Mathematical Model of Parametric Domain Design Approach for Filament Winding Path on Complex Composite Parts

Filip Kochoski[1], Vladimir Dukovski[2], Samoil Samak[1]
[1]Mikrosam DOO, Prilep, N. Macedonia;
[2]Ss. Cyril and Methodius University, Skopje, N. Macedonia;

Abstract—**Producing complex composite parts is a problematic issue in the composite materials industry. That is why, designing such a complex part must precede production. Design is usually the first step of production. When it comes to composite parts, especially complex ones, their design is the most difficult hurdle one must cross. For composites produced with resin fibers or prepared tapes, it is absolutely necessary to initially design the path of those fibers, i.e. tapes. The design of such paths is not an easy task for complex composite parts with a polysurface shape (multiple basic surfaces joined together). The algorithm model presented here shows how to design the paths of composite complex parts through the parametric space of the surface**

Keywords*—Complex surface; polysurface; parametric space; fiber path*

I. INTRUDUCTION

Filament winding is the application that continuous reinforcement filaments are wound around a rotating mandrel. A filament winding cell can consist in several components. Basically, the main ones are fiber supply spools, a resin impregnator (if needed), fiber delivery or delivery eye unit, and a mandrel. The delivery eye places fibers on a specified path on the mandrel. The strands can either be pre-impregnated with polymeric resin or impregnated during the process. Once the winding is completed, the resin is cured and, then, the mandrel is removed [1-10].

Because of the mechanical restrictions, part shapes that are produced with filament winding techniques are limited. Commercial applications of filament wound parts; wind-turbine blades, shafts, reinforced tubes, and special storage tanks. Today, many complex parts are produced with this technology.

A filament winding pattern for elbows and a new data processing method are presented in [4,5], and new method was proposed to measure the slippage coefficient between fiber tows and a mandrel surface for a non-geodesic filament-winding process by mathematical model to wind complex shapes [7-12]

In papers [13, 14] the method of winding pattern design of filament wound composite pressure vessel with unequal openings was proposed using geodesics and non-geodesics, which also satisfy the winding principles. The Lotus process [15] solves a key problem to automating continuous motion and access around complex shapes. The new variable curvature quasi-linear function to describe curvilinear fiber path is proposed for complex parts [16-19]. The results from Lei Zu et al. [20-22] also reveal that the structural efficiency of circular toroidal vessels can be significantly improved using non-geodesic winding. Examples of winding angle, mandrel rotation and non-geodesic path in cylindrical and non-cylindrical surfaces of revolution are shown and discussed [19, 23]. S Koussios et al. in the paper [24] are using the basic equations supporting such a path description between basic geometric quantities (metrics and curvatures) and the resulting fibre path orientation (winding angle distribution), but numerical example is given for geodesic and non-geodesic windings to clarify the solution from Faissal Abdel-Hady [26]. Wang Xianfeng et al. [27] are proposes a winding pattern design method combined with patch winding method and traditional winding method for the S-elbow composed of two elbows with different radii.

Winding (or placement) fiber path design is a key step in designing a composite product. This path design along which the product will later be produced first needs to be modeled using existing mathematical models and tools. The most common model when designing fiber paths of winding (or placement) is the model by which paths and the composite model are replaced by objects with curves and surfaces, respectively. Then, with these objects the mathematical modeling is built, based on differential geometry and numerical analysis, having in mind previously clearly defined conditions for the objects, such as geodetic curves, surface curves and so on, depending on the surface or technology to be applied in its production. In practice, the most commonly produced composite parts are those in the form of a tube or vessel, so the models for these surfaces are best built, but when we want to produce a complex composite part, by which we mean non-axis-symmetrical body, then mathematical models for this problem become much more complex and much more difficult to define and realize the model itself.

