Mathematical Approaches in Evaluation of Circularity Measurement

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***Abstract*--Measurement of form features becomes reliable and a necessity for conformance of accuracy defined on the modeled geometry with the manufactured geometry. Evaluation of form errors in manufactured parts is essential in determining its conformance to the tolerance specification. These specifications are established to provide acceptable limits on part variation in order to ensure functional equivalence. In the present work, the form errors are evaluated for circularity by Three Point Method. The results are compared with the CMM results. The part or component considered for the work is an eccentric shaft. The shaft is divided into different datum (Z) levels. The input data for the part (component) is taken by inspecting the part by advanced measuring instrument viz., Co-ordinate Measuring Machine.**

Keywords --- Form Tolerance, Circularity, Three Point Method, Coordinate Measuring Machine (CMM)

I. INTRODUCTION

Geometric dimensioning and tolerancing provides a means for specifying the shape requirements and the interrelationships between part features. Shape requirements include not only functional needs (the suitability for assembly with its designed counterpart(s) and the proper functioning of a mechanical system) but also issues such as manufacturability, aesthetics and conformance to regulations. Because no manufacturing process can make dimensionally perfect parts, designers must specify a region to allow dimensional variation in actual parts. This region is called tolerance zone. The traditional view of tolerancing is that when the dimensional variation is within the allowable region, the part meets the shape requirements; that is, the actual part is functionally acceptable.

After the machining, it is necessary to verify the possible occurrence of manufacturing errors, in terms of dimensional or geometrical variations of the designed shape. The inspection method for a manufactured part implies several steps:

i. The inspection scheme with the determination of sample space i.e., selection of data points in appropriate area with appropriate quantity.
ii. The identification of an ideal geometric element with known analytical expression that best fits the measured data.
iii. Mathematical models are attempted over the empirical data to determine and predict the geometry.
iv. The final step is the comparison of the evaluated deviations with the imposed tolerances, thus deciding the quality of the manufactured part.

Circularity plays a vital role in building cylindrical features. The parameters extended on ability of the centers (circles at different datum), parallelism of the planes, perpendicularity of the axis to the plane of traced circle contributes to the building of cylindricity. The cylindricity tolerance establishes a volume between two coaxial cylinders whose difference of radii is the value of the tolerance zone [1, 2]. The machined surface must lie between these two cylinders. Evaluating cylindricity is applied on components such as axles, piston cylinders, torque shaft and eccentric shaft. Since cylindrical surfaces are ubiquitous in industrial machining and the realization of high-quality cylinders is a crucial technological objective, evaluation of cylindricity with stringent tolerancing is executed to avoid costly rejections.

Mathematical models are adopted with suitability using a best fit approach to accommodate the traced points of the specimen from the Coordinate Measuring Machine (CMM) [3]. Further, a comparison of soft inspection against the hard inspection i.e., CMM is carried out on circularity shall attempt a predictive model in validation of the geometry.

II. PROBLEM DESCRIPTION

Specifying the shape requirements and the interrelationships between the eccentric shaft (part) features is an important task because the shape requirements include not only the functional needs but also issues such as manufacturability, aesthetics, and conformance to regulations to accommodate manufacturing reality. Designer must specify a region to allow dimensional variation in actual part. This region is called as tolerance zone. After the machining, it is necessary to verify the possible occurrence of manufacturing errors, in terms of dimensional or geometrical variations of the designed shape. Fitting technique such as Three Points Method is used to determine the substitute geometry,

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Three Point Method gives easiness in construction of a circle equation with having "n" data points from the sample space. The construction of circle is possible by seven approaches, out of which the formulation of the procedure through a matrix brings easiness and time reduction in evaluation of circle.

III. EXPERIMENTAL DETAILS

The measurement of form geometry (e.g. straightness, circularity, cylindricity etc.), is of vital importance in different applications. In principle, such measurement consists of two basic aspects:

1. Instruments are used (e.g., Co-ordinate Measuring Machines) to measure the coordinates of points at the surface of the given work piece - Hard inspection.
2. Mathematical techniques / software is used to include the measured points in the mathematical procedures – Soft Inspection.

The part considered for the evaluation of circularity tolerance is the eccentric shaft as shown in the Fig.1. The part is measured by the Coordinate Measuring Machine (CMM) as shown in Fig.1.

Fig. 1: The cylindrical component/ part during data points collection

The facility was provided by the Metrology Group, Aircraft Prototype Manufacturing Facility (APMF) of CSIR-NAL. The CMM used for the inspection is **Wenzel µ Star** and the probe diameter used for the measurement of the eccentric shaft is 5mm.

