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Mathematical Model of Burglary: A Review

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Abstract: **We review an agent based model of urban residential burglary initiated by Short et al [6].In this model residential houses are considered to be situated in one or two dimensional square lattice. Each site or target houses which are to be burglarized are characterized by a dynamic attractiveness to burglary and where burglars are represented by random walkers. The dynamics of the criminal agents and the target attractiveness field are as described in short et al model [6]. The discrete model of density of burglars and the dynamic attractiveness of target house are derived with certain modifications.**

Keywords*: *Burglar, attractiveness, discrete model

I. INTRODUCTION

Burglary is the most common property crime. Burglary occurs everywhere. In recent decades it is a major issue in every big city. Some good neighbourhoods in a city approximately free from burglary events. However there are some bad neighbourhoods where dense agglomerated of burglary or other crimes commit [7],[8],[9].

There is a spatio-temporal correlation between burglary and victims or their close neighbours. Some neighbours repeatedly victimized within short interval of time. Thus burglary often is clustered densely which tends to be spatial localized into a regions. These spatio-temporal clusters of burglary events occurrence are often referred to as burglary "hotspots" [10],[11].

Burglary hotspots are observed to vary depending upon the particular geographic, economic or environmental conditions present. Moreover, hotspots are seen to emerge or diffuse depending on specific category of crime. The emergence of hotspots is connected to repeat victimization. A successful burglar have tendency to commit repeat burglary in the same house or nearby house.

Some specific theory has been discussed in this model to understand why hotspots increase in some locations rather than others. How they evolve and how their different sizes and lifetime features are connected the different behavior of burglars, victims and cops on dots.

To discuss the model some specific theories and hypothesis like Routine activity theory[1] [2], Crime pattern theory[3], rational choice theory[3], repeat and near repeat victimization theory [5] and broken window theory [4] are referred.

In a previous work [6] a discrete model of burglary was initiated. The main purpose of this work was to describe a mathematical model of burglary to study the emergence, dynamics and evolving patterns of the criminal activity. The essential components of the model are the criminal agents termed as Burglars and houses of target for committing burglary termed as Attractiveness.

In the Short et al. model [6], burglary is included by allowing burglars to perform one of two actions during each time interval in the grid point. A burglar may either burgle the residence (target) he presently located or move to a neighbouring targets or if burglars fail to commit burglary at current location, they will move to one of the four neighbouring target of the grid point. If burglary is successfully completed, the burglar quickly leaves the site for keeping looted goods at their safety place and abstained from burglary for the time being.

Besides the above two actions in the short et al model [6] of burglary, another action is assumed during burglary in our modified model as the burglar is simply removed from the grid without having committed a burglary. Then density of burglars can be accounted by adding a new parameter as the probability that a burglar removed from the grid without committing burglary.

Two difference equations will be derived in this model -the equation of density of burglars and the equation of dynamic attractiveness of the target house.

II. DISCRETE MODEL OF BURGLARY

In the discrete model [6] of burglary consists of two elements (*i*) Target houses or houses of victims considered as **Attractiveness** (*ii*) **Burglars** who commit burglaries.

Let us consider the houses are located on any graph G_l , which can closely reflect that of an actual city. For simplicity, we locate the target houses at point on the one or two dimensional location (square) with grid spacing l .We denote the location by $x=(i)$ or $x=(i, j)$, where *i* and *j* are any positive integers and $x=(i)$ is as in the Fig. 3.4.

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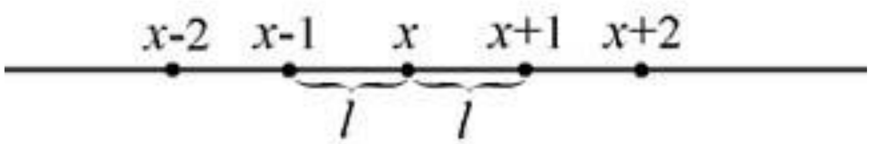

Fig. 3.4. Grid (Site)

