Mathematical Model Design & Validation of Planer Fourbar Mechanism for Path Point Synthesis Problem using Genetic Optimization Technique

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***Abstract*—An analytical approach for path point generation of planer fourbar mechanism is illustrated in this work, Mathematical model of planer fourbar mechanism is designed based on loop closer and Freudenstein's equation, here genetic optimization technique is used to find global optimum solution and validated with similar work carried out by different authors using different optimization technique, also illustrated how better solution can be obtained by modifying design space for the objective function. Here modification is carried out with the starting position of crank, it can start from any angular position with respect to fixed link of mechanism and also angular position for each trace point on desired path is left on algorithm to decide so that will further increase the search space for the algorithm.**

Keywords—Synthesis of Mechanism; Path synthesis; four bar Mechanism; Genetic Algorithm.

I. INTRODUCTION

In the synthesis of planer four bar mechanism. Three categories are there, namely function generation, path generation and body Guidance. In function generation, considered point on the coupler will generate definite function as an output when motion provided to the input link. In path Generation, Considered point on the coupler link will pass through predefines guide point or trace points as an output motion. And lastly in body guidance coupler link will pass through predefined point and orientation of the body.

Firstly. Cabrera and et al [1] had used genetic optimization technique to find the optimum solution for synthesis problem of the four-bar mechanism. Here Grashof's criteria and sequence of the input angles taken as constraint. Also Kinzel and et al [2] had proposed geometric constrained programming approach for synthesis problem of planar fourbar mechanism and later on Acharaya & Mandal [3] employed particle swarm optimization (PSO) & differential evolution (DE) approach for synthesis of 4-bar linkages. Zadah and et al [4] used multi-objective genetic algorithms (GA) for pareto optimal synthesis of 4-bar linkages. Two objectives namely tracking error and transmission angle deviation from 90 degree is accounted. Erkaya and Uzmay [5] presented a joint clearance influence on path generation and transmission angle by adapting GA. Todorov [6] described a new dimensional synthesis method. The position function of the four-bar mechanism is presented by the Freudenstein's equation and it is minimized by the Chebyshev's best approximation theory. Khare & Dave[7] developed a closed form equations for the synthesis of the 4-bar crank rocker mechanism in which the angle between dead-centre positions of the rocker and the corresponding angle turned by the crank are prescribed. Ahmed and Waldron [8] outlined synthesis techniques for 4-bar linkages, having adjustable driven crank pivots, for different motion generation problems. The method of solution is analytical in nature, and, therefore well suited for use on a digital computer. Levitskii et al [9] considered the general problem of determining five parameters 17 specifying a for 4-bar linkages which synthesizes a given function and at the same time satisfies some limiting conditions. Hobson & Torfason [10] presented the design of mechanism, which approximate desired centrodes and the applied to two prosthetic knee mechanisms. Geem & Kim [11] presented a new structural optimization method based on the harmony search algorithm. Mahdavi, Fesanghary & Damangir [12] presented an improvised HS algorithm for solving optimization problem.

