Mass Loading Effect on Natural Frequency of Cracked Beam in Free-Free Condition

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Abstract **- In this article the mass loading effect due to accelerometer on the natural frequency of cracked beam in free-free configuration has been investigated. Free-free configuration is selected as it is easier to replicate these boundary conditions in both experiment and analytical model. Also it is customary to validate the geometric model by comparing results with experiment in free-free condition. Natural frequencies obtained using accelerometer and FFT analyze are lower than obtained analytically or numerically using software based modal analysis. The difference could be attributed to the change in the boundary conditions, variation in geometric model simulating the real system, change in material properties and effect of accelerometer mass with its location on the beam. Therefore, can natural frequency alone be a detection parameter is an elusive question as on date. Nevertheless research is going on for qualitative assessment of health monitoring of the structure with natural frequency as the detection parameter due to its ease in measuring. Present work develops an analytical formulation to compute modal properties of a mass loaded beam, cracked beam and finally mass loaded cracked beam. The beam is considered as Euler-Bernoulli beam with additional mass effect is modeled by considering jump in shear force at the location. Crack is modeled as a mass less rotational spring and its flexibility parameter is obtained invoking concepts of fracture mechanics. Effects of additional mass location, crack depth and crack location on the modal properties are investigated.**

Keywords— Mass loading, modal analysis, FEM, free- free, cracked beam

I INTRODUCTION

In an attempt to extend the possibility of using modal parameters like natural frequency for detection of crack in the turbine blade, the process is validated firstly for simple beam. The intent eventually is to perform various analyses on the turbine blade model using ANSYS software so as to study mistuning effect due to cracking of blade.

As sensitivity of the natural frequency to the loss of stiffness needs to be ascertained and measured accurately for it to be become indicator for presence of crack and subsequently for location and severity of the crack. All the parameters which might affect the natural frequency need to be considered. One such parameter for small and slender structure is a mass of an accelerometer itself that lowers the measured natural frequencies. Therefore, in this research paper two modeling techniques analytical and numerical using ANSYS s/w are discussed so as to consider loading effect of an accelerometer mass on the natural frequencies of the beam. It is found that the mass of an accelerometer affected the natural frequencies according to its location and the ratio of its mass to the mass of beam. The effect was studied by performing experiment using two beams of different mass whereas material is same and accelerometer of same mass.

For validation of the geometric model before its use for further analyses, results of software based modal analysis have to agree closely with the experimental modal analysis of the physical or real system. It is therefore must to simulate the effect of additional mass of accelerometer in analytical and numerical i.e. software based modal analysis.

Many researchers have used the vibration response to detect cracks in a structure. Dado et.al [1] have tried to figure out not only the presence of crack by studying change in modal parameters but also the crack characterization like its depth and location. These detection schemes are based on the fact that the presence of a crack in a structure reduces the stiffness of the structure, hence reducing the natural frequencies.

Gounaris and Dimarogonas [2] have developed the Euler–Bernoulli beam cracked element based on the fracture mechanics approach. They have used coefficients of the compliance matrix that are computed based on available expressions of the stress intensity factor (SIF) and associated expressions of the strain energy density function (SEDF) by using the linear elastic fracture mechanics (LEFM) approach.

Ostachowicz and Krawczuk [3] have considered effect of two open cracks upon natural frequencies of flexural vibration of cantilever beam. They have observed that when two cracks are near to each other then drop in natural frequency is more and if two cracks distances from each other then frequency tend to be similar to single crack beam.

On the similar line of various researchers the loss of stiffness in the vicinity of the crack is estimated by calculating the additional flexibility coefficients from the relation between strain energy release rate, stress intensity factors using Castigliano's theorem and Paris Law. Then the stiffness matrix is obtained by taking inverse of the flexibility coefficient matrix as reported by Kotambkar [4].

In many situations, the mass of accelerometer is ignored in the analytical and numerical modeling process based on a usual assumption that the accelerometer mass is negligible compared to that of the structure under test. However, when lighter structures are investigated this effect can be significant and it may be necessary to eliminate this undesirable side effect before the measured data are used for future analyses. Cakar and Sanliturk [5] discussed the new method based on Sherman Morrison identity for elimination of mass loading effect of accelerometer from the measured FRF.

Wang [6] has done a comprehensive study of Euler-Bernoulli beam loaded with lumped mass of both translational and rotary inertias. Frequency sensitivity is performed with respect to location of lumped mass.

Low [7] has done a comparative study of eigenfrequency analysis for an Euler-Bernoulli beam carrying concentrated mass at an arbitrary location. The differential equation of motion along with corresponding boundary conditions and compatibility condition is converted to dimensionless frequency equation. The model does not consider rotary inertia of the beam and additional mass placed on it.