and $x=(i,j)$ is as in the Fig. 3.5

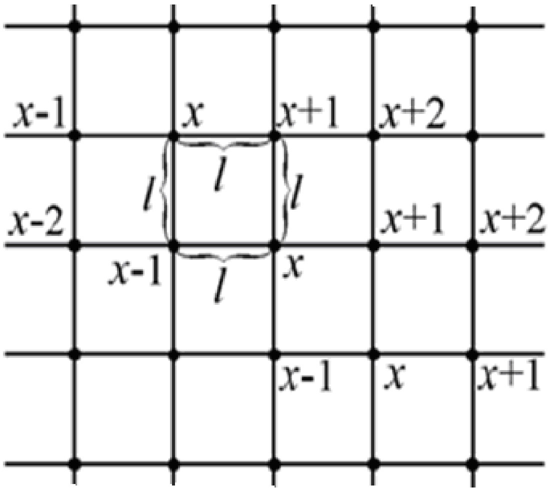

Fig. 3.5 Square Grid

Each grid site x, there is a target house with **Attractiveness**, defined as a scalar field $R(x,t)$ at time t. $R(x,t)$ is also called the measure of the burglars perception of the **Attractiveness** of the residence at each grid point $x=(i,j)$ at time *t*. i.e. $R(x,t)$ is a desirable location on the grid $x=(i,j)$ as a target for burglary event.

Thus the **Attractiveness** field can be expressed as the sum of two components –

$$R(x,t)=R_0+Q(x,t) \tag{3.1}$$

where R_0 represents the background **Attractiveness** , called the **static** component, which does not change with respect to time, but may vary in spatial location, such as economy of the area, traffic level, escaping facilities and other geographic factors. The other variable $Q(x,t)$ represents the **dynamic** component of **Attractiveness** of burglary, associated with repeat and near-repeat victimization effect.

III. DISCRETE MODEL OF DENSITY OF BURGLARS

Model assumptions : In this model, burglary is included by allowing burglars to perform one of two actions during each time interval in the grid point x

(*i*) A burglar may either burgle the residence (target) he presently located or move to a neighbouring targets.

(*ii*) If burglars fail to commit burglary at current location, they will move to one of the four neighbouring target of the grid point $x'=(i,j)$ as in the Fig. 3.5

If burglary is successfully completed, the burglar quickly leaves the site for keeping looted goods at their home or at safety place. He abstained from burglary for the time being.

To pretend the removed burglars returning to active status for the next burglaries, they are also generated at each site of the grid at the constant rate per unit of time.

There is a four adjacent site to x' of x. And $\dfrac{R(x,t)}{R(x-2,t)+R(x,t)}$ gives the probability that a burglar at $(x-1)$ will move to x instead of moving other three site adjacent to $(x-1)$ or move to $(x-2)$. This movement of burglars will be considered as a random walk biased towards areas of high **Attractiveness** $R(x,t)$.

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Because (1) burglars search for their victims in their routinely visited location (2) journey to crime distributions generally show that the distances that burglars are willing to travel away from their main home to engage in burglary is a monotonically decreasing function [12] and (3) In case of burglary the tendency stay close to home often outweighs gains that might be had in traveling further to victimize more desirable targets. Thus burglary is assumed to be a random event occurring with some probability $\mathrm{P}(x,t)$ during time interval t and $t+h$ for each burglar on the site x.

The decisions of burglar are influenced by $R(x,t)$, whose dynamics are coupled to the burglars' dynamics. During time step *t*, a burglar at grid, $x=(i,j)$ strikes at $x=(i,j)$ with probability $P(x,t)=1-e^{-R(x,t)h}$ (3.2)
where h is a small time step and $P(x,t)>0$ for higher $R(x,t)$. The burglars who strikes at $x=(i,j)$, exit the systems, if they can not rewarded, move to a neighbouring grid, say $x'=(i,j)$. Where x' is the neighbouring site to x and denoted by $x' : x$.

A burglar moves from x to x' during small time step h with probability $P_m(x,t;x')$ which is proportional to $R(x',t)$ .That is $P_m(x,t;x')=hR(x',t)$.

Again, when they moved to a neighbouring site after failing of burglary, the adjacent grid will be choosen randomly, but biased in the direction of target at high **Attractiveness** . In this case probability of a burglar will move from grid x to a neighbouring site x' is

$$P_m(x',t;x)=\frac{R(x,t)}{R(x',x)+R(x,t)}=\frac{R(x,t)}{\sum_{x' : x} R(x',t)} \quad (3.3)$$

Here $x' : x$ means that any one of neighbouring sites $x-1$, $x+1$, $x-2$ and $x+2$ of the site x as in Fig. 3.5.