II. MATHEMATICAL MODEL

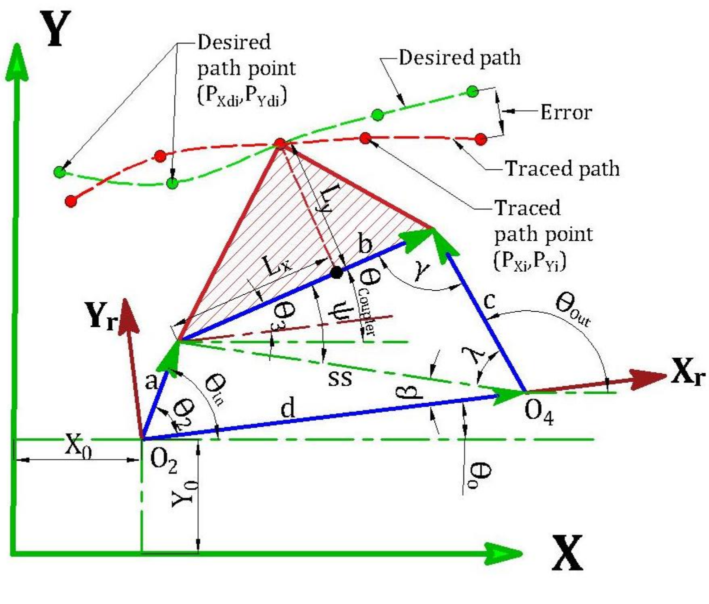


Fig. 1. Configuration of fourbar mechanism

In a four bar linkage synthesis problem need to trace number of desired points on a coupler curve generated by a mechanism. For this purpose configuration of four bar coupler mechanism shown in figure – 1 is considered in which a, b, c, d, L_x, and L_y are the basic linkage dimensions used in mechanism, here XY is the global co-ordinate system and X_rY_r is the reference co-ordinate system on mechanism, and point P (P_x, P_y) is the coupler point on the mechanism.

Here for tracing desired points by coupler point P need to evaluate the values of X_0, Y_0, Θ_0, a, b, c, d, L_x, L_y and as many input angles (Θ_2) as Number of desired points given to be trace. For formulating mathematical model loop closure method and Freudenstein's equation is used.

The vector position of trace point P is defined as

$$\vec{r}_P = \vec{a} + \vec{L}_x + \vec{L}_y \quad (1)$$

So the position of coupler point or trace point P with respect to reference coordinate system X_rY_r as

$$P_{X_r} = acos\theta_2 + L_x cos\theta_3 - L_y sin\theta_3 \quad (2)$$

$$P_{Y_r} = asin\theta_2 + L_x sin\theta_3 + L_y cos\theta_3 \quad (3)$$

For evaluation of angle Θ_3 vectors loop considered as

$$\vec{a} + \vec{b} - \vec{c} - \vec{d} = 0 \quad (4)$$

So its X and Y components about reference axis X_rY_r will become zero

$$a\cos\theta_2 + bcos\theta_3 - ccos\theta_4 - d = 0 \quad (5)$$

$$a\sin\theta_2 + bsin\theta_3 - csin\theta_4 = 0 \quad (6)$$

According to Freudenstein's equation with the use of above two equation eliminating angle Θ_4 so we can obtain

$$K_1 cos\theta_3 + K_4 cos\theta_2 + K_5 = \cos(\theta_2 - \theta_3) \quad (7)$$

Where,

$$K_1 = {}^{d}/_{a} \quad (8)$$

$$K_4 = {}^{d}/_{b} \quad (9)$$

$$K_5 = \frac{c^2 - a^2 - b^2 - d^2}{2ab} \quad (10)$$

From above equation by known value of input angle Θ_2 value of angle Θ_3 can be evaluate as following.

$$\theta_3{}^1 = 2\tan^{-1}\left(\frac{-E + \sqrt{E^2 - 4DF}}{2D}\right) \quad (11)$$

$$\theta_3{}^2 = 2\tan^{-1}\left(\frac{-E - \sqrt{E^2 - 4DF}}{2D}\right) \quad (12)$$

Where,

$$D = cos\theta_2 - K_1 + K_4 cos\theta_2 + K_5 \quad (13)$$

$$E = -2sin\theta_2 \quad (14)$$

$$F = K_1 + (K_4 - 1)cos\theta_2 + K_5 \quad (15)$$

Now position of coupler point P with respect to global coordinate system as

$$P_x = X_0 + P_{X_r} cos\theta_0 - P_{Y_r} sin\theta_0 \quad (16)$$

$$P_y = Y_0 + P_{X_r} sin\theta_0 - P_{Y_r} cos\theta_0 \quad (17)$$

III. GENETIC OPTIMIZATION ALGORITHM

There is several global optimization techniques are available likewise stochastic optimization, genetic algorithm, neural networks, particle swarm optimization algorithm, Tabu search optimization, ant colony method etc. Here out of these algorithms genetic optimization algorithm is selected because of following reasons. It takes the encoding of decision variables as the operational objects. It takes the objective function directly as the searching information. It uses searching information of several searching points at the same time. It uses probability searching technology.