The study to determine the effect of accelerometer mass on natural frequencies was triggered when it was observed that the first bending mode natural frequency of the turbine blade in free-free condition obtained experimentally did not match with the modal analysis result obtained by ANSYS software. Although for geometric model validation, modal analysis results in free-free condition has to agree closely with experimental values. It was concluded that the value of material parameters i.e. density and modulus of elasticity considered during modal analysis have to be accurate and effect of accelerometer as an additional mass also needs to be taken in consideration. It was decided to carry out the experiment and modal analysis using ANSYS on simple rectangular cross sectional beam with and without crack so as to study effect of crack on natural frequencies with due consideration to additional mass effect of accelerometer.

Thus this paper discusses the findings of an attempt done to consider effect of additional mass of accelerometer in the modal analysis of the beam with and without crack using ANSYS 11.0. There are two ways to model the additional mass in ANSYS using point mass element mass21 [8].

II. ANALYSIS OF MASS LOADING EFFECT ON BEAM IN FREE-FREE CONDITION

In order to validate the finite element model of the beam it has become customary to compare it with experimental measurement of natural frequency of the system under study in free-free condition as these boundary conditions can be replicated during experiment without any difficulty. Fig 1 below shows the mass loaded beam in free-free end conditions.

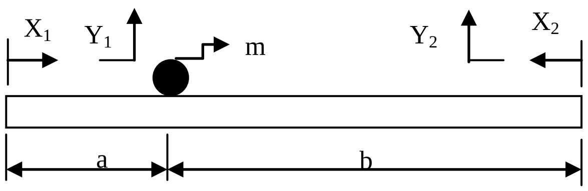

Figure 1: Beam in free-free condition with accelerometer mass

The beam is assumed to be divided into two segments at the mass location. Two different coordinate systems are taken at each end of the beam for reducing the number of integration constants. There are four constants for each segments resulting into total eight constants for the beam. Whereas there are four boundary conditions i.e. zero moment and force at each of the two free ends are used.

The equations of motion of free vibration for the beam is given by

$$EI\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} = 0 \qquad (1)$$

Above equation is well known as Euler-Bernoulli equation for a uniform cross section beam, for which, the solution exists in the form.

$$y(x,t) = Y(x)\cdot\eta(t) \qquad (2)$$

where the mode shape function Y(x) is expressed as

$$Y(x) = A_1\cosh\beta x + B_1\sinh\beta x + C_1\cos\beta x + D_1\sin\beta x \qquad (3)$$

with

$$\beta = \left(\omega^2\frac{\rho A}{EI}\right)^{1/4} \qquad (4)$$

Such that natural frequency, can be obtained as

$$\omega = \beta^2\sqrt{\frac{EI}{\rho A}} \qquad (5)$$

At the left end of the beam i. e. at $x_1 = 0$ and right end of the beam i.e. at $x_2 = 0_{\text{at } x=L}$, the displacement fields $y_1(x,t)$ and $y_2(x,t)$ satisfies the boundary conditions

Bending moment (BM): $EI\frac{\partial^2 y}{\partial x^2} = 0$ and Shear Force (SF): $\frac{\partial}{\partial x}\left[EI\frac{\partial^2 y}{\partial x^2}\right] = 0$ which allows the mode shape for the left segment 'a' to be reorganized as

$$Y_1(x_1) = A_1(\cosh\beta x_1 + \cos\beta x_1) + B_1(\sinh\beta x_1 + \sin\beta x_1) \qquad (6)$$

and for the right segment 'b' can be reorganized as

$$Y_2(x_2) = A_2(\cosh\beta x_2 + \cos\beta x_2) - B_2(\sinh\beta x_2 + \sin\beta x_2) \qquad (7)$$

The four equations due to compatibility conditions for displacement, slope, BM and SF at the location of additional mass for the beam is as follows. Due to the presence of additional mass there is sudden rise in shear force. [9]

$$y_{1\text{ at } x_1 = a} = y_{2\text{ at } x_2 = b}$$
$$y'_{1\text{ at } x_1 = a} = y'_{2\text{ at } x_2 = b}$$
$$y''_{1\text{ at } x_1 = a} = y''_{2\text{ at } x_2 = b} \quad \text{and}$$
$$y'''_{1\text{ at } x_1 = a} + m * \ddot{y}_{1\text{ at } x_1 = a} = y'''_{2\text{ at } x_1 = b} \qquad (8)$$

Substituting Eqs. 6-7 into the compatibility equations in Eq. 8, one gets the frequency equation in determinant form as

$$\begin{vmatrix} P_a & Q_a & -P_b & Q_b \\ S_a & P_a & S_b & -P_b \\ R_a & S_a & -R_b & S_b \\ Q_a + P_a * \psi\beta L & R_a + Q_a * \psi\beta L & Q_b & -R_b \end{vmatrix} = 0 \qquad (9)$$

Where

$P_a = \cosh\beta L\mu + \cos\beta L\mu$, in which $\mu = a/L$ is non dimensional mass location parameter

$$Q_a = \sinh\beta L\mu + \sin\beta L\mu$$

$$R_a = \cosh\beta L\mu - \cos\beta L\mu \quad \text{and}$$

$$S_a = \sinh\beta L\mu - \sin\beta L\mu$$

$P_b = \cosh\beta L(1-\mu) + \cos\beta L(1-\mu)$, in which $1-\mu = b/L$

$$Q_b = \sinh\beta L(1-\mu) + \sin\beta L(1-\mu)$$

$$R_b = \cosh\beta L(1-\mu) - \cos\beta L(1-\mu) \quad \text{and}$$

$$\psi = m/\rho AL \qquad (10)$$

Where ψ is ratio of additional mass to mass of the beam

The numerical simulations are done by solving characteristic equation obtained from the determinant in (9) to find the natural frequency (βL). The modulus of elasticity E is considered as 200 GPa and material density was measured 7835 Kg/m3. The results of simulations with sample beam of size 610.3 x 25.3 x 3.4 mm are presented in table 1.