Since burglars' movement is a random walk behaviour, therefore $P_m(x,t+h)$ denote the probability that a burglar (walker) is at a grid x after time step $t+h$. Further, since the burglars (walker) have an equal (fifty-fifty) probability to move or walk left and right, it is clear that (as in Fig. 3.6)

$$P_m\left(x,t+h\right)=\frac{1}{2}P_m\left(x+1,t;x\right)+\frac{1}{2}P_m\left(x-1,t;x\right) \quad (3.4)$$

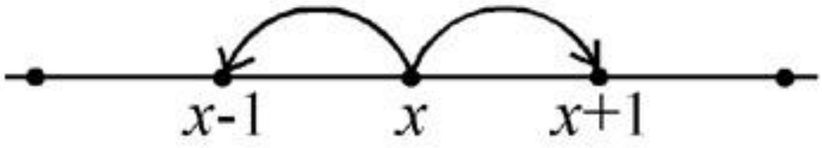

Fig. 3.6 Random Walk

where

$$P_m\left(x+1,t;x\right)=\frac{R\left(x,t\right)}{R\left(x+2,t\right)+R\left(x,t\right)}=\frac{R\left(x,t\right)}{\sum_{(x+1) : x} R\left(x+1,t\right)} \quad (3.5)$$

$$P_m\left(x-1,t;x\right)=\frac{R\left(x,t\right)}{R\left(x-2,t\right)+R\left(x,t\right)}=\frac{R\left(x,t\right)}{\sum_{(x-1) : x} R\left(x-1,t\right)} \quad (3.6)$$

In $P_m\left(x+1,t;x\right)$, $(x+1) : x$ means that burglars at site $x+1$ moves to either x or $x+2$ and in $P_m(x-1,t;x)$, $(x-1) : x$ means that burglars locating at site $x-1$ moves to either x or $x-2$ as in the Fig. 3.7

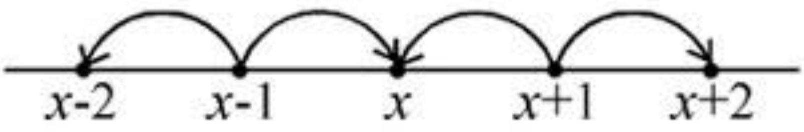

Fig. 3.7 Burglar's movement

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Thus, if we assume that the density (or number) of burglars at $x=(i,j)$ at time t be $\rho(x,t)$, which is represented as the statistical rate of burglary at grid $x=(i,j)$, at current location. If this grid x is burgled, the corresponding burglar is removed from this grid. To fill up his vacancy, new burglars are generated at the constant growth rate G on that grid.

However, the grid x is not burgled, the burglar will move to one of its neighbouring grid x' (*i.e.* $x-1$, $x-2$, $x+1$ or $x+2$) with probability $1-P(x,t)=e^{-R(x,t)h}$

Here the probability of burglar to move with biased random walk from site x to the neighbouring site x' as in equation (3.4), (3.5) and (3.6).

As per our assumption, the burglars that were at site x at time t must have removed the site either by committing burglary or by moving to a neighbouring site. For this reason any burglars that are present after one time step h must have either arrived there from a neighbouring site after failing to burgle the neighbour or have been generated there at rate G .

To derive the equation for density of burglars $\rho(x,t)$, we have to express the expected numbers of burglars at a site after time step site after time step h as

$$E\left(\rho\left(x,t+h\right)\right) = \sum \rho(x',t)P_m\left(x,t;x'\right)\left(1-P(x,t)\right)+Gh$$

or

$$\rho(x,t+h) = \left\{\rho\left(x-1,t\right)P_m\left(x-1,t\right) +\rho\left(x+1,t\right)P_m\left(x+1,t\right)\right\}\left(1-P(x,t)\right)+Gh$$

Using the relations (3.5) and (3.6) in the above relation and with the above assumptions (*i.e.* entrance and exit rules), we can model the movement of burglars on the gride site at the time $(t+h)$ as

$$\rho\left(x,t+h\right)=\left\{\frac{R(x,t)}{R(x-2,t)+R(x,t)}\rho(x-1,t) + \frac{R\left(x,t\right)}{R\left(x+2,t\right)+R\left(x,t\right)}\rho\left(x+1,t\right)\right\}\left\{1-P\left(x,t\right)\right\}+Gh \qquad (3.7)$$

where $R(x,t)/\sum_{x'\sim x} R(x',t)$ is the probability that a burglar at x' moves to x

The equation (3.7) is the simplest discrete model of movement of burglars.

IV. DISCRETE MODEL OF THE DYNAMICS ATTRACTIVENESS

Usually a previously victimized house is at high risk to be re-victimized within a small time step. Because burglars return to previously victimized location with pretending.

Model Assumptions: We model such repeat victimization assuming the dynamic **Attractiveness** $Q(x,t)$ depending upon previously burglary events at site x .