The aim of this paper is to give a different approach from the known models of differential geometry and to address the so-called parametric approach to the problem. With this we expect to skip the complexity of previous mathematical models and give a more intuitive design look. By discretising this problem with a different model of parametric approach, a larger deviation in accuracy is normally expected compared to the models based on differential geometry.

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II. PATH DESIGN FOR WINDING PROCESS

A. *Problem description: path design for winding a complex composite part*

In the production of composite parts, regardless of the technology we want to apply, we are faced with the first problem, and that is the form of the product that we want to produce. As already mentioned at the beginning, for some forms a good mathematical model has already been set, and that is above all the simplest forms of pipe and vessel. However, in practice there is a need for more complex parts such as the so-called connectors or fittings that can be in the form of a toroidal section, the so-called elbows, in the form of the letter T, S and so on. Many authors have already offered a solution for these standard connectors, such as the authors Hai-Sheng L. and You-Dong L[28] they offer a solution with geodesic curves on the torus part, also in another paper in Seereeram S. and Wen J.TY.[29] a solution of a T-connector with geodetic lines is considered. With the advancement of technology for the production of these parts, there is a need for more complex parts of these so-called connectors. The first more complex challenge is the so-called pipe system, Figure 1.

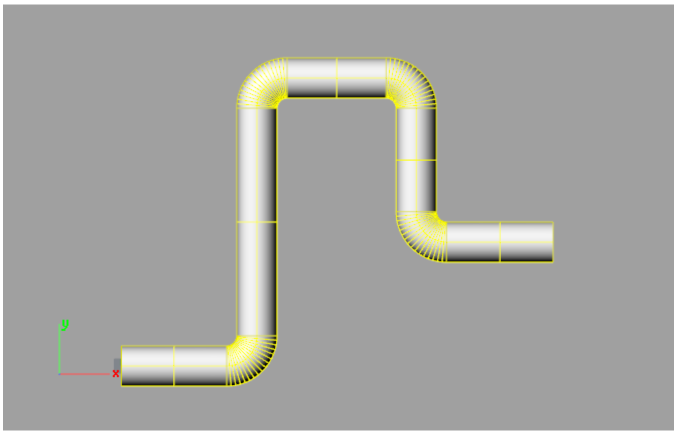

Fig. 1. Tube system

To produce this complex composite part, as in the example shown in Figure 1, it is necessary to make several separate parts (9 in the figure) as a combination of pipes and connectors. There are already mathematical models and solutions for these parts, but their connection remains a major problem. The best composite characteristics still remain in the production of this part as a whole.

In addition to this problem, there are other forms of composite parts that are more often required for their application, such as connectors with non-circular cross-section, including elbows, T and S shapes with triangular, square, polygonal cross-section and cross-section with an irregular geometric shape. These are complex shapes in which it is very difficult to analytically and explicitly present the surface of that model, so the use of differential geometry is practically impossible. The two main objects of path design come down to the object-surface and the object-curve that lies on the surface.

The question of mathematical model is: how to find a way to generate these paths in a complex geometric shape designed to produce a composite with fiber winding technology.

B. *Defining variables and their relationship in the model*

We set the problem by using an approximation of the surface of the part, from Figure 1, and from a part that is composed of several basic geometric parts we present it as a connected surface. For that purpose, we equate this problem with the so-called "Curved tube" as in Figure 2.

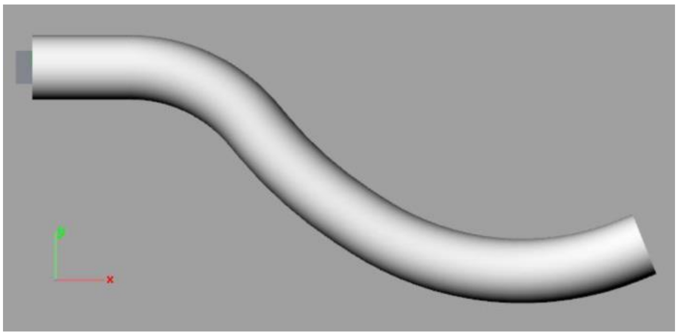

Fig. 2. Curved tube

It is important to emphasize that the approximate part we are analyzing does not have to have a circular cross-section or a tubular longitudinal part. Hence we can assume that we know the following parameters: the length of the cross section (on tubular models it is on models with different cross sections we will take the largest), the length of the mandrel in the longitudinal direction (usually the central axis, but depending on the purpose of winding , the longest or shortest such curve can be chosen).

