logic Controller (PLC), pantograph was also used in control the movement of cutting tool in milling process.



(Figure 1- Rough Diagram of the mechanism)

## II. MATHEMATICAL MODELING



(Figure 2 – Main Reference figure of the simulation.)

### A. Nomenclature:

| Sr no | Symbol   | Interpretation  |
|-------|----------|---|
| 1     | $\alpha$ | Angle AOD   |
| 2     | $\beta$  | Angle between line EO and x-axis  |
| 3     | $\theta$ | Angle subtended by line formed by joining center of drawing circle and the point tracing the circle w.r.t x-axis. |
| 4     | dOA      | Euclidean distance between origin and point A   |
| 5     | dOB      | Euclidean distance between origin and point B   |
| 6     | dOD      | Euclidean distance between the origin and point D   |
| 7     | Pd       | Center point of a drawing circle  |
| 8     | Pe       | Center point of a Tracing circle  |
| 9     | w        | Angular speed   |

(Table – 1)

### B. Calculations:

The Mathematical modeling and simulation method proposed in this paper is for the engraving or drawing the circle in the enhanced format with the ratio r. Where r being the radius of the enhanced circle.

In this case we are considering the user input for link length OA and OB and AD.

Based on these three links we can calculate the entire enhancement ratio of the circle drawn by the tracing point (i.e the E point)

Using trigonometric relations in the above geometry, we calculate the angles  $\alpha$ ,  $\beta$  &  $\theta$ .

Calculation of angle  $\alpha$

$$\alpha = \cos^{-1} \left( \frac{OA^2 + OD^2 - AD^2}{2 * OA * OD} \right)$$

Calculation of angle  $\beta$

$$\beta = \tan^{-1} \left( \frac{pd_y + \sin \theta}{pd_x + \cos \theta} \right)$$

For  $\theta$  –

Since the simulation consist of a circle enhancement the angle  $\theta$  will vary from 0 to  $2\pi$  with the interval of 1 radian  
 Considering,  $w = 0.1$  radian per time instance  
 And  $t = 1:1000$  - time instance

$$\theta = w \times t$$

Then we find the coordinates of all the points in the mechanism by mapping them in X-Y plane.

$$A = OA * [\cos(\alpha + \beta), \sin(\alpha + \beta)]$$

$$B = (OA + OB) * [\cos(\alpha + \beta), \sin(\alpha + \beta)]$$

$$C = [pd_x + \cos \theta + AB * \cos(\alpha + \beta), pd_y + \sin \theta + AB * \sin(\alpha + \beta)]$$

$$D = [pd_x + \cos \theta, pd_y + \sin \theta]$$

In MATLAB the coordinates of the points are stored in an array, so to calculate the distance between them, we have to consider the Euclidean distance.

Euclidean Distance formula is as follows:

$$d(\mathbf{p}, \mathbf{q}) = \sqrt{\sum_{i=1}^n (q_i - p_i)^2}$$

$\mathbf{p}, \mathbf{q}$  = two points in Euclidean n-space

$q_i, p_i$  = Euclidean vectors, starting from the origin of the space (initial point)

$n$  = n-space

The Euclidean Distance between points is :  
 In MATLAB there is an inbuilt function for the calculating Euclidean distance. The below equation gives distance between the point origin and point A.

$$dOA = \sqrt{[(OA \cos(\alpha + \beta)) - 0]^2 + [(OA \sin(\alpha + \beta)) - 0]^2}$$

Similarly, we calculate the distance between the OB, BC which are required for the further calculations.