TABLE 1: EFFECT OF MASS (ψ = 0.065) LOADING LOCATION ON FIRST THREE BENDING MODES

		βL values		
SN	***a/L***	***Mode I***	***Mode II***	***Mode III***
1	0.05	4.58985	7.72128	10.8987
2	0.1	4.65753	7.83232	10.994
3	0.15	4.70428	7.84705	10.8719
4	0.2	4.72741	7.78027	10.7473
5	0.25	4.72726	7.69779	10.777
6	0.3	4.70895	7.65988	10.905
7	0.35	4.68136	7.68328	10.9944
8	0.4	4.65405	7.75018	10.9286
9	0.45	4.63462	7.82191	10.7721
10	0.5	4.62765	7.8532	10.6952

It is observed from the table 1 that natural frequency of all the modes is sensitive to additional mass and its location. The drop in natural frequency due to additional mass is more if the location of the mass is near to the anti node of the mode and it is unaffected if the location is at the node of the mode. There are two anti nodes for half the beam considered due to symmetry in Mode I, one at free end and other at the middle of beam due to which drop observed is more as shown in fig 2 below.

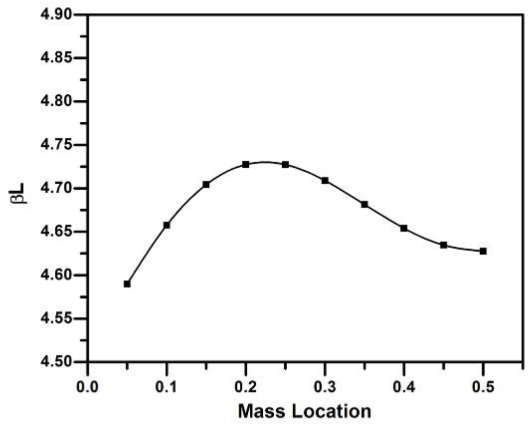

Figure 2: Effect of mass loading on Mode-I

Similar trend is observed in higher modes II and III. Figs. 3 and 4 show mass loading location effect on natural frequency of second and third mode

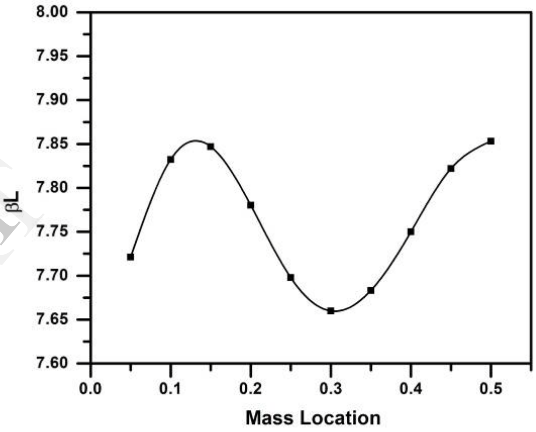

Figure 3: Effect of mass loading on Mode-II

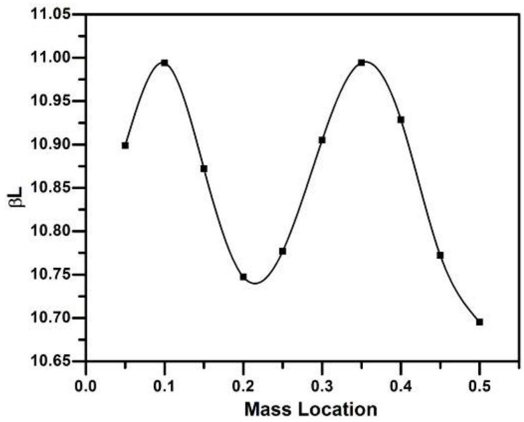

Figure 4: Effect of mass loading on Mode-III

III. ANALYSIS OF CRACK EFFECT ON BEAM IN FREE-FREE CONDITION

In order to find out drop in natural frequency of the beam in free-free condition due to crack alone, the analysis is carried out by considering crack of varying depth at different locations. Fig 5 below shows the cracked beam.