Fig.2: Wenzel µ Star CMM
[Courtesy: Metrology Group, APMF, CSIR-NAL]

The total length of the eccentric shaft is 263.50mm with the interest of diameter of 30mm for the length of 50.1mm respectively as shown in Fig. 3. The part is placed horizontally as shown in the figure. The coordinate measurement is traced at different Z-levels with respect to the machine co-ordinate at 445mm, 450mm, 460mm, 470mm, 480mm respectively for circularity (roundness) evaluation by the circumferential method of tracing the coordinates.

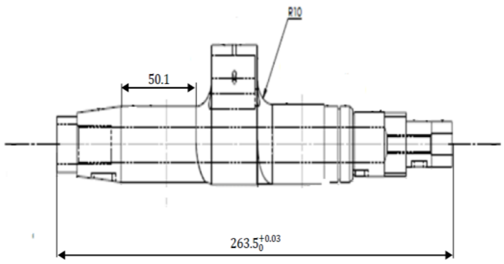
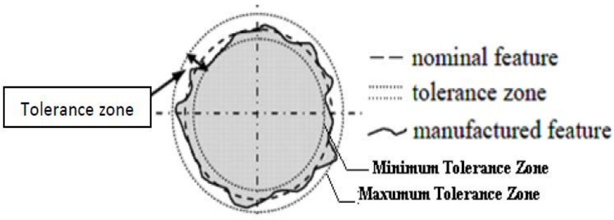


Fig. 3: Front View of the part / component considered for the evaluation of circularity.

IV. METHODOLOGY

A component is described as round if all points of a cross section are equidistant to a common centre. Therefore, to measure roundness, rotation of the component is necessary coupled with the ability to measure change in radius.



In the above figure the features of circularity are shown. Tolerance is introduced, to accommodate the manufactured circular feature within the tolerance zone.

A. Evaluation of Form Errors

The following flow chart gives the sequence of stages in evaluating the form errors for circularity. Validity of the form tolerance is presented by comparing results by mathematical procedures and CMM enabled software tool.

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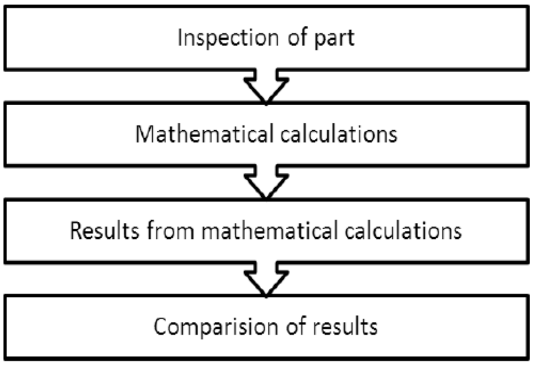

Fig. 4: Flow chart representing the method followed in evaluating form tolerance

- Inspection of the part:

The part considered for inspection (eccentric shaft) is inspected using the Coordinate Measuring Machine (CMM). The required data for mathematical calculations is obtained.

- Mathematical calculations:

The data obtained from the inspection of part are subjected to the mathematical calculations for the evaluation of circularity.

- Results from mathematical calculations:

The results are obtained by substituting the variable in the mathematical formulation.

- Comparison of results:

The obtained results from the mathematical calculations are compared with the results obtained from the CMM software.

B. Mathematical Evaluation

For (x_i, y_i) , i = 1,2,…, n and n > 3.

1) Least Squares Method

The procedure to fit a circle having "n" data points (x, y) distributed on the x-y plane, for n ≥3, the least squares regression is used to find the equation of the circle that best fits the data. That is, to determine the values of h, k, and r such that the curve provides a good fit around the data points.

(x –h) 2 + (y - k) 2 = r2

The least squares function for the circle equation is given by

$F(h, k, r) = \Sigma[(xi - h)^2 + (yi - k)^2 - r^2]^2$

The equation of the circle can be linearized as follows:

