

# Mathematical and Simulation Approach for Synchronization of Two Asynchronous Grids

Bhushan H Band

Department of Electrical Engineering  
PRMCEAM, Badnera  
Amravati, India

Anuradha D Ingole

Department of Electrical Engineering  
PRMCEAM, Badnera  
Amravati, India

**Abstract**— Electrical power is transmitted over the large distances from one power system to another by means of high voltage transmission lines. The interconnection of various power systems has depended upon the efficient allocation of available power over wide areas. For system stability, such interconnected systems are synchronized in time and frequency of generation and thus are said to be "synchronously tied".

There are many systems available to interconnect asynchronous grids like variable frequency transformer (VFT), HVDC but with nearly same value of grids. The idea is to connect the large different values of grids and for this two induction motors are used and connect them in cascade. The mathematical and simulation results are provided.

**Index Terms**—Induction Motor, Power System, Asynchronous Grid

## I. INTRODUCTION

With the almost universal adoption of A.C. system for distribution of electric energy of light and power, the field of application of A.C. motors has widen considerably during recent years. The two induction motors are connected with cascade, in which rotors of two motors are mechanically couple with each other and stator is energized with the help of power supply of two different values of grids.

The three phase induction motors run at constant speed from no load to full load, however, the speed is frequency dependent. The two grids which are operated on different value of frequency and connected to stator of two motor will operate with different rotating speed of rotor hence to make synchronization of two grids the rotor of two motors must run at predetermined speed.

## II. INDUCTION MOTOR

In this connection shafts of the two induction motor are mechanically coupled for different rotation and the rotor windings of the two motors are electrically connected in a reverse phase sequence. The cascaded induction motors are geared to a relatively low power variable speed drive which accurately controls the position and rotation of the shafts. One of the benefits of the cascade connection is it gives synchronous behavior at a user set speed. The cascaded consist of two nominally identical wound rotor induction motor. The rotors of the motors are mechanically coupled and the rotor windings are connected so as to produce contra rotating magnetic fields in the separate motor sections.

Let the frequency of supply given to the first motor A be  $f_1$ , its number poles be  $p_1$ , and its slip of operation be  $S_1$ . Let  $f_2$ ,  $p_2$  and  $S_2$  be the corresponding quantities for the second motor B. The frequency of currents flowing in the rotor of the first motor and hence in the rotor of the second motor is  $S_1 f_1$ . Therefore  $f_2 = S_1 f_1$ . Since the motors are coupled at the shaft, the speed of the rotor is common. Hence, if  $\omega$  is the angular speed of the rotor (for both motor) in rad/sec,

$$\omega = f_1 / p_1 (1 - S_1) = \pm S_1 f_1 p_2 (1 - S_2) \quad (1)$$

Now while giving the rotor output of the first motor to the rotor of the second, the resultant stator mmf of the second motor may set up an air-gap flux which rotates in the same direction as that of the rotor or opposes it. This results in values for speed as,

$$\omega = f_1 / (p_1 + p_2) \text{ or } \omega = f_1 / (p_1 - p_2) \quad (2)$$

Consider the following different cases:

Table 1: Operating Conditions of Cascaded I.M

Case	Common Rotor frequency	Motor A direction	Motor B direction
1	$\omega > \omega_1 > \omega_2$	Reverse	Reverse
2	$\omega_1 > \omega > \omega_2$	Forward	Reverse
3	$\omega_1 > \omega_2 > \omega$	Forward	Forward
4	$\omega$ opposite than $\omega_1$ and $\omega_2$	High speed forward	High speed forward

$\omega$  is the angular speed of the rotor (for both motor) in rad/sec

$\omega_1$  - the angular speed of the rotor A in rad/sec

$\omega_2$  - the angular speed of the rotor B in rad/sec

For both motor A and motor B stators are connected to different grids, provided with variable voltage and frequency to provide current at any power factor. The motor three phase windings are considered symmetrical and distributed around a uniform air gap for both stators and rotors. Only the fundamental components of the voltages and currents are considered. Also only the fundamental space MMF waves are considered for the stators and rotors. Friction, windage and core losses are neglected.