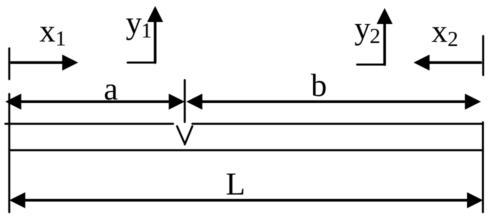

Figure 5: Beam with crack

Similar approach is used for modeling the effect of crack except the four equations due to compatibility conditions for displacement, slope, BM and SF at the location of crack for the blade given below. Due to the presence of crack there is sudden jump in slope. [10][11]

$$y_{1\ \text{at}\ x_1=a} = y_{2\ \text{at}\ x_2=b},$$
$$y'_{1\ \text{at}\ x_1=a} + EIc_f\, y''_{2\ \text{at}\ x_2=b} = y'_{2\ \text{at}\ x_2=b},$$
$$y''_{1\ \text{at}\ x_1=a} = y''_{2\ \text{at}\ x_2=b}\ \text{and}$$
$$y'''_{1\ \text{at}\ x_1=a} = y'''_{2\ \text{at}\ x_1=b} \qquad (11)$$

Where cf is local flexibility coefficient obtained from fracture mechanics based strain energy density function

$$f(h/H)=0.6384-1.035*n+3.7201*n^2-5.1773*n^3+7.553*n^4-7.332*n^5+2.4909*n^6 \qquad (12)$$

$$\frac{1}{c_f}=k=\frac{b*h^2*E}{72*\Pi*n^2*f(h/H)} \qquad (13)$$

The frequency determinant can be obtained as given in Eq. 14 below

$$\begin{vmatrix} P_a & Q_a & -P_b & Q_b \\ S_a & P_a & S_b+R_b*\gamma\beta L & -P_b-S_b*\gamma\beta L \\ R_a & S_a & -R_b & S_b \\ Q_a & S_a & Q_b & -R_b \end{vmatrix}=0 \qquad (14)$$

Where $\gamma = EIc_f / L$ is dimensionless crack flexibility coefficient

The blade crack severity ratio $\phi = h/H$ which is crack depth to thickness of blade is introduced. The ratio varying from 0 to 0.5 has been used at different locations and accordingly flexibility 'cf' due to crack and $\gamma = EIc_f / L$ values are obtained. The material for the beam is considered to be steel with modulus of elasticity 200 GPa and density was measured to be 7835 Kg/m3. The results of simulations is presented in tables 2-5 for some typical locations x/L = 0.05, 0.15, 0.3 and 0.5.

TABLE 2: CRACK EFFECT ON NATURAL FREQUENCIES (βL VALUES) FOR CRACK LOCATION X/L = 0.05

Mode	βL values					
	ϕ = 0	ϕ =0.1	ϕ =0.2	ϕ =0.3	ϕ =0.4	ϕ =0.5
1	4.73004	4.73004	4.73003	4.73002	4.73001	4.72998
2	7.8532	7.85318	7.85312	7.85301	7.85284	7.85257
3	10.9956	10.9955	10.9952	10.9947	10.9939	10.9926

TABLE 3: CRACK EFFECT ON NATURAL FREQUENCIES (βL VALUES) FOR CRACK LOCATION X/L = 0.15

Mode	βL values					
	ϕ = 0	ϕ =0.1	ϕ =0.2	ϕ =0.3	ϕ =0.4	ϕ =0.5
1	4.73004	4.72994	4.72963	4.72909	4.7282	4.72685
2	7.8532	7.85236	7.84994	7.84559	7.83849	7.82761
3	10.9 956	10.993	10.9854	10.9719	10.9499	10.9165

TABLE 4: CRACK EFFECT ON NATURAL FREQUENCIES (βL VALUES) FOR CRACK LOCATION X/L = 0.30

Mode	βL values (11)					
	ϕ = 0	ϕ =0.1	ϕ =0.2	ϕ =0.3	ϕ =0.4	ϕ =0.5
1	4.73004	4.72919	4.72677	4.72242	4.71536	4.70463
2	7.8532	7.85056	7.843	7.82958	7.80816	7.7764
3	10.9956	10.9944	10.9909	10.9847	10.975	10.9608

TABLE 5: CRACK EFFECT ON NATURAL FREQUENCIES (βL VALUES) FOR CRACK LOCATION X/L = 0.50

Mode	βL values					
	ϕ = 0	ϕ =0.1	ϕ =0.2	ϕ =0.3	ϕ =0.4	ϕ =0.5
1	4.73004	4.72826	4.72319	4.71413	4.69957	4.67774
2	7.8532	7.8532	7.8532	7.8532	7.8532	7.8532
3	10.9956	10.9924	10.9832	10.9669	10.9411	10.9032

It is quite clear from the Tables (2-5) that all the natural frequencies of all the modes are sensitive to the presence of crack. As the severity of crack increase, there is further drop in the natural frequency.