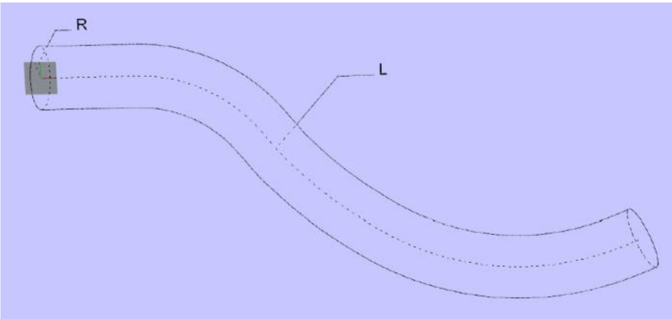

Fig.3. Geometric property of "Curved tube"

Other parameters that affect the winding itself will be defined as w the width of the fiber (threads, tape, etc.) and the winding angle α_w defined in relation to the longitudinal curve.

The idea of this algorithm is to "unfold" the surface as a parametric surface that would be a parametric rectangle $L_C \times L$ in order to generate the necessary curves here and to later copy them on the 3D surface. Of course, it goes without saying that if the surface is not so-called developable (such as cylinder, cone, etc.) in the general case we would expect deviations of the curves with respect to the angle we have previously defined. If the goal in any path generation of a given mandrel is to be as close as possible to the geodetic (important in the filament winding process), to the development surfaces (with zero Gaussian curvature) this is not a problem when applying this algorithm, but in all other cases the deviation from the geodetic lines is more than clear.

The variables in this model will be divided into mandrel parameters, those that are directly related to the geometry of the model under consideration, as L_C and L, and path parameters, those that are directly related to the geometry of the path being generated, as well. It can be seen that these input parameters in the algorithm affect the geometry of either the mandrel (model) or the path (fiber). To simplify the basis of the approach of this algorithm, only the geometric parameters that affect the production process of the selected composite part are taken into account, while the process parameters that affect the process and the technology of composite production are excluded from this algorithm. The aim is to dwell on the main idea that we have set as a problem for solving the mathematical problem in modeling, and that is to generate paths of complex geometric shapes.
It can also be concluded that these parameters are independent of each other, neither parameter affects the other. The only connection is the end result, the 3D dots generated by this algorithm. Any change in these input parameters only affects the change in the end result

III. EXPERIMENTAL INVESTIGITION

A. *Assumptions included in the model*

The main assumption of the model is mentioned in the previous chapter, and it is the deviation of the curves from the geodesic principle in undevelopable surface areas. Given the intention to first construct mathematical lines in the parametric space, those lines are straight lines in that space, so these lines in the developable surfaces would be geodetic lines because the geodetic curve remains unchanged. In contrast, in the general case, the generated straight lines in the parametric space when mapping the 3D surface will deviate from the geodetic ones and this is the main assumption in the model.
Another assumption is that the constant distance between two straight lines in the parametric space will not be maintained when mapping onto surface, in the general case. Again, developable surfaces are excluded from this assumption.
The result we expect from this model are points on the surface whose interpolation will result in a curve on the surface in space. A mathematical model for fitting a curve through these points is not part of the problem of this model.