In genetic search algorithm number of solution generated randomly is known as population and which is repetitively applied on objective function is known as iteration. In every iteration reproduction, crossover and mutation operation carried out for making new generation of population and that newly generated population again applied on objective function, when objective criteria satisfied then this iterative procedure will stop and algorithm will gives a solution. Here in this work size of population is taken as 100, crossover fraction considered as 0.8 and mutation fraction considered as 0.1. At this values algorithm gives a better performance checked by author Hitesh Patel and J. R. Mevada [13] in their work based on genetic optimization technique.

IV. OBJECTIVE FUNCTION AND CONSTRAINTS FOR OPTIMIZATION

Here the main objective of this work is to minimize the error between the position of desired guide points (P_{xdi},P_{ydi}) and position of coupler or trace point (P_{xi},P_{yi}) on the mechanism. The error function is considered as

$$F(X) = \sum_{i=1}^{n}\left[\left(P_{Xd_i} - P_{X_i}\right)^2 - \left(P_{Yd_i} - P_{Y_i}\right)^2\right] \quad (18)$$

Where n is the number of desired points considered for synthesis problem.

Here mechanism needs to satisfy some constraint like Grashof's law for full rotation of input link, also Input link 'a' need to make small so it can be work as a crank and continuous increment require in input angle θ_{In} of crank for continuous one direction rotation so it can be possible to maintaining sequences of desired points to be trace by the mechanism.

In mechanism requirement is that having one of the link will act as a crank and become an input link, so mechanism need to satisfy Grashof's criterion. By Grashof's law summation of shortest link length (L_s) and longest link length (L_l) is less than the summation of remaining two links from fig. link O_2A is considered as a shortest link so

$$(Ls + Ll) < (Summation\ of\ remaining\ two\ links)$$

$$2 * (Ls + Ll)$$
$$< (Ls + Ll) + (Summation\ of\ remaining\ two\ links)$$

$$2 * (Ls + Ll) < (a + b + c + d)$$

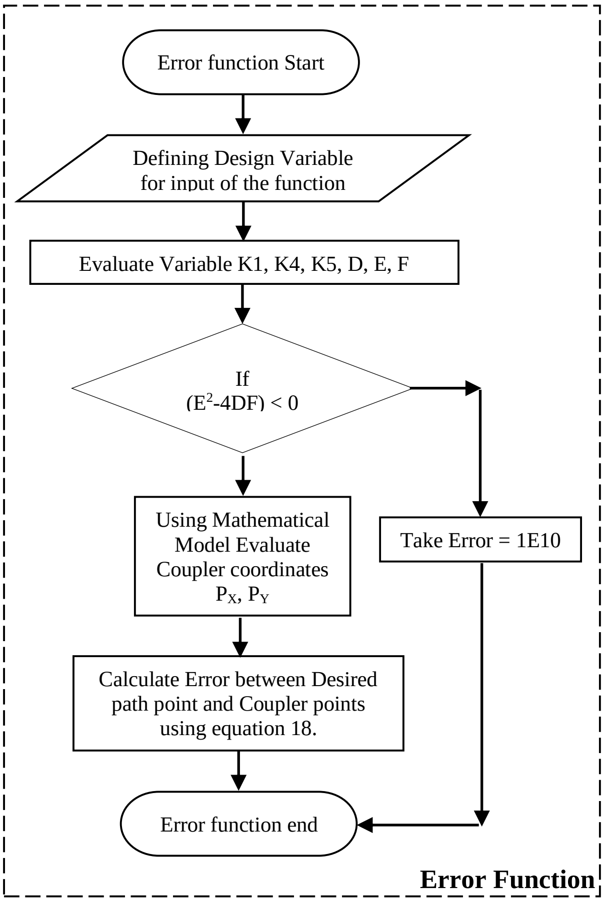
For maintaining sequences of desired points to be trace by the mechanism constraint considered as following.
If values of $\theta_2^i - \theta_2^{i-1} < 0$ then add penalty to the objective function.

