Love Waves Propagation in a Transverse-Isotropic Fluid Saturated Porous Layer Lying Over a Self-Reinforced Half Space

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Abstract:- **This paper reveals propagation of Love waves in a fluid saturated anisotropic porous layer lying over a self- reinforced half space. Transverse isotropic fluid flow resistivity is taken into consideration for porous layer. The equations of motion have been formulated separately for different media under suitable boundary conditions at the interface of porous layer and self-reinforced half-space. We obtain the dispersion relation for assuming model and the result shows there is attenuation in propagation of Love waves which arises due to the resistivity, porosity and reinforced parameters. The effect of porosity, resistivity and reinforced parameter in phase velocity is studied and shown graphically. Some particular cases are also derived and compared with results obtained earlier.**

Keywords: Love waves, Porous Layer, Transverse Isotropic, Self-reinforced, Attenuation, Phase velocity.

1. INTRODUCTION

Studies of propagation of elastic waves in layered media have long been a research subject because of its practical importance in geophysics, earthquake engineering and exploration of oil and underground water. A good amount of information about the propagation of seismic waves in layered media is available in the well known book of Ewing et al.(1957). Konczak Z.(1989) discussed the propagation of Love waves in a fluid saturated porous anisotropic layer and he showed that attenuation in the propagation of surface waves occurs due to porosity of the layer. The propagation of Love wave in layered anisotropic media has been discussed by Kuznetsov S V.(2006), Yongqiang Guo et al.(2008) and others. Recently the study of wave propagation in porous medium has gained prime interest.

The earth contains fluid saturated porous rocks on or below its surface in the form of sandstone and other sediments permeated by groundwater or oil. Water saturated ocean sediments may also be considered as porous material. Biot M.A.(1956a,b) developed the theory of plane wave propagation in fluid saturated porous media in his two classical papers. Later Deresiewicz H.(1961), studied the effect of boundaries on the propagation of waves in liquid filled porous solid and derived the dispersion equation for the Love waves in a porous solid. Practically, the saturated porous materials are anisotropic due to bed, compaction and the presence of aligned micro cracks. Anisotropy may

have significant effects on wave characteristic in layered media. Ghorai et al.(2010) studied the effect of gravity on propagation of Love wave in a porous layer under rigid boundary. Chattaraj et al(2013) discussed the dispersion of Love wave propagating in irregular anisotropic porous stratum under initial stress. Again Chattaraj et al(2013) shows the dispersion of torsional surface waves in an anisotropic layer over the porous half space under gravity. Pradhan et al (2002) discuss the dispersion of shear waves in a fluid saturated elastic plate, taking resistivity of fluid and obtained some useful result. Samal et al discussed surface wave propagation in fiber-reinforced anisotropic elastic layer between liquid saturated porous half-space ad uniform liquid layer. Wang et al. (1998) studied the propagation of Love wave in a transversely isotropic fluid saturated porous layered half space. Later Ke et al. (2006) studied the same problem with a linearly varying elastic property.
Maradudinad et al. (1976) studied the attenuation of Rayleigh surface waves by surface roughness. Billy M.de et al. (1987) analyzed the attenuation measurements of an ultrasonic Rayleigh wave propagation along rough surfaces.

Fiber reinforced materials are widely used in engineering structures due to its high strength-to-weight and stiffness-to-weight ratio, which makes them ideally suited for use in weight sensitive structures. As the mechanical behavior of many fiber-reinforced composite materials is adequately modeled by the theory of linear elasticity for transversely isotropic materials in the preferred direction, coinciding with the fiber direction, the fiber reinforced composites are assumed to exhibit anisotropy. Also, some hard and soft rocks or minerals beneath the earth surface exhibit fiber reinforcement property. Pipkin and Rogers(1971) developed the plane strain theory of finite deformation for fiber reinforced materials. Spencer(1972) introduced the constitutive equation for a fiber reinforced linearly anisotropic elastic medium with respect to preferred direction. The concept of continuous reinforcement at every point of an elastic solid was given by Belfield et al. (1983). Hashing and Rosen(1964) gave the elastic moduli for fiber reinforced materials. Chattopadhyay and Michel(2006) studied a model for spherical second harmonic *SH* wave propagation in self-reinforced linearly elastic media. A study of torsional waves in a fiber reinforced composite material has been done by Bao et al. (2006). Chattopadhyay et al.(2009) discussed the propagation of Torsional waves in semi infinite fiber reinforced cylindrical rod. Chattopadhyay et al. (2012) studied the torsional surface waves in fiber reinforced layer lying over inhomogeneous half space with linearly varying rigidity and density and showed that the presence of reinforcement increases the phase velocity.

In the present paper, we discuss the propagation of Love waves in a fluid saturated anisotropic porous layer lying over a reinforced half space. The transverse- isotropic fluid flow resistance in porous layer is considered in the porous medium which first of all taken by Konczak Z.(1989). Further, very few have considered the resistivity in fluid saturated porous layer over a reinforced halfspace due to complexity in getting solution. The equations of motion have been formulated for both the media under suitable boundary conditions at the interface. Following Biot (1956a, b) we derive the dispersion equation for love waves. In this problem we observed that the attenuation of propagation of Love waves occurred due to the effect of anisotropy, porosity and resistivity of the medium. Love waves for a particular model has been discussed and shown graphically. It is observed that the phase velocity gradually decreases with increase in wave number for a fixed value of attenuation parameters. The attenuation also decreases with the increase of frequency.

