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Lower Level Subsets of Anti L-Fuzzy Subfield of a Field

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ABSTRACT: In this paper, we made an attempt to study the algebraic nature of lower level subsets of anti L-fuzzy subfield of a field under homomorphism.

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KEY WORDS: L-fuzzy set, anti L-fuzzy subfield, anti L-fuzzy (a,b)-coset, lower level subset pseudo anti L-fuzzy coset.

INTRODUCTION: After the introduction of fuzzy sets by L.A.Zadeh[16], several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subgroups, anti-fuzzy subgroups, fuzzy fields and fuzzy linear spaces was introduced by Biswas.R[4, 5]. In this paper, we introduce the some theorems in lower level subsets of anti L-fuzzy subfield of a field under homomorphism.

1. PRELIMINARIES:

1.1 Definition: Let X be a non-empty set and L be a complete lattice. A L-**fuzzy subset A** of X is a function $A : X \rightarrow L$.

1.2 Definition: Let (F, +, ·) be a field. A L-fuzzy subset A of F is said to be an **anti L-fuzzy subfield(ALFSF)** of F if the following conditions are satisfied:

(i) $A(x+y) \leq A(x) \vee A(y)$, for all x and y in F,

(ii) $A(-x) \leq A(x)$, for all x in F,

(iii) $A(xy) \leq A(x) \vee A(y)$, for all x and y in F,

(iv) $A(x^{-1}) \leq A(x)$, for all x in $F-\{0\}$, where 0 is the additive identity element of F.

1.3 Definition: Let (F, +, ·) and (F^{l}, +, ·) be any two fields. Let $f : F \rightarrow F^{l}$ be any function and A be an anti-fuzzy subfield in F, V be an anti L-fuzzy subfield in $f(F) = F^{l}$, defined by $V(y) = \inf_{x \in f^{-1}(y)} A(x)$, for all x in F and y in F^{l}.

Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.4 Definition: Let A be an anti L-fuzzy subfield of a field (F, +, ·). For any a and b≠0 in F, ${}_aA_b$ is defined by (a+A)(x) = A(–a+x), for all x in F and (bA)(x) = A(b^{-1}x), for all x in F, is called an anti L-fuzzy (a,b)-coset of F.

1.5 Definition: Let A be an anti L-fuzzy subfield of a field (F, +, ·) and a in F. Then the pseudo anti L-fuzzy coset $(aA)^p$ is defined by $((aA)^p)(x)$ = p(a)A(x), for every x in F and for some p in P.

1.6 Definition: Let A be a fuzzy subset of X. For α in L, the lower level subset of A is the set A_α = { x∈X: A(x) ≤ α }.

2 – PROPERTIES OF ANTI L-FUZZY SUBFIELDS:

2.1 Theorem: Let (F, +, ·) and (F', +, ·) be any two fields. The homomorphic image of an anti L-fuzzy subfield of F is an anti L-fuzzy subfield of F'.

Proof: Let (F, +, ·) and (F', +, ·) be any two fields and f : F→F' be a homomorphism. That is f(x+y) = f(x)+f(y), for all x and y in F and f(xy) = f(x)f(y), for all x and y in F. Let V= f(A), where A is an anti L-fuzzy subfield of F. We have to prove that V is an anti L-fuzzy subfield of F'. Now, for f(x) and f(y) in F', we have V(f(x)–f(y)) = V(f(x–y)) ≤ A(x–y) ≤ A(x)∨A(y), which implies that V(f(x)–f(y)) ≤ V(f(x)) ∨ V(f(y)), for all f(x) and f(y) in F'. And V(f(x)(f(y) $)^{-1}$) = V(f(xy^{-1})) ≤ A(xy^{-1}) ≤ A(x) ∨ A(y), which implies that V(f(x)(f(y) $)^{-1}$) ≤ V(f(x)) ∨ V(f(y)), for all f(x) and f(y) ≠ 0^1 in F'. Hence V is an anti L-fuzzy subfield of a field F'.

2.2 Theorem: Let (F, +, ·) and (F', +, ·) be any two fields. The homomorphic pre-image of an anti L-fuzzy subfield of F' is an anti L-fuzzy subfield of F.