B. *Experimental work on parametric domain design model*

Let U and V be the parametric intervals $U=[U_0,U_1]$ and $V=[V_0,V_1]$ with which a new reparametrication of the space in $U=[0,L]$ and $V=[0,L_C]$ is made. The L and L_C marks remain the same as in section 2, the length of the selected longitudinal curve and the length of the selected transverse curve, respectively. Further in this model we will understand that such a change of the parametric space has been made.
Next we will define the lengths of the domain by U and V:

$$L_U = |U_1 - U_0| = L$$
$$L_V = |V_1 - V_0| = L_C \qquad (1)$$

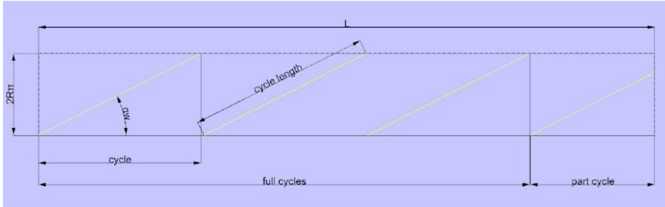

Fig.4. Parametric space and curve path

Figure 4 shows the construction of a curve at a given angle in the parametric space. We define a cycle, i.e the length of a cycle as the length required for the curve to change the parameter U by one circle around it. We define it as follows:

$$L_{cycle} = \left|\frac{L_V}{\tan\alpha_w}\right| = \frac{L_C}{\tan\alpha_w} \qquad (2)$$

We define the number of whole cycles needed to construct a path from one end of the parametric space to the other:

$$N_{cycle} = \left\lceil \frac{L_U}{L_{cycle}} \right\rceil = \left\lceil \frac{L}{L_{cycle}} \right\rceil \qquad (3)$$

We define the number of rotations around the V parameter needed to fill the width of the fiber / fibers / tape (w). This number shows us how many times we need to "rotate" the path around the body (along V) to cover the whole body:

$$N_{rot} = \left\lfloor \frac{L_V \cos\alpha_w}{w} \right\rfloor + 1 \qquad (4)$$

Since we have an integer number of such rotations (as many as there should be fibers in one direction a smaller part of an entire rotation remains (which is included in the number of N_{rot}) we need to make a computational correction of w, so to have a full coverage we need to have a little overlap of the mandrel fibers, which is normal for composites produced with this technology. We make the correction as follows:

$$m = \frac{w}{\cos\alpha_w}$$
$$\Delta m = L_V - (N_{rot} - 1)m = L_C - (N_{rot} - 1)m \qquad (5)$$
$$m_{new} = m - \frac{m - \Delta m}{N_{rot}}$$
$$w_{new} = m_{new} \cos\alpha_w$$

The number m in the equations above represents the part occupied by the tape at a given angle on a U parametric iso-

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curve, i.e the part of the length occupied by L_C. Furthermore, in the model and in the algorithm, when used, m and w we will understand that they are already corrected as above.

Let the part of the curve required for one cycle be interpolated by N points, then for each rotation indexed by $r=\{0,1,\ldots,N_{rot}-1\}$, for each whole cycle indexed by $i=\{0,1,\ldots,N_{cycle}-1\}$ and for each such point of the surface indexed by $j=\{0,1,\ldots,N\}$ we define parametric variables with:

$$t_U = L_{cycle}(i+\frac{j}{N-1})$$
$$t_V = (\frac{jL_{cycle}}{N-1}\tan\alpha_w + rm)\bmod(L_V) \quad (6)$$