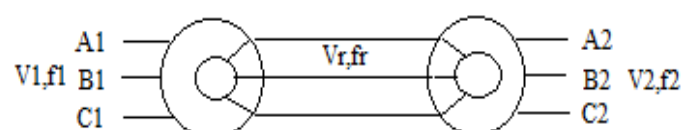


Fig. 1 Cascade Connection of I.M

### III. MATHEMATICAL MODEL

The Fig. 1, is considered, for  $\omega_1 > \omega > \omega_2$ . The rotating field in the airgap of motor A developed by the stator voltage  $V_1$  of motor A has an angular speed  $\omega_1$

$$\omega_1 = 2 \pi f_1 / p \quad (3)$$

Where  $p$  is the number of pole and  $f_1$  is the stator supply voltage phasor and  $f_1$  frequency of motor A. The voltage phasor  $V_1$  induced in the motor A rotor windings by the air-gap rotating field has an angular speed  $\omega$  equal to the difference between the air-gap field angular speed  $\omega_1$  and (motor A) rotor angular speed  $\omega_{m1}$ . In this case  $\omega_{m1}$  has the same (clockwise) direction as motor 1 field rotation.

$$\omega = \omega_1 - \omega_{m1} \quad (4)$$

The corresponding rotor frequency is,

$$f_r = [p(\omega_1 - \omega_{m1}) / 2 \pi] \quad (5)$$

Motor B is fed with the voltage  $V_r$  from the rotor having rotor frequency  $f_r$  and with the voltage  $V_2$  from the stator having the frequency  $f_2$ . The air gap rotating field angular speed  $\omega_2$  of motor 2 determined by the voltage  $V_2$  is,

$$\omega_2 = 2 \pi f_2 / p \quad (6)$$

Thus, in order to maintain synchronization, the motor B rotor is forced to turn with an angular velocity  $\omega_{m2}$ , so that the rotating field developed by the rotor supply superimposed on the rotor B angular velocity  $\omega_{m2}$  gives a rotating field of angular speed equal to  $\omega_2$ .

$$\omega_2 = \omega + \omega_{m2} \quad (7)$$

Since  $\omega_2 < \omega$ , from equation (7) it is concluded that the rotor angular velocity  $\omega_{m2}$  of motor B has a negative value and rotates in a direction opposite (anticlockwise) to the stator rotating field  $\omega_2$  of motor 2.

From equation (4) and (7),

$$\omega_{m1} - \omega_{m2} = \omega_1 - \omega_2 \quad (8)$$

The two rotor angular speed  $\omega_{m1}$  and  $\omega_{m2}$  may take various values however the value  $\omega_{m1} - \omega_{m2}$  should be constant and equal to the difference of the two stator angular speed  $\omega_1 - \omega_2$ .

The differential drive speed control may be obtained by varying the frequency difference  $f_1 - f_2$  of the voltage phasors supplying the motor stators. Then the value  $\omega_{m1} - \omega_{m2}$  of both rotor angular speed increases when the frequency difference increases.

For motor A slip is  $S_1$ ,

$$S_1 = \omega / \omega_1 \quad (9)$$

For motor B slip is  $S_2$ ,

$$S_2 = \omega / \omega_2 \quad (10)$$

When  $\omega_1 > \omega > \omega_2$ , the cascaded motor operate in forward direction (clockwise). Then according to equation (9) motor A slip is  $S_1 < 1$ , rotor 1 rotates in rotating field direction giving motor A forward (clockwise) direction. For motor B, according to equation (10), slip is  $S_2 > 1$  and rotor B rotates opposite to its rotating field direction, giving reverse (anticlockwise) motor direction.