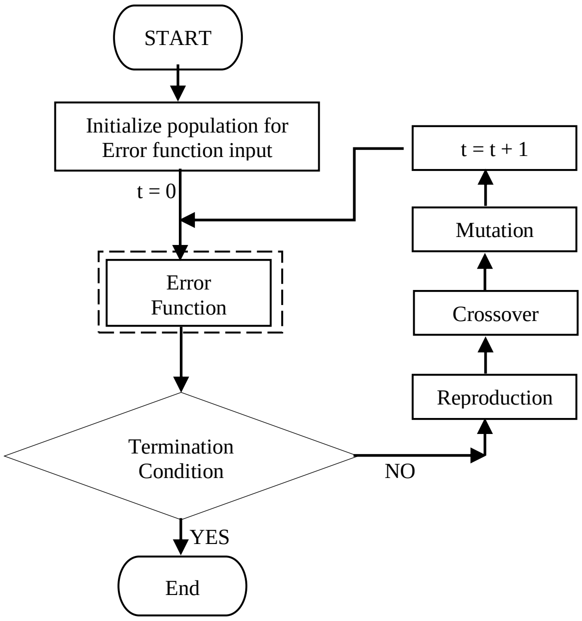
So the total number of Design variables and its boundary limits with this optimization problem are as following.

TABLE I. BOUNDARY LIMITS OF DESIGN VARIABLE.

Sr. No.	Description	Symbol and boundary limits	No. of Design Variable.
1	Links Length	$0.05 \le a, b, c, d \le 5$	4
2	Coordinates of reference coordinate system with respect to global coordinate system,	$-5 \le X_0, Y_0 \le 5$	2
3	Coupler Point position on connecting rod.	$-5 \le L_X, L_Y \le 5$	2
4	Rotation of reference coordinate system with respect to global coordinate system	$-180 \le \Theta_0 \le 180$	1
5	Input crank angles	$-360 \le \Theta_2 \le 540$	Equal to number of desired points

In below figure shown a Flow chart for error function which is used to calculate error and this error function is utilized in genetic algorithm for find out optimum solution.





V. RESULTS AND DISCUSSION

Here, firstly 18 points considered to trace by coupler point firstly proposed by Kunjur and Krishnamurthy (KK) [14] to compare the performance of genetic algorithm with central difference and exact gradient method also Cabrera et al. [1] had modified operators and tested same problem with genetic algorithm, also same problem is discussed by A. Smaili, N. Diab [15] had discussed same problem using ACO algorithm, all the authors have taken fixed values of input angle, here in this solution input angle values were to decide by algorithm itself, also constraints discussed above are applied to the algorithm and then obtained results are compared with existing work.

TABLE II. DESIRED 18 POINTS [14]

Desired point i	1	2	3	4	5	6	7	8	9
X_{di}	0.5	0.4	0.3	0.2	0.1	0.05	0.02	0	0
Y_{di}	1.1	1.1	1.1	1	0.9	0.75	0.6	0.5	0.4
	10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
X_{di}	0.03	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.6
Y_{di}	0.3	0.25	0.2	0.3	0.4	0.5	0.7	0.9	1

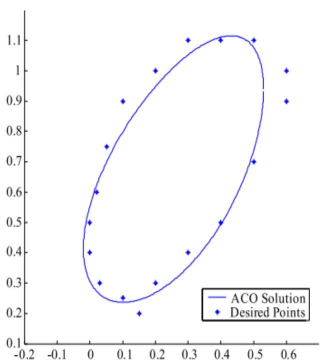


Fig. 2. Coupler curve generated by ACO Optimal Mechanism [15]

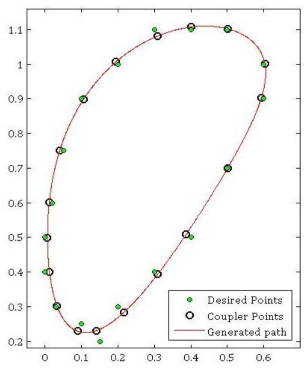


Fig. 3. Coupler curve obtained by GA in current work

after applying Genetic algorithm obtained results are compared with the results of A. Smaili, N. Diab [15] comparative results graph was shown in figure 2 & 3, also the configuration for obtained mechanism is shown in below figure – 4,