Proof: Let (F, +, ·) and (F', +, ·) be any two fields and f : F→F' be a homomorphism. That is f(x+y) = f(x)+f(y), for all x and y in F and f(xy) = f(x)f(y), for all x and y in F. Let V = f(A), where V is an anti L-fuzzy subfield of F'. We have to prove that A is an anti L-fuzzy subfield of F. Let x and y in F. Then, A(x–y)= V(f(x–y)) = V(f(x)–f(y)) ≤ V(f(x)) ∨ V(f(y)) = A(x) ∨A(y), which implies that A(x–y) ≤ A(x) ∨ A(y), for all x and y in F. And, A(xy^{-1}) = V(f(xy^{-1})) = V(f(x)f(y^{-1})) = V(f(x)(f(y) $)^{-1}$) ≤ V(f(x)) ∨ V(f(y)) = A(x) ∨ A(y), which implies that A(xy^{-1}) ≤ A(x) ∨ A(y), for all x and y ≠ 0 in F. Hence A is an anti L-fuzzy subfield of a field F.

In the following Theorem ◦ is the composition operation of functions :

2.3 Theorem: Let A be an anti L-fuzzy subfield of a field H and f is an isomorphism from a field F onto H. Then $A \circ f$ is an anti L-fuzzy subfield of F.

Proof: Let x and y in F and A be an anti L-fuzzy subfield of a field H. Then we have $(A\circ f)(x-y) = A(f(x-y)) = A(f(x)+f(-y)) = A(f(x)-f(y)) \leq A(f(x)) \vee A(f(y)) \leq (A\circ f)(x) \vee (A\circ f)(y)$, which implies that $(A\circ f)(x-y) \leq (A\circ f)(x) \vee (A\circ f)(y)$, for all x and y in F. And, $(A\circ f)(xy^{-1}) = A(f(xy^{-1})) = A(f(x)f(y^{-1}))$ $= A(f(x)(f(y))^{-1}) \leq A(f(x)) \vee A(f(y)) \leq (A\circ f)(x) \vee (A\circ f)(y)$, which implies that $(A\circ f)(xy^{-1}) \leq (A\circ f)(x) \vee (A\circ f)(y)$, for all x and $y \neq 0$ in F. Therefore $(A\circ f)$ is an anti L-fuzzy subfield of a field F.

2.4 Theorem: If A is an anti L-fuzzy subfield of a field (F, +, .), then the pseudo anti L-fuzzy coset $(aA)^p$ is an anti L-fuzzy subfield of a field F, for every $a \in F$ and p in P.

Proof: Let A be an anti L-fuzzy subfield of a field (F, +, .). For every x and y in F, we have $((aA)^p)(x-y) = p(a)A(x-y) \leq p(a)\{A(x) \vee A(y)\} = p(a)A(x) \vee p(a)A(y) = ((aA)^p)(x) \vee ((aA)^p)(y)$. Therefore, $((aA)^p)(x-y) \leq ((aA)^p)(x) \vee ((aA)^p)(y)$, for all x and y in F. And for every x and $y \neq 0$ in F, $((aA)^p)(xy^{-1}) = p(a)A(xy^{-1}) \leq p(a)\{A(x)\vee A(y)\} = p(a)A(x) \vee p(a)A(y) = ((aA)^p)(x)\vee((aA)^p)(y)$. Therefore, $((aA)^p)(xy^{-1}) \leq ((aA)^p)(x) \vee ((aA)^p)(y)$, for all x and $y \neq 0$ in F. Hence $(aA)^p$ is an anti L-fuzzy subfield of a field F.

2.5 Theorem: Let A be an anti L-fuzzy subfield of a field (F, +, .), then the anti L-fuzzy (0, 1)-coset ${}_0A_1$ is an anti L-fuzzy subfield of a field F, where 0 and 1 are identity elements of F.

Proof: Let A be an anti L-fuzzy subfield of a field (F, +, .). For every x and y in F, we have, $(0+A)(x-y) = A(0+x-y) = A(x-y) \leq A(x) \vee A(y)$. Therefore $(0+A)(x-y) \leq A(x) \vee A(y)$, for all x and y in F. And for x and $y \neq 0$ in F, we have $(1A)(xy^{-1}) = A(1.xy^{-1}) = A(xy^{-1}) \leq A(x) \vee A(y)$. Therefore $(1A)(xy^{-1}) \leq A(x) \vee A(y)$, for all x and $y \neq 0$ in F. Hence the anti L-fuzzy (0, 1)-coset ${}_0A_1$ is an anti L-fuzzy subfield of a field F.