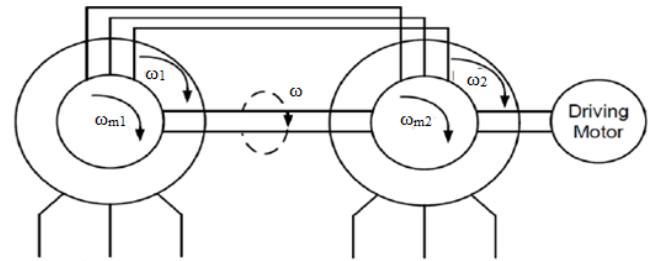


Fig. 2 Mechanical Connection of Combined Rotor

#### Case I

The magnetic fields of two stators are rotating in the same direction as driving speed  $\omega$ .  $\omega_1$  and  $\omega_2$  have the same direction of rotation as the rotor speed  $\omega$ ,

$$\omega_{m1} = f_1 [\omega_1 - \omega / \omega_1] \quad (11)$$

$$\omega_{m2} = f_2 [\omega_2 - \omega / \omega_2] \quad (12)$$

The condition for synchronization of two asynchronous grids is when  $\omega_{m1} = \omega_{m2}$ . Equating the two frequencies, we get the predetermined speed  $\omega$ , at which rotor frequencies are same.

$$\omega = 60 [f_1 - f_2 / p_1 - p_2]$$

Where,  $p_1$  and  $p_2$  are the number of poles of motors A and motor B respectively.

#### Case II

The magnetic fields of two stators are rotating in the same direction to each other but in opposite direction to rotor speed  $\omega$ .  $\omega_1$  and  $\omega_2$  have the same direction of rotation but opposite to the rotor speed  $\omega$ .

$$\omega_{m1} = f_1 [\omega_1 + \omega / \omega_1] \quad (13)$$

$$\omega_{m2} = f_2 [\omega_2 + \omega / \omega_2] \quad (14)$$

The condition for synchronization of two asynchronous grids is when  $\omega_{m1} = \omega_{m2}$ . Equating the two frequencies, we get the predetermined speed  $\omega$  at which rotor frequencies are same.

$$\omega = 60 [f_1 - f_2 / p_1 - p_2] \quad (15)$$

Where  $p_1$  and  $p_2$  are the number of poles of motors A and motor B respectively

#### Case III

The stators magnetic fields of one motor is rotating in the same direction as rotor speed  $\omega$  and the other motor stator field is rotating in the opposite direction to the speed  $\omega$ .  $\omega_1$  has the same direction as  $\omega$  and  $\omega_2$  has an opposite direction of rotation to speed  $\omega$ .

$$\omega_{m1} = f_1 [\omega_1 - \omega / \omega_1] \quad (16)$$

$$\omega_{m2} = f_2 [\omega_2 + \omega / \omega_2] \quad (17)$$

The condition for synchronization of two asynchronous grids is when  $\omega_{m1} = \omega_{m2}$ . Equating the two frequencies, we get the predetermined speed  $\omega$  at which rotor frequency are same.

$$\omega = 60 [f_1 - f_2 / p_1 + p_2] \quad (18)$$

Where  $p_1$  and  $p_2$  are the number of pole of motors A and B respectively.

Comparing above three cases if motor A having 2 pole and connected to 50Hz frequency grid and motor B having 2 pole and connected to 36Hz frequency grid. The predetermined calculated rotor speed  $\omega$  for case I, case II and case III are for case I and case II, speed ( $\omega$ ) is 420 rpm. For case III, speed ( $\omega$ ) is 140 rpm. This is lowest speed at which rotors of two motors

are rotate and lead to same rotor frequency. Hence case III is used in simulation.

In the mathematical analysis we have calculated rotor frequency, rotor electrical power, rotor speed of motor A and motor B generated in induction motor. The two induction motors are used here with 220 V, 2 pole and frequency 60 Hz for motor A and 50 Hz for motor B.