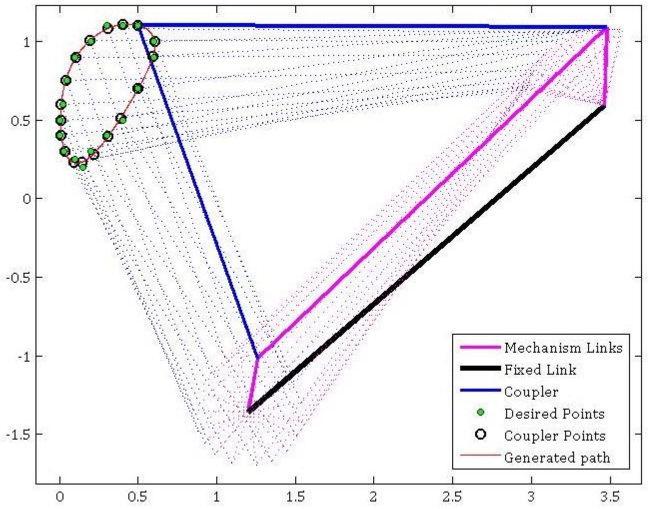


Fig. 4. Obtained Mechansim configuration for 18 desired points

For above configuration of mechanism the linkage parameters are tabulated in below table – III,

TABLE III. OBTAINED PARAMETERS OF 18 POINTS PATH

Variable	Value	Variable	Value	Variable	Value
X_0	1.201835	Θ_2 - 1	39.10128	Θ_2 – 10	211.0341
Y_0	-1.35544	Θ_2 – 2	57.2049	Θ_2 – 11	237.6793
L_x	0.904845	Θ_2 – 3	73.09223	Θ_2 – 12	254.0793
L_y	2.060674	Θ_2 – 4	94.02072	Θ_2 – 13	273.3342
a	0.34529	Θ_2 – 5	114.6813	Θ_2 – 14	292.9728
b	3.057001	Θ_2 – 6	137.6361	Θ_2 – 15	307.8849
c	0.497724	Θ_2 – 7	159.6725	Θ_2 – 16	329.3006
d	2.983992	Θ_2 – 8	174.8861	Θ_2 – 17	353.1682
Θ_0	40.71874	Θ_2 - 9	191.0274	Θ_2 – 18	368.4727

Here, error obtained in this problem is compared with same work carried out by different authors using different optimization techniques are tabulated in below table – IV, error generated in this work is comparatively very lower as compared to the work of other authors.

TABLE IV. COMPARISON OF OBTAINED RESULTS

Optimization algorithm	No. of Evaluation	Final Error
Exact Gradient [14]	240	0.0168
GA – KK [14]	5000	0.043
GA – CSP [1]	5000	0.0245
Tabu-gradient [15]	550	0.0137
Ant [15]	50000	0.0526
Ant colony [15]	479	0.0109
GA (Current work)	**5000**	**0.0035**

One more example is considered here to trace 25 points by coupler point of four bar planer mechanism, the position of all 25 desired points are tabulated in below table – 5, same example is considered by Mc Garva[16] which is used for a packaging machinery. And later on it is utilized by Laribi and et al [17] by using genetic algorithm.

TABLE V. DESIRED 25 POINTS [16]

Desired point i	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Xdi	7.03	6.95	6.77	6.4	5.91	5.43	4.93	4.93	4.67
Ydi	5.99	5.45	5.03	4.6	4.03	3.56	2.94	2.6	2.2
	10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
Xdi	4.39	4.04	3.76	3.76	3.76	3.76	3.76	3.76	3.8
Ydi	1.67	1.22	1.97	2.78	3.56	4.34	4.91	5.47	5.98
	19	**20**	**21**	**22**	**23**	**24**	**25**		
Xdi	4.07	4.53	5.07	5.05	5.89	6.41	6.92		
Ydi	6.4	6.75	6.85	6.84	6.83	6.8	6.58		

After applying genetic algorithm, results obtained results are compared with the same work carried out by author A. Smaili, N. Diab [15] as shown in figure 5 & 6. And the configuration for the obtained mechanism is shown in below figure – 7. And the dimensional parameters for the linkage of mechanism is tabulated in below table – VI.