2.6 Theorem: Let A be an anti L-fuzzy subfield of a field (F, +, .). Then for α in L such that $\alpha \geq A(0)$, $\alpha \geq A(1)$, A_α is a subfield of F, where 0 and 1 are identity elements of F.

Proof: For all x and y in $A_{\alpha,}$ we have, $A(x) \leq \alpha$ and $A(y) \leq \alpha$. Now, $A(x–y) \leq A(x) \vee A(y) \leq \alpha \vee \alpha = \alpha$, which implies that, $A(x–y) \leq \alpha$. And also, $A(xy^{-1}) \leq A(x) \vee A(y) \leq \alpha \vee \alpha = \alpha$, which implies that, $A(xy^{-1}) \leq \alpha$. Therefore, $A(x–y) \leq \alpha$, $A(xy^{-1}) \leq \alpha$, we get x–y, xy^{-1} in A_α. Hence A_α is a subfield of F.

2.1 Definition: Let A be an anti L-fuzzy subfield of a field (F, +, .). The lower level subset A_α, for α in L such that $\alpha \geq A(0)$, $\alpha \geq A(1)$, is called lower level subfield of A.

2.7 Theorem: Let A be an anti L-fuzzy subfield of a field (F, +, .). Then two lower level subfields $A_{\alpha 1}$ and $A_{\alpha 2}$, α_1 and α_2 in L and $\alpha_1 \geq A(0)$, $\alpha_2 \geq A(0)$, $\alpha_1 \geq \nu_A(1)$, $\alpha_2 \geq \nu_A(1)$ with $\alpha_2 > \alpha_1$ of A are equal if and only if there is no x in F such that $\alpha_1 < A(x) < \alpha_2$, where 0 and 1 are identity elements of F.

Proof: Assume that $A_{\alpha 1} = A_{\alpha 2}$. Suppose there exists $x \in F$ such that $\alpha_1 < A(x) < \alpha_2$. Then $A_{\alpha 1} \subseteq A_{\alpha 2}$, which implies that x belongs to $A_{\alpha 2}$, but not in $A_{\alpha 1}$. This is contradiction to $A_{\alpha 1} = A_{\alpha 2}$. Therefore there is no $x \in F$ such that $\alpha_1 < A(x) < \alpha_2$. Conversely, if there is no $x \in F$ such that $\alpha_1 < A(x) < \alpha_2$. Then $A_{\alpha 1} = A_{\alpha 2}$.

2.8 Theorem: Let (F, +, .) be a field and A be a fuzzy subset of F such that A_α be a lower level subfield of F. If α in L satisfying $\alpha \geq A(0)$, $\alpha \geq A(1)$, then A is an anti L-fuzzy subfield of F, where 0 and 1 are identity elements of F.

Proof: Let (F, +, .) be a field. For x and y in F. Let $A(x) = \alpha_1$ and $A(y) = \alpha_2$. **Case (i)**: If $\alpha_1 > \alpha_2$, then x and y in $A_{\alpha 1}$. As $A_{\alpha 1}$ is a lower level subfield of F, so x – y and xy^{-1} in $A_{\alpha 1}$. Now, $A(x–y) \leq \alpha_1 = \alpha_1 \vee \alpha_2 = A(x) \vee A(y)$, which implies that $A(x–y) \leq A(x) \vee A(y)$, for all x and y in F. Now, $A(xy^{-1}) \leq \alpha_1 = \alpha_1 \vee \alpha_2 = A(x) \vee A(y)$, which implies that $A(xy^{-1}) \leq A(x) \vee A(y)$, for all x and $y \neq 0$ in F. **Case (ii)**: If $\alpha_1 < \alpha_2$, then x and y in $A_{\alpha 2}$. As $A_{\alpha 2}$ is a lower level subfield of F, so x–y and xy^{-1} in $A_{\alpha 2}$. Now, $A(x–y) \leq \alpha_2 = \alpha_1 \vee \alpha_2 = A(x) \vee A(y)$, which implies that $A(x–y) \leq A(x) \vee A(y)$, for all x and y in F. Now, $A(xy^{-1}) \leq \alpha_2 = \alpha_1 \vee \alpha_2 = A(x) \vee A(y)$, which implies that $A(xy^{-1}) \leq A(x) \vee A(y)$, for all x and $y \neq 0$ in F. In all the cases, A is an anti L-fuzzy subfield of a field F.