Rotor speed

$$\omega = 60 [(f_1 - f_2) / (p_1 + p_2)] = 150 \text{ rpm}$$

The synchronous speed calculated as,

For motor A

$$\omega_1 = 120 f_1 / p_1 = (120 * 60) / 2 = 3600 \text{ rpm}$$

$$S_1 = (\omega_1 - \omega) / \omega_1 = 0.95$$

$$f_{r1} = S_1 * f_1 = 57 \text{ Hz}$$

For motor B

$$\omega_2 = 120 f_2 / p_2 = (120 * 50) / 2 = 3000 \text{ rpm}$$

$$S_2 = (\omega_2 + \omega) / \omega_2 = 1.05$$

$$f_{r2} = S_2 * f_2 = 52 \text{ Hz}$$

The common rotor frequency at which two rotors of TSIM operated,

$$f_r = (f_1 + f_2) / 2 = 55 \text{ Hz}$$

Since,  $f_{r1} > f_r > f_{r2}$  according to the table 1, frequency of the rotor has a value between the two stator frequencies. Hence two motors are rotating in opposite directions but the sum of the absolute values of their speeds is maintained constant.

The stator electrical power calculated as,

$$P = \sqrt{3} * V * I * \cos \Phi = 304 \text{ W}$$

The mechanical power  $P_{m1}$  and  $P_{m2}$  expressed as functions of the stator electrical powers  $P_1$ ,  $P_2$  and slips  $S_1$ ,  $S_2$ .

$$P_{m1} = - (1 - S) P_1 = -15.2 \text{ W}$$

$$P_{m2} = - (1 - S) P_2 = 15.2 \text{ W}$$

Since both motors are delivering mechanical power both  $P_{m1}$  and  $P_{m2}$  are considered negative.

For forward direction of rotation, slip obtained  $S_1 < 1$  and  $S_2 > 1$ , hence  $P_1$  is positive but  $P_2$  is negative.

The rotor slip power  $P_r$  expressed as a function of mechanical power  $P_{m1}$  and slip  $S_1$  for motor 1 and  $P_{m2}$  and slip  $S_2$  for motor 2.

$$P_{r1} = P_{m1} S_1 / (1 - S_1) = -289 \text{ W}$$

$$P_{r2} = P_{m2} S_2 / (1 - S_2) = 289 \text{ W}$$

From above it is deduced that the slip power  $P_{r1}$  is negative for motor 1 and  $P_{r2}$  is positive for motor 2. In this way synchronization of two asynchronous grids is possible at predetermined speed of 150 rpm.

#### IV. SIMULATION MODEL

The simulation of the system is carried out using the Simulink software package. The wound rotor induction motor is used to facilitate the rotors connection and to be able to measure the rotor currents. Also the two motors are mechanically coupled, so the two motors are forced to rotate at a predetermined speed. The two rotors are electrically connected and mechanically coupled, so there is one rotating speed for both rotors. The cascaded connection of two induction motor is used into the simulation model for the desired result.

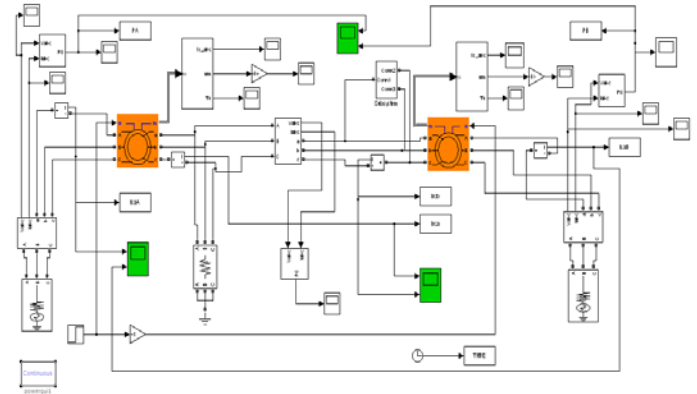


Fig. 3 Simulation Model

The electrical connection is provided between two rotors, from which current flow from one motor to other. The different parameters of motor are measured through the current measurement block, power measurement block and different waveforms on the scope. The simulation model consists of two similar 1.5 kW, three phases, wound rotor induction motors. The corresponding rotor phases are connected together. Stator of motor A connected to the three phase source (grid 1), of frequency 60 Hz. The stator of motor B connected to another three phase source (grid 2) of frequency 50 Hz.