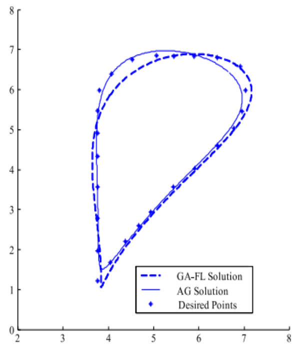


Fig. 5. Coupler curve generated by ACO Optimal Mechanism [15]

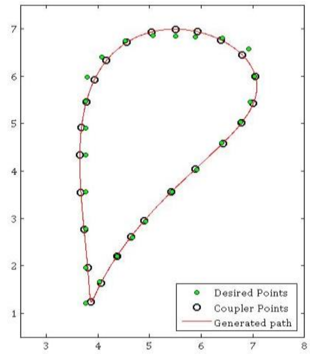


Fig. 6. Coupler curve obtained by GA in current work

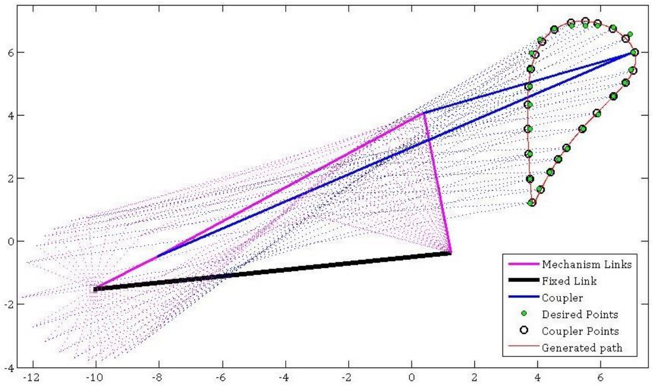


Fig. 7. Obtained Mechansim configuration for 18 desired points

TABLE VI. OBTAINED PARAMETERS OF 25 POINTS PATH

Variable	Value	Variable	Value	Variable	Value
X_0	-10.0758	Θ_2 – 4	71.68049	Θ_2 – 16	259.7496
Y_0	-1.51267	Θ_2 – 5	88.31685	Θ_2 – 17	270.6838
L_x	16.38188	Θ_2 – 6	102.1246	Θ_2 – 18	281.0881
L_y	-1.46501	Θ_2 – 7	117.8924	Θ_2 – 19	292.1885
a	2.273927	Θ_2 – 8	126.6046	Θ_2 – 20	305.7724
b	9.578672	Θ_2 - 9	137.0034	Θ_2 – 21	319.7508
c	4.516432	Θ_2 – 10	152.2369	Θ_2 – 22	330.7251
d	11.38357	Θ_2 – 11	180.744	Θ_2 – 23	340.1309
Θ_0	5.786575	Θ_2 – 12	205.8736	Θ_2 – 24	352.1487
Θ_2 - 1	21.32654	Θ_2 – 13	221.4302	Θ_2 – 25	365.061
Θ_2 – 2	42.6952	Θ_2 – 14	235.3666		
Θ_2 – 3	56.78272	Θ_2 – 15	249.0915		

TABLE VII. COMPARISON OF OBTAINED RESULTS

Optimization Algorithm	Error
GA FL [17]	0.902
Ant [15]	3.56
Ant-gradient [15]	0.682
GA (current work)	**0.1656**

Here comparative study being carried out based on mathematical modeling and optimization algorithm, Here using mathematical modeling in the program, change is considered like crank of four bar mechanism can start from any angular position with respect to fixed link so for tracing a path, restriction on the crank to start always from 0º initial position with respect to crank is removed and also the values of angular position of crank for every desired tracing points is to be left on the algorithm to decide so the solution space for the algorithm is increased and the algorithm can find more better solution. Due to this reasons in the obtained results you can see within a same number of generation, algorithm can find better solutions as compared to the results obtained for the same examples by different authors, comparative results are shown in above two examples. In example for 18 no's of desired points obtained results are shown in above figure 2 & 3 , also error is reduced to much more level as compared to the work being carried out by other authors. And the configuration of mechanism for the same is shown in figure - 4. Same as also in another example of 25 desired points work being carried and the obtained results are plotted in figure – 5 & 6, also comparative results in the terms of error is shown in table – VII, the configuration for obtained mechanism is shown in figure - 7.