The required point is $P_1 = S(t_U, t_V)$

However, these calculations can also be used to calculate the points of the cross path ($-\alpha_w$) for which we know that there will be the same U parameter after, so it only remains to calculate a new parameter V by defining the extension (or reduction) of the already calculated parameter. We define the extension (by sign) of a parameter as follows:

$$E_V = L_V(1-\frac{2j}{N-1}) \quad (7)$$

The new parameter V for the cross curve is

$$\overline{t_V} = (t_V + E_V + rm)\bmod(L_V) \quad (8)$$

The required point of the cross curve is

$$P_2 = S(t_U, \overline{t_V}) \quad (9)$$

It remains to define the points for the last cycle which is not a whole cycle (marked as part cycle in Figure 4). We calculate the residual value of that part of the parametric space as:

$$res = L_U - N_{cycle}L_{cycle} \quad (10)$$

We will define a variable J that is initially zero and increase it in each iteration for $\frac{L_{cycle}}{N}$ (uniform distribution of points also for whole cycles) until the condition $J + N_{cycle}L_{cycle} < L_U$ is met. The points obtained on the incomplete part of the cycle are:

$$t_U = J + N_{cycle}L_{cycle}$$
$$t_V = (J\tan\alpha_w + rm)\bmod(L_V) \quad (11)$$

Analogous to the above, the points of the cross curve are obtained by defining an extension (by sign):

$$E_V = L_V(1-\frac{2J}{L_{cycle}})$$
$$\overline{t_V} = (t_V + E_V + rm)\bmod(L_V) \quad (12)$$

When interpolating curves we want them to start and end exactly at the beginning of the edge of the surface until the other end (opening), so it is necessary to define the last points of the curve and the cross curve at the ends as follows:

$$t_U = L_U$$
$$t_V = (res\cdot\tan\alpha_w + rm)\bmod(L_V)$$
$$E_V = L_V(1-\frac{2res}{L_{cycle}}) \quad (13)$$
$$\overline{t_V} = (t_V + E_V + rm)\bmod(L_V)$$

Thus we have defined the points of all curves in the parametric space, and therefore the points on the surface whose interpolation gives the curves on it.

IV. RESULTS AND DISSCUSION

A. *Results Report of the solution on the mathematical model*

The mathematical model is implemented in application software for composite design and CAD, MikroPlace and some examples are shown on figures 5-11.

Figure 5 shows the pipe system mentioned earlier. This system consists of five tubes and four elbows. According to the previous discussion on the geodetic condition of the curves, the curves of the tubular parts are geodetic, while the curves of the knees or the torso parts will deviate from the geodetic curve. The following figure 5 compares paths generated on the knee with an algorithm that generates geodetic curves on the knee (torus) and curves generated by this algorithm.

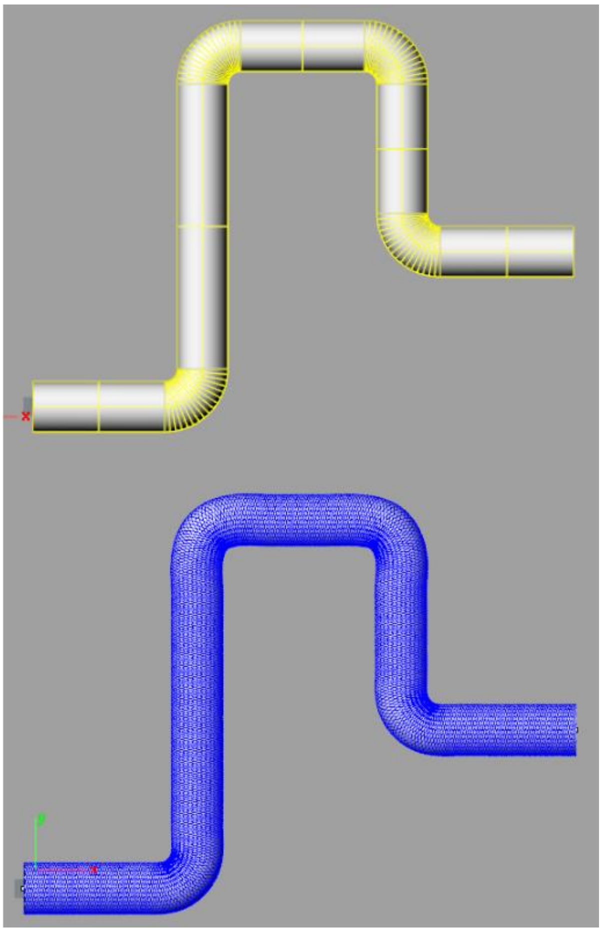

Fig.5. Tube system consisting of nine parts (upper) vs. as one-part tube system with winding paths (lower)