$(x - h)^2 + (y - k)^2 = r^2$

$x^2 - 2hx + h^2 + y^2 - 2ky + k^2 = r^2$

$x^2 + y^2 = 2hx + 2ky + r^2 - h^2 - k^2$

$x^2 + y^2 = Ax + By + C$

This equation is now linear with the undetermined coefficients A, B and C, viz., the system can be solved by the least squares with matrix formulation. A, B, and Care solved followed by h, k, and r. The matrix equation for circular regression is

$$\begin{bmatrix} \sum x_i^2 & \sum x_i y_i & \sum x_i \\ \sum x_i y_i & \sum y_i^2 & \sum y_i \\ \sum x_i & \sum y_i & n \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \sum x_i(x_i^2 + y_i^2) \\ \sum y_i(x_i^2 + y_i^2) \\ \sum x_i^2 + y_i^2 \end{bmatrix}$$

where "n" is the number of data points (x_i, y_i). The 3x3 matrix is inversed to obtain the unique set of values for A, B, and C there by subsequently solving for h, k, and r thus generating the best fit of circle.

h = -A/2

k = -B/2

$r = \frac{\sqrt{4C + A^2 + B^2}}{2}$

2) Three Point Best Fit Circle

The procedure to fit a circle having "n" data points (x, y) data distributed in a ring-shape on the x-y plane, where n ≥3 the 3 points method is used to find the equation of the circle that best fits the data. That is, to determine the values of h, k, and r where (h, k) is the center of the best fit circle and r is the radius of the best fit circle such that the curve provides a good fit around the data points. A, B, C are the points considered to formulate the circle amidst the given 'n' number of points from the CMM data.

The selection procedure for triplet (A, B, C) is as follows

a) Let (x_i, y_i) where i=1,2…n, and n≥ 3, be the CMM measured set of n points.

b) Select a point from the CMM measured set of n points and name it as A, which is the first point in triplet.

c) Calculate distance from point A to each point CMM measured points using equation below.

$A\,P_i = \sqrt{(x_a - x_i)^2 - (y_a - y_i)^2}$

Where, A Pi is distance from point A to point Pi

i = 1, 2 …n,

(x_a, y_a) are the coordinates of point A,

(x_i, y_i) are the coordinates of point Pi.

d) Select second point from P_i, where i= 1,2…n, (second point in triplet) for which A P_i is maximum. Name it as B.

e) Point C (third point in triplet) is selected from Pi, where i = 1, 2…n such that its normal distance from line AB is maximum. To determine maximum normal distance, the following procedure is followed.

i. Find the equation of the line for points A&B.

ii. Calculate the distance between all other corresponding points using the formula below.

Distance= $\frac{|Dx+Ey+F|}{\sqrt{D^2+E^2}}$

Where Dx +Ey +F=0 is the line equation obtained from the points AB.

f) Substitute points (A, B, C) in the general form of circle and solve for the coefficients.

g) Let the circle be $x^2 + y^2 + Gx + Hy + I = 0$

h) Built a coefficient matrix for finding the values of G, H, I.

$$\begin{bmatrix} x_a & y_a & 1 \\ x_b & y_b & 1 \\ x_c & y_c & 1 \end{bmatrix} \begin{bmatrix} G \\ H \\ I \end{bmatrix} = \begin{bmatrix} \Sigma x_a^2 + y_a^2 \\ \Sigma x_b^2 + y_b^2 \\ \Sigma x_c^2 + y_c^2 \end{bmatrix}$$

The 3x3 matrix is inversed to obtain the unique set of values for G, H, and I by subsequently solving for h, k, and r thus generating the best fit circle.

h = G/2

k = H/2

r = $\sqrt{h^2 + k^2 + I}$

where (h, k) is the center of best fit circle, 'r' is the radius of the best fit circle.

V. RESULTS AND DISCUSSIONS

The sample points are collected from the finalized sample space using the CMM technology. The eccentric shaft is placed on the CMM measuring surface table and the readings are taken from the computer supporting software of the CMM named as "Sceptre". The readings are taken on the entire surface of the circle at particular datum levels of the cylinder (eccentric shaft).

The best fit of the circles at different datum levels are generated. The deviation of the circles considered for the evaluation is calculated. This deviation is given by

Deviation (d) = $r_i - r_c$

where

d – Deviation

r_i– Manufactured radius

r_c– Manipulated radius

A graph representing the deviation of the considered circles is drawn. The deviation due to manufacturing aspects which is incurred in the circle is calculated.

A. Results on Least Squares Method

The least squares circle is calculated for the given sample points and is given below

$$(x - 356.6735157)^2 + (y - 206.9634138)^2 = (14.99492962)^2$$

The graph depicting the error deviations of the given data points for the least squares circle are given below.