In relation (3.1) **Attractiveness** is expressed as $R(x,t)=R_0+Q(x,t)$,

(*a*) Since the movement of burglars is considered as a random walk biased towards high $R(x,t)$. (*b*) Further, since $Q(x,t)$ denote the dynamic of **Attractiveness** that the burglar is at a site x after time t steps. Since the burglars have an equal probability to walk left and right for dynamic **Attractiveness** with diffusion rate η . Therefore, it is clear that,

$$Q(x,t+h)=\frac{1}{2}\left[Q(x-1,t)+Q(x+1,t)\right] \text{ for the grid (as in Fig.3.8)}$$

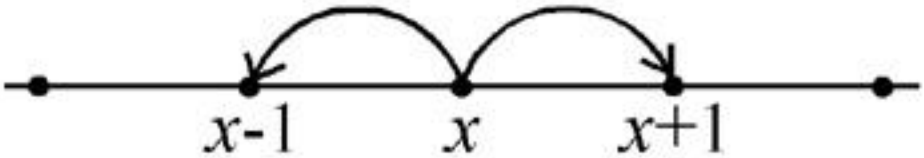

Fig 3.8 Probability to walk

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And $Q(x,t+h)=\frac{1}{4}\left[Q(x-1,t)+Q(x+1,t)+Q(x-1,t)+Q(x+1,t)\right]$

$=\frac{1}{2}\left[Q(x-1,t)+Q(x+1,t)\right]$ for the grid (as in Fig. 3.9)

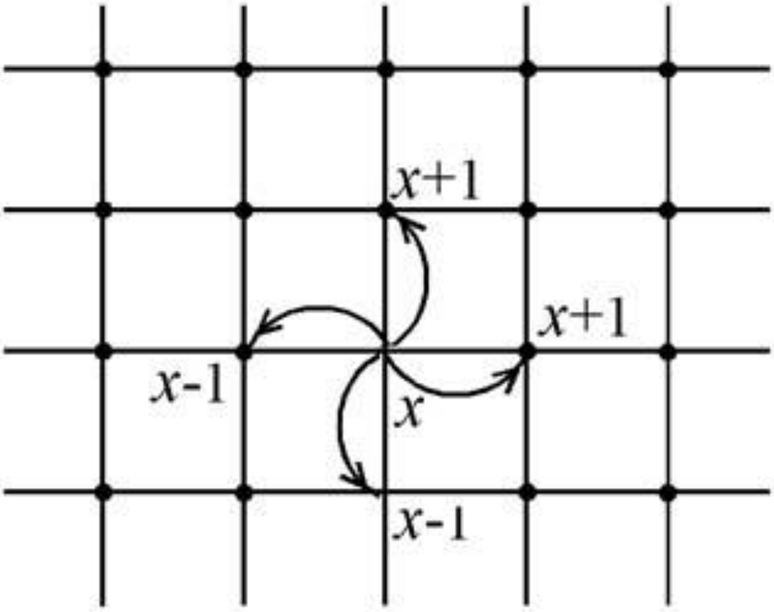

Fig. 3.9 Probability to walk in square grid

Therefore, $\eta Q(x,t+h)=\frac{\eta}{2}\left[Q(x-1,t)+Q(x+1,t)\right]$

Particularly, every time a targeted house is victimized, the $Q(x,t)$ of the target residence is increased for that site by a fixed amount α (say, constant). So that probability $P(x,t)$ for subsequent burglary at that site increased via $P(x,t)=1-e^{-R(x,t)h}$.

This increase in the dynamic $Q(x,t)$ at site x is modeled as

$$Q(x,t+h)=Q(x,t)+\alpha E(x,t) \quad (3.8)$$

where $\alpha>0$ is a rate of burglary and $E(x,t)$ is the expected number of burglary during time t and $t+h$ and

$$E(x,t)=\rho(x,t)P(x,t) \quad (3.9)$$

The **Attractiveness** of burglary rate decays over time and back to the background **Attractiveness** R_0. The decay **Attractiveness** is modeled as

$$Q(x,t+h)=Q(x,t)(1-\beta_1 h) \quad (3.10)$$

where $\beta_1>0$ is the decay rate and $\beta_1\in(0,1)$ sets a time scale over which repeat victimization are most likely to occur.