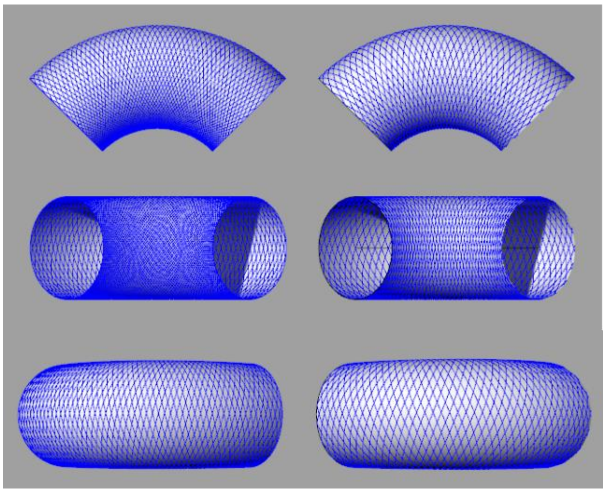

Fig. 6. Elbow comparison: geodesic curves (left) and curves generated through the algorithm (right)

The following example refers to curves generated on a spiral tube at an angle of 75°.

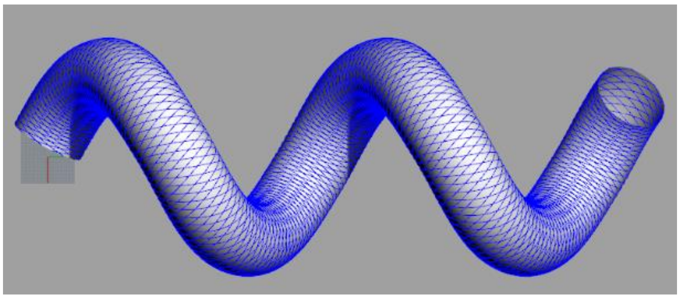

Fig. 7. *Spiral tube with curves under 75°*

The next few images show the results of generative curves of joints with unconventional cross-section, as well as pipes with non-circular cross-sections.