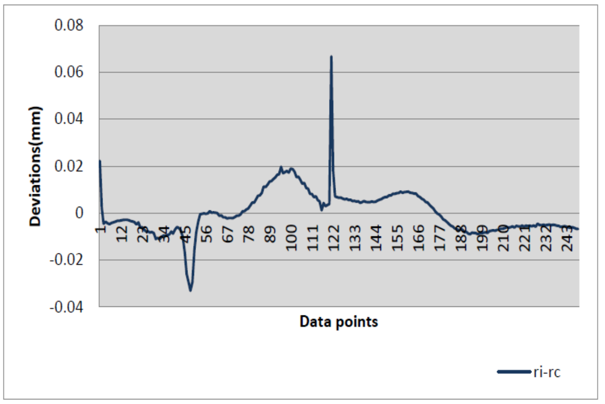

Fig. 5: Graph representing the deviations of the sample points from best fit circle at the datum level Z=445 mm by Least Squares Method.

B. Results on Three Point Method

The Three Point Method circle is calculated for the sample points using the methodology and the equation of circle obtained by the method is given

$$(x - 356.67675)^2 + (y - 206.972436)^2 = (14.99242969)^2$$

The graph depicting the error deviations of the given data points for the Three Points Circle are given below.

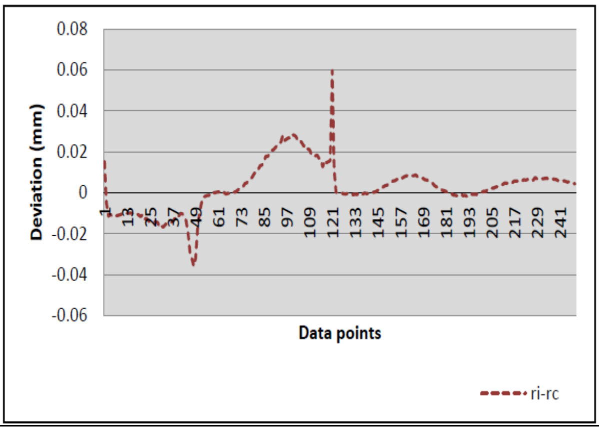

Fig. 6: Graph representing the deviations of the sample points from best fit circle at the datum level Z=445 mm by Three Points Method.

C. Results on Comparison of Least Squares Method and Three Point Method with CMM Data

A comparison of above two methods adopted for evaluation of circularity with respect to the form errors (deviations) is presented by plotting a graph.

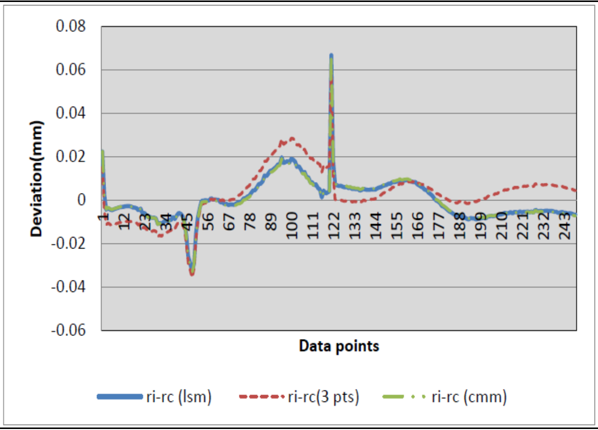

Fig. 7:Graph represents the comparisons of the deviations from the best fit circle calculated by different methods for the circle at the datum level of Z =445 mm.

Table 1 shows results of circularity evaluation for the dataset. The result of least squares method and three points method are expressed up to seven decimal places. It can be observed that circularity error obtained by three points method is less than that of LSM. It can also be observed that the same is more than that obtained by CMM result. The CMM results were available up to three decimal places. Table 1 also shows the comparison of errors. It can be observed that the errors are minimum amongst all.

VI. CONCLUSION

The present paper proposes an approach termed as Three points Method to determine dimensions of a circular feature from CMM measured point datasets. TPM is a simple method to understand and to implement amongst similar methods. It gives comparative results with Least Square Method (LSM) for CMM measured points (Table 1). The Three point methodology has the potential for implementation in CMM software for evaluation of circular features.

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TABLE 1: Results of circularity evaluation for measured points at datum level of Z=445 mm

	CMM Results	Least Squares Method	Three Points Method
Centre(x, y)	356.674	356.6735157	356.67675
	206.963	206.9634138	206.9724
Radius	14.995	14.9949296	14.9924297
Maximum Positive Error	0.066941707	0.049888743	0.05971999
Minimum Positive Error	0.0000946932	0.000180724	9.35E-05
Maximum Negative Error	-0.0000280085	-0.000276184	-1.75E-09
Minimum Negative Error	-0.03352235	-0.049888743	-0.035803249