Near repeat victimization and broken window effect is modeled by allowing $Q(x,t)$ to diffuse in spatial location from each grids to its neighbouring grids (either left or right of x; *i.e.* x move to $x-1$ or $x+1$) and this diffusion of **Attractiveness** is expressed as

$$Q(x,t+h)=\left[(1-\eta)Q(x,t)+\frac{\eta}{2}\left\{Q(x-1,t)+Q(x+1,t)\right\}\right](1-\beta_1 h) \quad (3.11)$$

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where $\eta \in (0,1)$ is the weight of the broken window effect and called the rate of diffusion and $x-1$ and $x+1$ are neighbouring site of x. With the above hypothesis (*a*) and (*b*) we already have $Q(x,t+h)=\frac{1}{2}\left[Q(x-1,t)+Q(x+1,t)\right]$

Now we may combine the relation (3.8), (3.9), (3.10) and (3.11) to obtain the resulting equation as

$$Q(x,t+h)=\Big[(1-\eta)Q(x,t)+\frac{\eta}{2}\{Q(x-1,t)+Q(x+1,t)\}\Big](1-\beta_1 h)+\alpha\rho(x,t)P(x,t) \qquad (3.12)$$

Since $P(x,t)=R(x,t)h$ so, we can rewrites the expected number of burglary as $E(x,t)=\rho(x,t)R(x,t)h$ Substituting these, the equation (3.12) can be re-written as

$$Q(x,t+h)=\left\{(1-\eta)Q(x,t)+\frac{\eta}{2}(Q(x-1,t)+Q(x+1,t))\right\}(1-\beta_1 h)+\alpha\rho(x,\text{t})R(x,t)h \qquad (3.13)$$

The term $\frac{\eta}{2}(Q(x-1,t)+Q(x+1,t))$ is the broken window effect, $(1-\beta_1 h)$ is the decay term and the last term $\alpha\rho(x,\text{t})R(x,t)h$ is represented as the 'repeat burglary'. The term $\alpha\rho(x,\text{t})R(x,t)h$ is called the expected number of burglary events in time $(t,t+h)$.

The equation (3.13) is called the simplest discrete model of **Attractiveness**.

The equations (3.7) and (3.13) together represent the discrete system of burglary and this system is rewritten as below:

$$Q(x,t+h)=\left\{(1-\eta)Q(x,t)+\frac{\eta}{2}((Q(x-1,t)+Q(x+1,t))\right\}(1-\beta_1 h)+\alpha\rho(x,\text{t})R(x,t)h$$

$$\rho(x,t+h)=\left\{\frac{R(x,t)}{R(x-2,t)+R(x,t)}\rho(x-1,t)+\frac{R(x,t)}{R(x+2,t)+R(x,t)}\rho(x+1,t)\right\}\{1-P(x,t)\}+Gh$$

V. THE MODIFIED MODEL:

In the Short *et. al*. model it was assumed that burglars are generated at the constant rate and leave the grid only after committing burglary and otherwise they move to another adjacent grid to commit burglary.

In order to modify this, we would like to remove the assumption that the burglars always removed the grid after committing burglary. But we assume that the burglars may simply remove from the grid without committing burglary due to security or other environmental problems.

In this case density of burglars can be derived by incorporating the additional probability that the burglar removed from the grid without having committed a burglary.

Thus the equation of density of burglars is as

$$\rho(x,t+h)=(1-\text{K})\left\{\frac{R(x,t)}{R(x-2,t)+R(x,t)}\rho(x-1,t)+\frac{R(x,t)}{R(x+2,t)+R(x,t)}\rho(x+1,t)\right\}\{1-P(x,t)\}+Gh$$

The parameter K represents the probability that a burglar may be removed from the grid without having committed a burglary. Thus the discrete system of burglary can be expressed as

$$Q(x,t+h)=\left\{(1-\eta)Q(x,t)+\frac{\eta}{2}\big((Q(x-1,t)+Q(x+1,t)\big)\right\}(1-\beta_1 h)$$
$$+\alpha\rho(x,t)R(x,t)h$$

$$\rho(x,t+h)=(1-K)\left\{\frac{R(x,t)}{R(x-2,t)+R(x,t)}\rho(x-1,t)\right.$$
$$\left.+\frac{R(x,t)}{R(x+2,t)+R(x,t)}\rho(x+1,t)\right\}\{1-P(x,t)\}+Gh$$

parameter	meaning
G	Growth rate of burglars at each grid.
β_1	Decay rate
α	Rate of burglary
η	Rate of diffusion
K	Probability that a burglar removed without committing burglary
Gh	Expected number of burglars generated in a time interval of length *h*

VI. CONCLUSION:

The value of η and β_1 have chosen between 0 and 1 and the value of α is considered always positive. The grid spacing is taken 1 unit length of each square grid. For high attractiveness (i.e $R(x,t)\to\infty$) , $e^{-R(x,t)h}\to 0$ and then the probability $P(x,t)=1$ which is the confirmation of burglary occurrence.

The discrete system of burglary has the spatial homogeneous solution. Considering constant attractiveness and constant density of burglars as the average attractiveness and average density of burglars respectively, then the solution can be obtain for the dynamic attractiveness and density of burglars of the discrete system of burglary. The solution of the systems exhibit the nature of burglary hotspots.

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