2.9 Theorem: Let A be an anti L-fuzzy subfield of a field (F, +, .). If any two lower level subfields of A belongs to F, then their intersection is also lower level subfield of A in F.

Proof: For α_1, α_2 in L, $\alpha_1 \geq A(0)$ and $\alpha_2 \geq A(0)$, $\alpha_1 \geq A(1)$ and $\alpha_2 \geq A(1)$, where 0 and 1 are identity elements of F. **Case (i):** If $\alpha_1 > A(x) > \alpha_2$, then $A_{\alpha 2} \subseteq A_{\alpha 1}$. Therefore, $A_{\alpha 1} \cap A_{\alpha 2} = A_{\alpha 2}$ but $A_{\alpha 2}$ is a lower level subfield of A. **Case (ii):** If $\alpha_1 < A(x) < \alpha_2$, then $A_{\alpha 1} \subseteq A_{\alpha 2}$. Therefore, $A_{\alpha 1} \cap A_{\alpha 2} = A_{\alpha 1}$, but $A_{\alpha 1}$ is a lower level subfield of A. **Case (iii):** If $\alpha_1 = \alpha_2$, then $A_{\alpha 1} = A_{\alpha 2}$. In all cases, intersection of any two lower level subfields is a lower level subfield of A.

2.10 Theorem: Let A be an anti L-fuzzy subfield of a field (F, +, .). If α_i in L, $\alpha_i \geq A(0)$, $\alpha_i \geq A(1)$ and $A_{\alpha i}$, i in I, is a collection of lower level subfields of A, then their intersection is also a lower level subfield of A.

Proof: It is trivial.

2.11 Theorem: Let A be an anti L-fuzzy subfield of a field (F, +, .). If any two lower level subfields of A belongs to F, then their union is also lower level subfield of A in F.

Proof: For α_1, α_2 in L, $\alpha_1 \geq A(0)$ and $\alpha_2 \geq A(0)$, $\alpha_1 \geq A(1)$ and $\alpha_2 \geq A(1)$, where 0 and 1 are identity elements of F. **Case (i):** If $\alpha_1 > A(x) > \alpha_2$, then $A_{\alpha 2} \subseteq A_{\alpha 1}$. Therefore, $A_{\alpha 1} \cup A_{\alpha 2} = A_{\alpha 1}$, but $A_{\alpha 1}$ is a lower level subfield of A. **Case (ii):** If $\alpha_1 < A(x) < \alpha_2$, then $A_{\alpha 1} \subseteq A_{\alpha 2}$. Therefore, $A_{\alpha 1} \cup A_{\alpha 2} = A_{\alpha 2}$, but $A_{\alpha 2}$ is a lower level subfield of A. **Case (iii):** If $\alpha_1 = \alpha_2$, then $A_{\alpha 1} = A_{\alpha 2}$. In all cases, union of any two lower level subfields is a lower level subfield of A.

2.12 Theorem: Let A be an anti L-fuzzy subfield of a field (F, +, .). If α_i in L, $\alpha_i \geq A(0)$, $\alpha_i \geq A(1)$ and $A_{\alpha i}$, i in I, is a collection of lower level subfields of A, then their union is also a lower level subfield of A.

Proof: It is trivial.

2.13 Theorem: Any two different anti L-fuzzy subfields of a field may have identical family of lower level subfields.

Proof: We consider the following example: Consider the field F = Z_5 = { 0, 1, 2, 3, 4 } with addition modulo 5 and multiplication modulo 5 operations. Define fuzzy subsets A and B of F by A = { ⟨0, 0.1⟩, ⟨1, 0.4⟩, ⟨2, 0.4⟩, ⟨3, 0.4⟩, ⟨4, 0.4⟩ } and B = {⟨0, 0.2⟩, ⟨1, 0.3⟩, ⟨2, 0.3⟩, ⟨3, 0.3⟩, ⟨4, 0.3⟩ }. Clearly A and B are two different anti L-fuzzy subfields of F. And, Im A = {0.1, 0.4}, then the lower level subfields of A are $A_{0.1}$ = {0}, $A_{0.4}$ = { 0, 1, 2, 3, 4 }= F. And, Im B = {0.2, 0.3}, then the lower level subfields of B are $B_{0.2}$= {0}, $B_{0.3}$ = { 0, 1, 2, 3, 4 } = F. Thus the two anti L-fuzzy subfields A and B have the same family of lower level subfields.