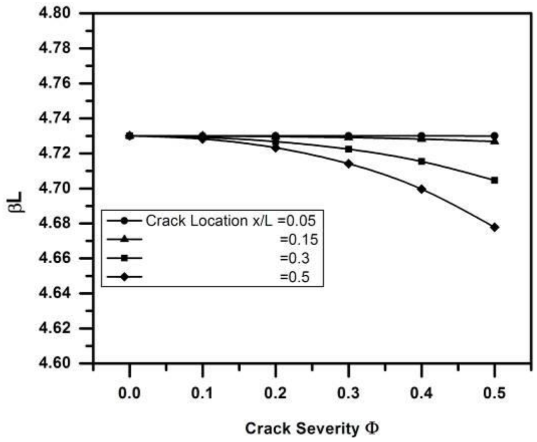

Figure 6: Crack severity effect on Mode-I

The figure 6 depicts effect of crack severity at different locations on Mode-I. It is observed that as the location of crack moves towards mid of the beam, the rate at which natural frequency drops gets increased.

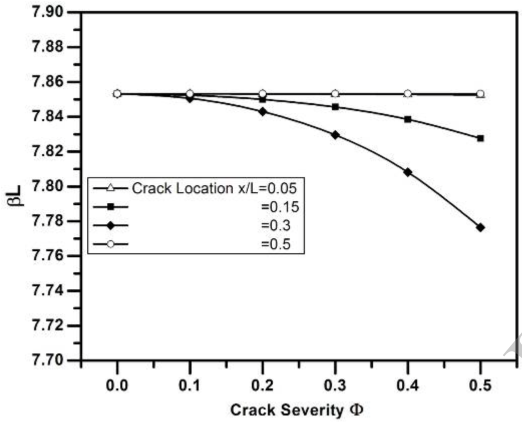

Figure 7: Crack severity effect on Mode-II

The fig 7 depicts effect of crack severity at different locations on natural frequency of Mode-II. It is noted that the drop is almost negligible when crack location is at free ends or at middle of the beam, rather it is unaffected for mid-location. If the crack location is nearer to anti node of the mode, the rate of drop in natural frequency increases with crack severity.

The figure 8 shows effect of crack severity at different locations on natural frequency of Mode III. It is observed that for crack location at free ends, natural frequency is unaffected whereas for all other locations, drop in natural frequency increases with increase in crack severity.

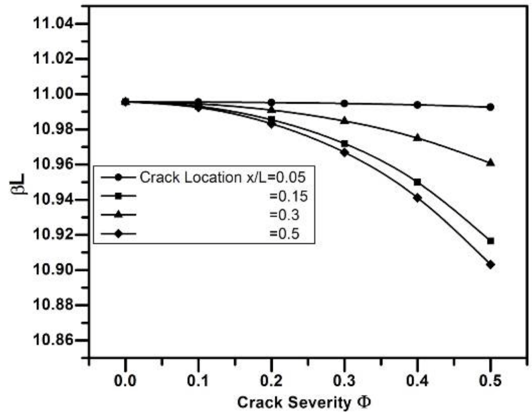

Figure 8: Crack severity effect on Mode-III

The figure 8 shows effect of crack severity at different locations on natural frequency of Mode III. It is observed that for crack location at free ends, natural frequency is unaffected whereas for all other locations, drop in natural frequency increases with increase in crack severity.

IV. ANALYSIS OF ADDITIONAL MASS AND CRACK EFFECT ON BEAM IN FREE-FREE CONDITION

In order to study the effect of mass loading on the natural frequencies of cracked beam as shown in the Fig.9, the previous two models i.e. beam with mass loading and cracked beam are combined appropriately and the determinant form of frequency equation is given below (Eq. 15).

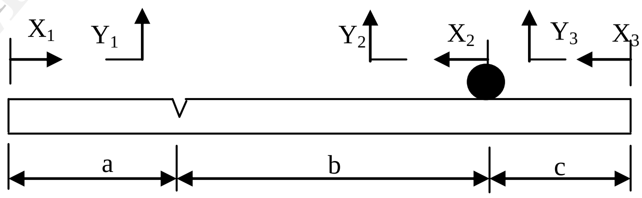

Figure 9: Cracked beam with mass loading.

$$\begin{vmatrix} P_a & Q_a & -T_b & U_b & V_b & W_b & 0 & 0 \\ S_a & P_a & U_b+T_b\gamma\beta L & -T_b-U_b\gamma\beta L & -W_b-V_b\gamma\beta L & -V_b+W_b\gamma\beta L & 0 & 0 \\ R_a & S_a & -T_b & U_b & V_b & -W_b & 0 & 0 \\ Q_a & R_a & U_b & -T_b & W_b & V_b & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & -P_c & Q_c \\ 0 & 0 & 0 & 1 & 0 & 1 & S_c & -P_c \\ 0 & 0 & 1 & 0 & -1 & 0 & -R_c & S_c \\ 0 & 0 & \psi\beta L & 1 & \psi\beta L & -1 & Q_c & -R_c \end{vmatrix} \quad (15)$$

Where $T_b = \cosh\beta L(1-\mu)$, $U_b = \cos\beta L(1-\mu)$, $V_b = \sinh\beta L(1-\mu)$, $W_b = \sin\beta L(1-\mu)$

The frequency equation will give characteristic root as βL which is function of crack severity and location of crack as well as additional mass and its location. The numerical simulation is done with typical mass m = 0.027 Kg located at c/L =0.05 and crack of severity ratio ϕ = 0 to 0.5 at varying location x/L = 0.05, 0.15, 0.3 and 0.5 and results are presented in tables (6-9).