While considering the coordinates of the drawing point (D) the constraints are –

$$0 < Pd(x, y) < OA + AD$$

D point is the Drawing point of the Mechanism.  
 Euclidean distance of point D from origin.

$$dOD = \sqrt{[(Pd_x + \cos \theta) - 0]^2 + [(Pd_y + \sin \theta) - 0]^2}$$

Enhancement Ratio of the circle is calculated by

$$r = \frac{dOB}{dOA}$$

Coordinates center of the Tracing Circle

$$Pe = r * Pd(x, y)$$

E is the Tracing Point and its coordinates are

$$E = r * [pd_x + \cos \theta, pd_y + \sin \theta]$$

### III. SIMULATION

For the simulation in MATLAB considering the following test case –

OA = 6 units  
 AB = 3 units  
 AD = 4 units  
 Pd = [5,5]  
 w = 0.1 rad/s  
 Radius of drawing circle = 1

Therefore, radius of tracing circle (r) = 1.5

#### III.1 MATLAB CODE:

```

clc;
clear all;

%Length of Link OA
a = input('input Length of Link OA');
%length of Link AB
b = input('Input length of link AB');
%length of Link AD
d = input('Input length of Link AC');
% Origin Point
O=[0, 0];
%center of the Drawing circle point
pd=[5,5];
%plot parameters
axis(gca, 'equal');
axis([-10 20 -5 20]);
%angular speed
w=0.1;
    
```

```
for t=1:1000 % Simulation run time
    theta=w*t; % theta changing w.r.t angular speed

    % Calculating Euclidean distance of a drawing point
    OD=[0,0;(pd(1)+cos(theta)), (pd(2)+sin(theta))];
    dOD = pdist(OD,'euclidean');

    %calculations of alpha and beta
    alpha = acos((a^2+(dOD*dOD) - d^2)/(2*a*dOD));
    beta = atan((pd(2)+sin(theta))/(pd(1)+cos(theta)));

    %Coordinates of points of the mechanism
    A = a*[cos(alpha + beta) sin(alpha + beta) ];
    B = (a+b)*[cos(alpha + beta) sin(alpha + beta)];
    D = [(pd(1)+cos(theta)) (pd(2)+sin(theta))];
    C = [(pd(1)+cos(theta)+b*cos(alpha+beta))
    (pd(2)+sin(theta)+b*sin(alpha+beta))];

    % Distance calculation for OA and OB
    OA = [0,0 ; A(1),A(2)];
    dOA = pdist(OA,'euclidean');
    OB=[0,0 ;B(2),B(1)];
    dOB = pdist(OB,'euclidean');

    %Calculation of the enhancement Ratio
    r=(dOB/dOA);

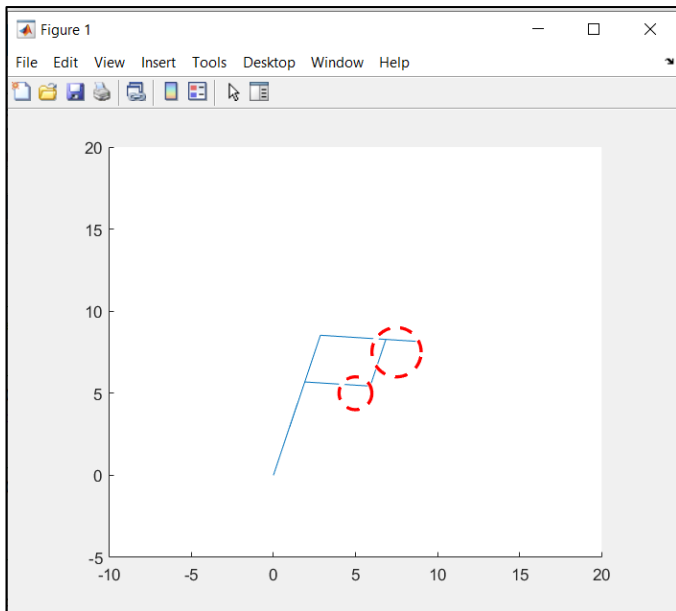
    %Center point of the Tracing Circle
    Pe=r*[pd(1),pd(2)];
    %coordinates of tracing point
    E=r*[D(1), D(2)];
    %distance between BE
    BE = [B(1),B(2);E(1),E(2)]
    dBE = pdist(BE,'euclidean');
    %The following lines represent the mechanical
    %links of the mechanism
    OA=line([O(1) A(1)], [O(2) A(2)]);
    AB=line([A(1) B(1)], [A(2) B(2)]);
    BC=line([B(1) C(1)], [B(2) C(2)]);
    CD=line([C(1) D(1)], [C(2) D(2)]);
    AD=line([A(1) D(1)], [A(2) D(2)]);
    CE=line([C(1) E(1)], [C(2) E(2)]);
    %Drawing the trajectories of the drawing and tracing circle
    Pd_circ=viscircles(pd,1);
    Pd_traj= viscircles([pd(1) pd(2)],1,'LineStyle','- -');
    Pe_circ =viscircles(Pe,r);
    Pe_traj= viscircles([r*pd(1) r*pd(2)],r,'LineStyle','- -');

    %interval for updation
    pause(0.001);
    %delete previous shapes
    delete(Pd_circ); delete(Pd_traj);
    delete(Pe_circ); delete(Pe_traj);
    delete(OA); delete(AB);
    delete(BC); delete(CD);
    delete(AD); delete(CE);

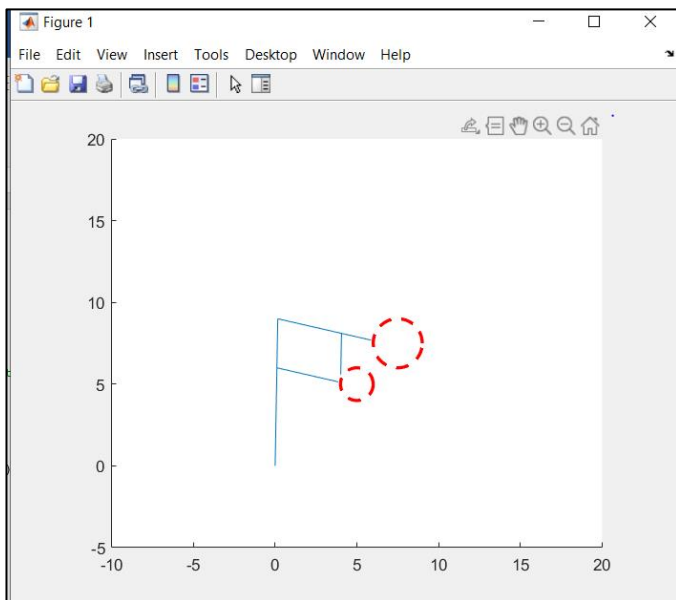
end
```