The rotors of two induction motors are made to run at the predetermined operating speed according to equation (18), so that they lead to same rotor frequency. The rotor speed obtained is 150 rpm and applied to the rotor of motor. The simulation results show the synchronization of grids by its different waveforms.

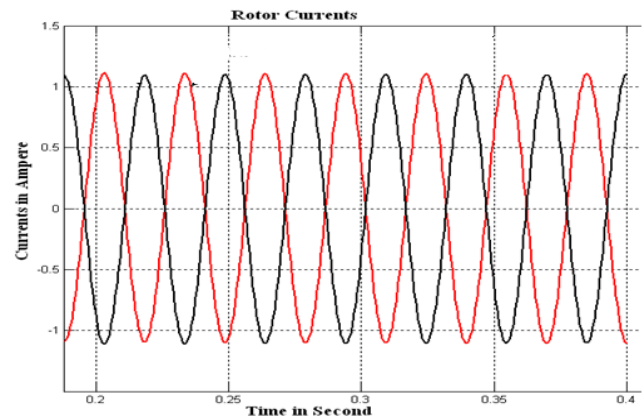


Fig. 4 Rotor Currents

The rotor currents for both the motors have same frequency. The rotor currents contain no harmonics. The condition for synchronization of two grids is satisfied at the calculated speed. The two rotor currents have a phase shift of 180°. The 60 Hz is connected to the motor A of (2-pole) and the 50 Hz is connected to the motor 2 of (2-pole). The system is driven at the calculated speed of 150 rpm. The voltage at the motor A is 220 V with an angle of -240°. The voltage at the motor B is 220 V with 0°. The angles of these two voltages cannot be related to each other due to the difference in the frequencies.

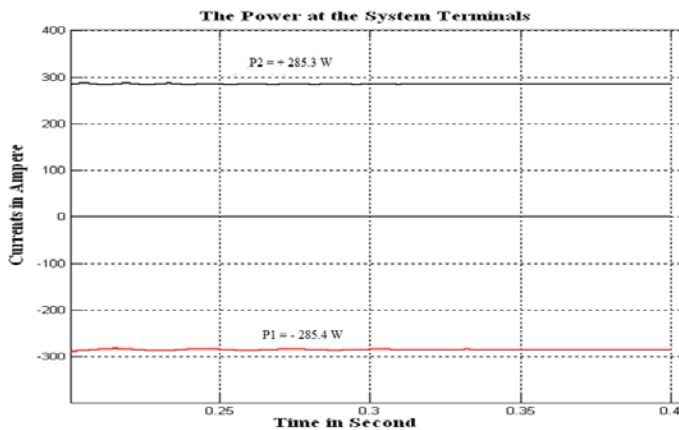


Fig. 5 Power at System Terminal

Figure 5 shows the steady state power at the system terminals. The power at the 60 Hz is about -285.4 W and the power at the 50 Hz is around +285.3 W. So it shows that the power has a constant value at steady state and hence it lead to synchronization of two grids at any value of frequencies. The simulated and mathematical results are verified.

#### V. CONCLUSION

Due to high rate of increase in power consumption as compared to generation, demand side management has become vital from system stability point of view. The power system management is important to fulfill the requirement of electricity. The grid interconnection plays vital role in power distribution to the various consumers at distribution end and hence providing the cascade connection of two motors which gives the synchronization of grids at any value of frequency. The mathematical and simulation calculation shows that two motor make the grid frequency same at predetermined value.

#### VI. FUTURE SCOPE

The experimental approach will be used for calculation of parameters of two induction motors and check how this parameters affect the machine values like efficiency, power factor and also how the stator to rotor turns ratios affect the performance.

The experimental setup can be used to calculate power flow between two induction motors and for determining different types of losses occurred in motor.

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