TABLE 6: NATURAL FREQUENCIES (βL VALUES) FOR CRACK LOCATION x/L = 0.05

Mode	βL values					
	ϕ = 0	ϕ =0.1	ϕ =0.2	ϕ =0.3	ϕ =0.4	ϕ =0.5
1	4.58985	4.58984	4.58983	4.58982	4.5898	4.58984
2	7.72128	7.72121	7.72111	7.72094	7.72069	7.72121
3	10.8987	10.8986	10.8983	10.8978	10.897	10.8958

TABLE 7: NATURAL FREQUENCIES (βL VALUES) FOR CRACK LOCATION x/L = 0.15

Mode	βL values					
	ϕ = 0	ϕ =0.1	ϕ =0.2	ϕ =0.3	ϕ =0.4	ϕ =0.5
1	4.58985	4.58975	4.58949	4.58902	4.58825	4.58707
2	7.72128	7.72049	7.7182	7.71408	7.70737	7.69709
3	10.8987	10.8961	10.8887	10.8753	10.8536	10.8206

TABLE 8: NATURAL FREQUENCIES (βL VALUES) FOR CRACK LOCATION x/L = 0.3

Mode	βL values					
	ϕ = 0	ϕ =0.1	ϕ =0.2	ϕ =0.3	ϕ =0.4	ϕ =0.5
1	4.58985	4.58909	4.58693	4.58304	4.57674	4.56716
2	7.72128	7.71865	7.71112	7.69775	7.67638	7.64467
3	10.8987	10.8974	10.8936	10.8869	10.8763	10.861

TABLE 9: NATURAL FREQUENCIES (βL VALUES) FOR CRACK LOCATION x/L = 0.5

Mode	βL values					
	ϕ = 0	ϕ =0.1	ϕ =0.2	ϕ =0.3	ϕ =0.4	ϕ =0.5
1	4.58985	4.58816	4.58334	4.57473	4.56089	4.54013
2	7.72128	7.72127	7.72124	7.72118	7.72109	7.72095
3	10.8987	10.8955	10.8862	10.8699	10.844	10.806

It is observed that drop in natural frequency due to crack is accentuated due to effect of additional mass. The drop is sensitive to the location of crack when it is near to the anti node of the mode. From the table 9, it is clear that the second mode natural frequency for crack location at mid of beam span is showing some sensitivity to crack severity whereas it remains unaffected (Table 5) in the absence of additional mass. The drop in natural frequency with crack severity at different locations with and without mass loading effect is shown in the figs. 10-12.

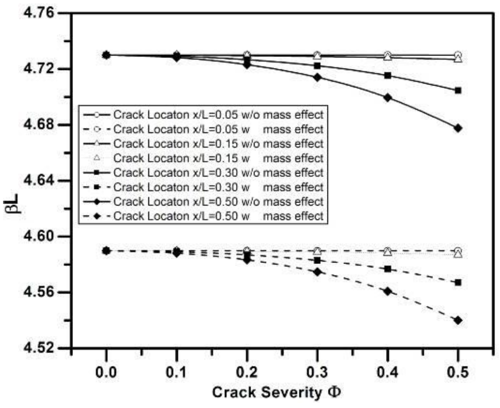

Figure 10: Effect of crack severity at different locations with and without additional mass on natural frequency of Mode-I

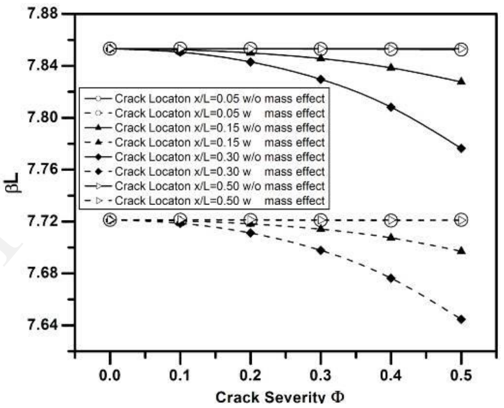

Figure 11: Effect of crack severity at different locations with and without additional mass on natural frequency of Mode-II

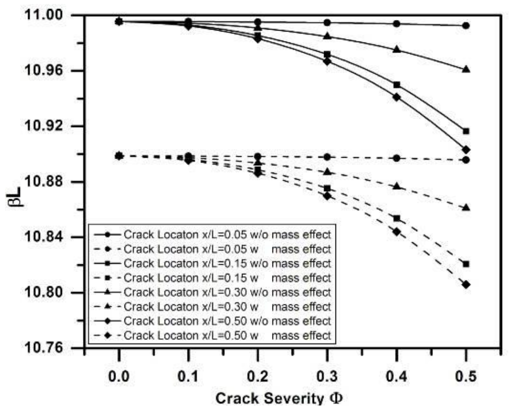

Figure 12: Effect of crack severity at different locations with and without additional mass on natural frequency of Mode-III

It is quite evident from the figs 10-12 that there is definite ratio by which all the modes natural frequency get shifted from without mass loading effect to with mass loading effect. The ratio does not influence the trend in drop of natural frequency due to crack severity.