2.14 Theorem: Let (F, +, .) be a finite field and A be an anti L-fuzzy subfield of F. If α, β are elements of the image set of A such that $A_\alpha = A_\beta$, then α = β.

Proof: It is trivial.

2.15 Theorem: Let (F, +, •) and (F^{l}, +, •) be any two fields. If f : F→ F^{l} is a homomorphism, then the homomorphic image of a lower level subfield of an anti L-fuzzy subfield of F is a lower level subfield of an anti L-fuzzy subfield of F^{l}.

Proof: Let (F, +, •) and (F^{l}, +, •) be any two fields and f : F→ F^{l} be a homomorphism. That is, f(x+y) = f(x)+f(y), for all x and y in F and f(xy) = f(x)f(y), for all x and y in F. Let V = f(A), where A is an anti L-fuzzy subfield of F. Clearly V is an anti L-fuzzy subfield of F^{l}. If x and y in F, then f(x) and f(y) in F^{l}. Let A_α be a lower level subfield of A. Suppose x, y and x−y, xy^{-1} in A_α. That is, $A(x) \leq \alpha$ and $A(y) \leq \alpha$, $A(x-y) \leq \alpha$, $A(xy^{-1}) \leq \alpha$. We have to prove that $f(A_\alpha)$ is a lower level subfield of V. Now, $V(f(x)) \leq A(x) \leq \alpha$, implies that $V(f(x)) \leq \alpha$; $V(f(y)) \leq A(y) \leq \alpha$, implies that $V(f(y)) \leq \alpha$, $V(f(x)-f(y)) = V(f(x)+f(-y)) = V(f(x-y)) \leq A(x-y) \leq \alpha$, which implies that $V(f(x)-f(y)) \leq \alpha$, for all f(x) and f(y) in F^{l}. And $V(f(x)(f(y))^{-1}) = V(f(x)f(y^{-1})) = V(f(xy^{-1})) \leq A(xy^{-1}) \leq \alpha$, which implies that $V(f(x)(f(y))^{-1}) \leq \alpha$, for f(x) and $f(y) \neq 0^{l}$ in F^{l}. Therefore, $V(f(x)-f(y)) \leq \alpha$, $V(f(x)(f(y))^{-1}) \leq \alpha$. Hence $f(A_\alpha)$ is a lower level subfield of an anti L-fuzzy subfield V of a field F^{l}.

2.16 Theorem: Let (F, +, •) and (F^{l}, +, •) be any two fields. If f : F→ F^{l} is a homomorphism, then the homomorphic pre-image of a lower level subfield of an anti L-fuzzy subfield of F^{l} is a lower level subfield of an anti L-fuzzy subfield of F.

Proof: Let (F, +, •) and (F^{l}, +, •) be any two fields and f : F→F^{l} be a homomorphism. That is, f(x+y) = f(x)+f(y), for all x and y in F and f(xy) = f(x)f(y), for all x and y in F. Let V = f(A), where V is an anti L-fuzzy subfield of F^{l}. Clearly A is an anti L-fuzzy subfield of F. Let x and y in F. Let $f(A_\alpha)$ be a lower level subfield of V. Suppose f(x), f(y) and f(x)−f(y), $f(x)(f(y))^{-1}$ in $f(A_\alpha)$. That is, $V(f(x)) \leq \alpha$ and $V(f(y)) \leq \alpha$; $V(f(x)-f(y)) \leq \alpha$,

$V(f(x)(f(y))^{-1}) \leq \alpha$. We have to prove that A_α is a lower level subfield of A. Now, $A(x) = V(f(x)) \leq \alpha$, implies that $A(x) \leq \alpha$; $A(y) = V(f(y)) \leq \alpha$, implies that $A(y) \leq \alpha$, we have $A(x-y) = V(f(x-y)) = V(f(x)+f(-y)) = V(f(x)-f(y)) \leq \alpha$, which implies that $A(x-y) \leq \alpha$, for all x and y in F. And $A(xy^{-1}) = V(f(xy^{-1})) = V(f(x)f(y^{-1})) = V(f(x)(f(y))^{-1}) \leq \alpha$, which implies that $A(xy^{-1}) \leq \alpha$, for all x and $y \neq 0$ in F. Therefore, $A(x-y) \leq \alpha$, $A(xy^{-1}) \leq \alpha$. Hence A_α is a lower level subfield of an anti L-fuzzy subfield A of F.

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