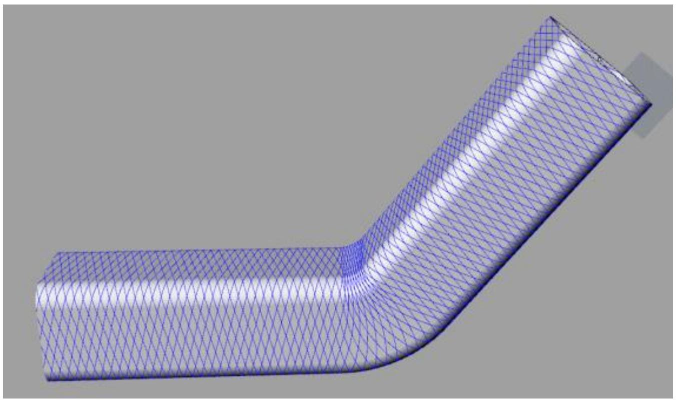

Fig. 8. *Curves on elbow with rectangular curved cross section*

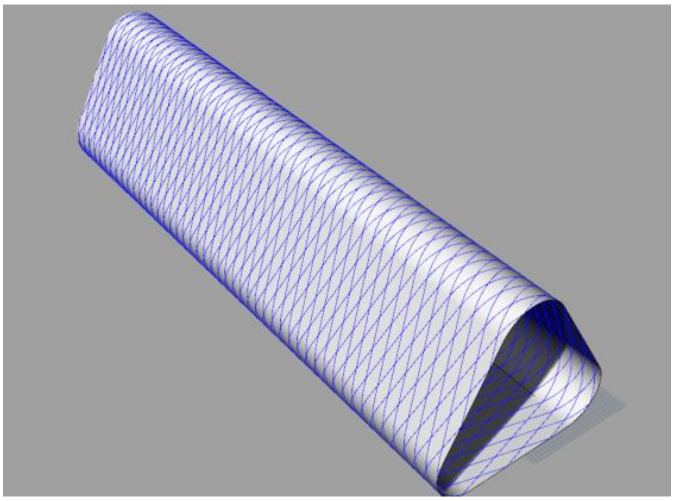

Fig. 9: *Curves on tube with triangular curved cross section*

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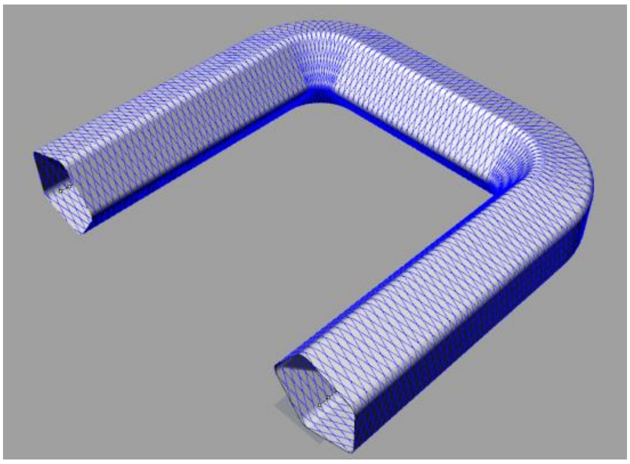

Fig. 10: Curves on horseshoe fitting shape with pentagonal curved cross section

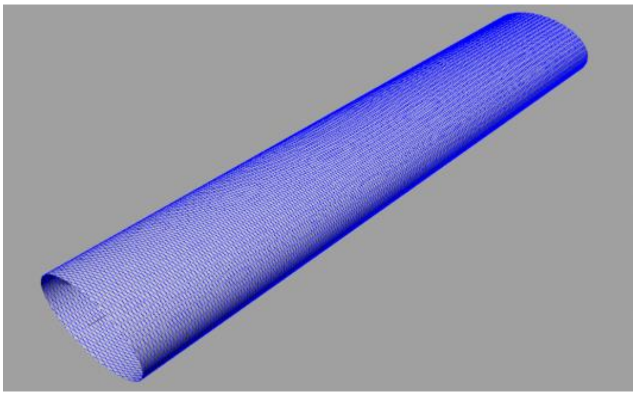

Fig. 11: *Curves on tube with elliptical cross section*

V CONCLUSION

According to the research in this paper, we can conclude:

- Complex geometric shapes are too complex indeed to represent a mathematical model for generating paths on them. The complexity is further increased by the fact that these parts are usually represented as a set of multiple surfaces, or as a whole represent the so-called polyfaces. Due to the violation of the geodetic principle (on undevelopable surfaces) the algorithm presented here is an easy way to generate curves on complex surfaces. On any surface with the help of its parametric space, curves can be generated as in 2D space and generated (mapped) on the surface.
- After defining the problem and its construction as a model, the algorithm was verified through a computer simulation in MikroPlace, which confirmed the correct generation of the model curves. The classification criteria for the model are in its linearity and statics when setting the mathematical formulations of the problem, as well as explicitness in performing the formulas. The formulas are clear and intuitive, their nature is linear and does not depend on time (static).
- The model is presented in a discrete way, the curves are generated through points with interpolation (the nature of the curves themselves is not subject to this model), which is another criterion for classification of the model. Due to the way the problem of the model is posed, and above all the clearly predicted value of the input parameters (as well as their independence between them), we also consider this model to be deterministic.

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