V. EXPERIMENT:

Figure 13 shows schematic of experimental set up used which consists of two sample beams of dimensions Lx B x H in mm (SB1: 610.3 x 25.3 x 3.4; and SB2: 452 x 39.4 x5.7), accelerometer (B & K make, weigh 27 gm), FFT analyzer (DI 2200). The sample beams were suspended with threads in free-free condition. Accelerometer fitted with magnetic base was mounted on the beam and the response was sent to FFT after initial disturbance given to the beam. The first three bending mode natural frequencies were measured. The density of mild steel was measured to be 7835 Kg/m3.

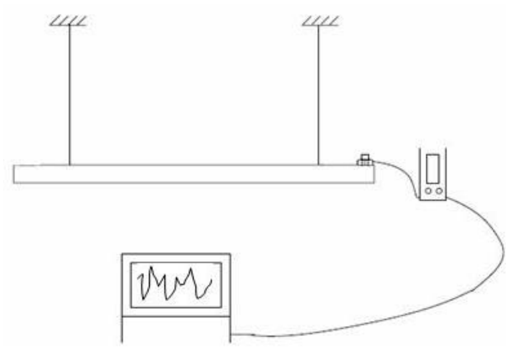

Figure 13: Experimental set up for measuring natural frequencies.

The table 10 shows the first three bending mode natural frequencies in Hz obtained experimentally in free-free condition.

TABLE 10: EXPERIMENTAL NATURAL FREQUENCIES OF SB1 AND SB2

Mode	Frequency in Hz	
	SB1	***SB2***
1	43.500	144.25
2	118.250	386.25
3	250.500	765

A crack of 0.4 mm width and 2 mm depth was cut in the (SB2) beam, henceforth referred as SBC2 (Sample beam with crack) using EDM Wire Cut Machine. The crack location was decided so that it didn't coincide with any one of the nodes of the first three modes of vibration as observed in (FEA) modal analysis.

Table 11 depicts comparison between measured natural frequencies for un-cracked (SB2) and cracked beam (SBC2).

TABLE 11: EXPERIMENTAL NATURAL FREQUENCIES OF UN-CRACKED SB2 AND CRACKED SBC2 BEAM

Mode	Frequency in Hz	
	SB2	***SBC2***
1	144.25	143.875
2	386.25	382.175
3	765	760

VI. NUMERICAL SIMULATION USING ANSYS

The numerical investigation is carried out using modal analysis module of ANSYS software. The finite element model is created using solid 45, brick element with 8 nodes having all the three translational degrees of freedom and mass of accelerometer is simulated using mass21 element. The fig. below shows the FE model of the beam (SB1) with distributing mass load on nodes that are coming in contact of circular magnetic base of accelerometer by using CERIG command. The command created constrained region between master and slave nodes thus ensuring connectivity between the two as rigid link. The global size of the brick element is 6.8 mm for all the analysis result presented here, however, the FE model shown in Fig. 14 is with element size 3.4. The number of elements is 360 with size 6.8 mm and it is 1440 with size 3.4 mm. The frequency values converge with element size 6.8 mm itself and hence this size is kept.

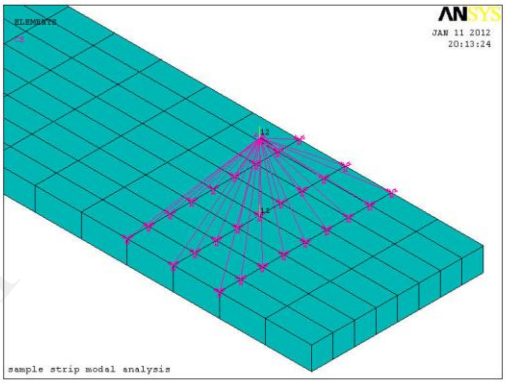

Figure 14: FE model of SB1 with mass loading using CERIG

The results of modal analysis are presented in the table 12 below.

TABLE 12: COMPARISON OF FIRST THREE MODE FREQUENCIES WITH EFFECT OF CONCENTRATED AND DISTRIBUTED MASS FOR SB1

Frequency in Hz of beam SB1		
Without mass effect	***Mass concentrated on one node***	***Mass distributed over nodes***
46.920	43.672	43.635
129.42	123.33	123.05
253.98	245.85	244.86

It is obvious from the values depicted in Table 12 that both the methods of simulating the accelerometer mass load effect gives pretty much the same result. Thus the method of adding mass at one node is used for the analysis.

The comparison between natural frequency for the first bending mode of vibration of the sample beam (SB1), obtained by FEA and analytical model is given in the table 13. In both the models accelerometer mass effect with its location has been considered.