VI. CONCLUSION

Genetic algorithm is a global optimum search technique and can find a solution from the available search field, by considering constraints to the design variable participated in formulation of objective functions, as search space increased there is a more chances to find better solution. Here in above discussed examples restriction to the crank is to start angular displacements always with respect to fixed link and also fixed angular position for each desired trace point is removed so the design space for search algorithm is increased and also chances to obtain better and better results are increased, in above discussed examples same fundamental is applied to synthesis a mechanism for a given path which can pass through given number of desired points and obtained a better results as compared with work being carried out by different authors.

VII. REFERENCES

[1] J.A .Carbrea. A. Simon & M.Prado, Optimal synthesis of mechanism with GA, Mechanism & Machine Theory, Vol.37, pp 1165-1177, 2002.
[2] E.C.Kinzel,J.P.Schniedeler & G.R.Pennock, Kinetic synthesis for finitely separated positive any geometric coordinate programming, J. Machine design, Trans.ASME, Vol.128,pp.1070-1079,2006
[3] S.K.Acharyya and M.Mandal, Performance of EAs for four-bar linkage synthesis, Mechanism & Machine Theory, vol. 44, pp.1784-1794,2009.
[4] N.N.Zadah, M.Fklezi,A.Jamalize M.Garj, Pareto optimal synthesis of four-bar mechanism for path generation, Mechanism & Machine Theory, Vol.44, pp.180-191, 2009.
[5] S.Erkaya & I.Uzmay, Determining parameters using GA in mechanism with joint clearance, Mechanism & Machine Theory, Vol.44, pp.222-234, 2009.
[6] T.S.Todorov, Synthesis of four-bar mechanisms by Freudenstein–Chebyshev theory, Mechanism and Machine Theory, Volume 37,Pages 1505-1512,2002.
[7] A.K. Khare and R. K. Dave, Optimizing 4-bar crank-rocker mechanism, Mechanism and Machine Theory, Volume 14, Pages 319-325,1979.
[8] A.Ahmad and K.J.Waldron, Synthesis of adjustable planar 4bar mechanisms, Mechanism and Machine Theory, Volume 14, pp.405-411,1979.
[9] N.I.Levitskii, Y.L.Sarkissyan and G.S.Gekchian, Optimum synthesis of for 4-bar function generating, Mechanism and Machine Theory, Volume 7, Pages 387-398,1972.
[10] D.A. Hobson and L.E. Torfason, optimization of four-bar knee mechanisms A computerized approach, Journal of Biomechanics, Volume 7, Pages 371-376,1974.
[11] Z.W.Geem & J.H.kim, ' new heurisrc optimization algorithm H ', imulation, vol.76, pp.60-68, 2001.
[12] M.Mahdavi, M.Fesanghary & E.Damangir, An improved HS algorithm for solving optimization problem, Applied Mathematical computation, Vol.188, pp-1567-1579, 2007.
[13] Hitesh R. Patel, J. R. Mevada, Shape Control and Optimization using Cantilever Beam, International Journal of Engineering Research and Application, Vol. 3, Issue 3, May-June 2013, pp 155-161.
[14] S. Kunjur, S. Krishnamurty, Genetic algorithms in mechanism synthesis, J. App. Mech. Rob. 4 (2) (1997) 18–24
[15] A. Smaili, N. Diab, Optimum synthesis of hybrid-task mechanisms using ant-gradient search method, Mechanism and Machine Theory 42 (2007) 115–130.
[16] J.R. McGarva, Rapid search and selection of path generating mechanism from a library, Mech. Mach. Theory 29 (2) (1994) 223–235.
[17] M.A. Laribi, A. Mlika, L. Romdhane, S. Zeghloul, A combined genetic algorithm-fuzzy logic method (GA-FL) in mechanism synthesis, Mech. Mach. Theory 39 (2004) 717–735.