III.2 SIMULATION RESULTS :

MATLAB Workspace:



(Figure 3 – MATLAB simulation -1)



(Figure 4- MATLAB Simulation -2)

| Workspace |                     |
|-----------|---------------------|
| Name ^    | Value               |
| a         | 6                   |
| A         | [2.0318,5.6455]     |
| AB        | 1x1 Line            |
| AD        | 1x1 Line            |
| alpha     | 0.5713              |
| b         | 3                   |
| B         | [3.0476,8.4683]     |
| BC        | 1x1 Line            |
| BE        | [3.0476,8.4683;8... |
| beta      | 0.6540              |
| C         | [6.8782,7.3164]     |
| CD        | 1x1 Line            |
| CE        | 1x1 Line            |
| d         | 4                   |
| D         | [5.8623,4.4936]     |
| dBE       | 6.0000              |
| dOA       | 6                   |
| dOB       | 9                   |
| dOD       | 7.3864              |
| E         | [8.7935,6.7405]     |
| O         | [0,0]               |
| OA        | 1x1 Line            |
| OB        | [0,0;8.4683,3.0476] |
| OD        | [0,0;5.8623,4.4936] |
| pd        | [5,5]               |
| Pd_circ   | 1x1 Group           |
| Pd_traj   | 1x1 Group           |
| Pe        | [7.5000,7.5000]     |
| Pe_circ   | 1x1 Group           |
| Pe_traj   | 1x1 Group           |
| r         | 1.5000              |
| t         | 1000                |
| theta     | 100                 |
| w         | 0.1000              |

(Figure 5 – MATLAB workspace displaying all the variables )

We have successfully done the mathematical modeling of pantograph to trace a circle in MATLAB, which can be seen in above figures

VI. REFERENCES :

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