TABLE 13: FIRST BENDING MODE FREQUENCY OF SB1 WITH FEA AND ANALYTICAL MODEL

Sr. No.	Location of mass a/L	Natural frequency in Hz	
		FEA	***Analytical***
1	0.1	45.974	45.988
2	0.2	47.364	47.379
3	0.3	46.995	47.009
4	0.4	45.907	45.920
5	0.5	45.389	45.400

The values of natural frequencies by both the methods are in close agreement.

Further, modal analysis is carried out for sample beam SB2 by considering crack similar to what is cut with EDM in the actual beam. The geometric model is as shown in the fig. 15

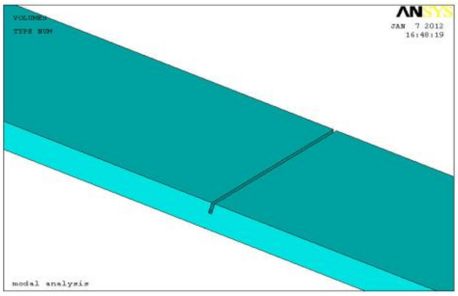

Figure 15: ANSYS model of a beam (SBC2) with crack

The finite element model is shown in the fig. 16, due to presence of crack in the model, numbers of finite elements and nodes increases significantly.

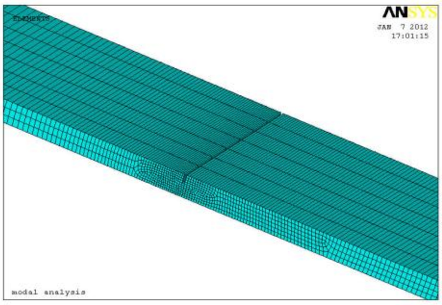

Figure 16: ANSYS FE model of a beam (SBC2) with crack

The result of modal analysis is presented in the table 14 along with experimentally obtained natural frequency values.

TABLE 14: COMPARISON OF NATURAL FREQUENCIES OF UN-CRACKED (SB2) AND CRACKED BEAM (SBC2) (EXPERIMENTAL, FEA W AND W/O ACCELEROMETER MASS)

Mode	Freq. in Hz Without crack			Freq. in Hz With crack		
	Experimental	***FEA w/o A.M.***	***FEA w. A.M.***	***Experimental***	***FEA w/o A.M.***	***FEA w. A.M.***
1.	144.25	144.91	144.58	143.875	143.90	143.59
2.	386.25	399.79	389.75	382.175	392.95	383.34
3.	765	784.70	771.84	760	777.77	764.25

(w/o: without, w.: with, A. M.: Accelerometer Mass)

Thus with simulating mass loading effect with ANSYS, natural frequency values of all the three mode are coming close to the measured values.

VII. CONCLUSIONS

The natural frequency for all the modes is reducing with rise in mass loading whereas its location affects the drop more if close to the anti node of the mode. The drop in natural frequency due to crack in the beam gets amplified due to mass loading effect. Therefore to estimate the drop in natural frequency due to crack alone, it is essential to know the extent of frequency drop caused by mass loading. The correction factor due to mass loading can be obtained from software based modal analysis and used to modify experimentally measured value of natural frequency.

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REFERENCES

[1] Mohammad H. Dado, "A Comprehensive Crack Identification Algorithm for Beams under Different End Conditions" Applied Acoustics, Vol. 51, No. 4, pp. 381-398, 1997
[2] G. Gounaris and A. Dimarogonas, "A Finite Element of a Cracked Prismatic Beam for Structural Analysis" Computer and Structures, 1988, 28(3).
[3] W. M. Ostachowicz and M. Kraawczuk, "Analysis of the effect of cracks on the natural frequencies of a cantilever beam", J. of Sound and Vibration, (1991), 150(2), 191-201
[4] M. S. Kotambkar " Exploring the Vibration Based Method for Crack Detection in Turbine Blade" The 2010 International Conference on Mechanical and Electrical Technology（ ICMET 2010)
[5] O. Cakar, K. Y. Sanliturk, "Elimination of Transducer Mass Loading Effects from Frequency Response Functions" Mechanical Systems and Signal Processing 19 (2005) 87-104
[6] D. Wang , "Vibration and Sensitivity Analysis of a Beam With a Lumped Mass of Translational and Rotary Inertias " Journal of Sound and Acoustics, June 2012, Vol. 134
[7] K. H. Low, "A comparative study of the eigen solutions for mass-loaded beams under classical boundary conditions", Int. Journal of Mechanical Sciences 43 (2001) 237-244
[8] ANSYS 11.0
[9] V. Mermertas and H. Erol, "Effect of mass attachment n the free vibration of cracked beam", The 8th International congress on Sound and Vibration, July 2001, Hong Kong, China
[10] T.G. Chondros and A.D. Dimarogonas, "Vibration of a cracked cantilever beam", ASME J. of Vibration and Acoustics, Vol. 120 (1998), 742-758.
[11] C-S Wang and L-T Lee, "Modified and simplified sectional flexibility of a cracked beam", J. of Applied Mathematics, Vol 2012, Article ID 543828
[12] DI 2200 FFT manual