2. FORMULATION

We consider a fluid saturated anisotropic porous layer of thickness 'h' lying over a self-reinforced half-space. The free surface of the porous layer is assumed to be traction free. A coordinate system (x, y, z) has been taken with origin at the interface and positive axial (z) direction is directed downward. The wave is assumed to propagate along x −direction (Fig-1).

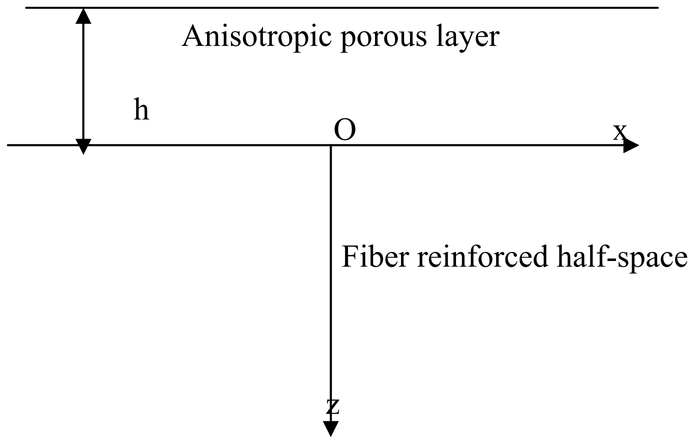

Fig.1: Geometry of the problem

3. Dynamics of Porous layer

The equations of motion for the fluid-saturated porous layer without body forces are (1962)

$$\sigma_{ij,j} = \rho_{11}\ddot{u}_i + \rho_{12}\ddot{U}_i - b_{ij}(\dot{U}_j - \dot{u}_j) \quad (1)$$

$$\sigma_{,i} = \rho_{12}\ddot{u}_i + \rho_{22}\ddot{U}_i - b_{ij}(\dot{U}_j - \dot{u}_j) \quad (2)$$

Where σ_{ij} are stress components of solid skeleton, $\sigma = -fp$ is the reduced pressure of the fluid(p is the pressure of the fluid and f is the porosity factor of the medium), u_i and U_j are displacement components of solid and fluid in porous media, $\rho_{11}, \rho_{12}, and\ \rho_{22}$ are dynamics coefficients arise due to the inertia effect of fluid flow and are related to the mass density of solid ρ_s and fluid ρ_f .

The components of the flow resistance tensor **b** , for the transverse-isotropy are

$$[b_{ij}] = \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{11} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \quad (3)$$

Using the conventional Love waves conditions

$u_1 \equiv 0,\ \ u_2 = u_2(x, z, t),\ \ \ u_3 \equiv 0$

$U_1 \equiv 0,\ \ U_2 = U_2(x, z, t),\ \ \ U_3 \equiv 0$

Equation (1) and (2) reduces to

$$N\frac{\partial^2 u_2}{\partial x^2} + G\frac{\partial^2 u_2}{\partial z^2} = \frac{\partial^2}{\partial t^2}(\rho_{11}u_2 + \rho_{12}U_2) - b_{11}\frac{\partial}{\partial t}(U_2 - u_2) \quad (4)$$

$$0 = \frac{\partial^2}{\partial t^2}(\rho_{11}u_2 + \rho_{12}U_2) + b_{11}\frac{\partial}{\partial t}(U_2 - u_2) \qquad (5)$$

By eliminating U_2 from these equations, we obtain

$$N\frac{\partial^2 u_2}{\partial x^2} + G\frac{\partial^2 u_2}{\partial z^2} - \rho_{11}\frac{\partial^2 u_2}{\partial t^2} - b_{11}\frac{\partial u_2}{\partial t} - \frac{\left(\rho_{12}\frac{\partial^2 u_2}{\partial t^2} - b_{11}\frac{\partial u_2}{\partial t}\right)^2}{\rho_{22}\frac{\partial^2 u_2}{\partial t^2} + b_{11}\frac{\partial u_2}{\partial t}} = 0 \qquad (6)$$

For the waves changing harmonically, we take $u_2 = \emptyset(z)e^{i(kx-\omega t)}$ (7)

The equation (6) takes the form $\left(\frac{\partial^2}{\partial z^2} + \aleph_1^2\right)\varphi = 0$ (8)

Where $\aleph_1^2$ is a complex quantity and is defined by

$$\aleph_1 = k\sqrt{\frac{c^2}{c_G^2}F + i\frac{c^2}{c_G^2}R - \frac{N}{G}} = k\xi \text{ , and } \xi = \sqrt{\frac{c^2}{c_G^2}F + i\frac{c^2}{c_G^2}R - \frac{N}{G}} \qquad (9)$$

$$F = F(\omega) = \frac{\omega^2}{c_G^2}\left[\frac{(c_1\gamma_{22}\Omega^2+1)\gamma_{22}}{(1+\Omega^2\gamma_{22}{}^2)c_1}\right],\ R = R(\omega) = \frac{\omega^2\Omega}{c_G^2}\left[\frac{(-c_1+\gamma_{22})\gamma_{22}}{(1+\Omega^2\gamma_{22})c_1}\right],\ c_1 = \gamma_{11}\gamma_{22} - \gamma_{12}^2,$$

$$\gamma_{kl} = \rho_{kl}, k, l = 1,2.\ c_G^2 = G/\bar{\rho},\ \bar{\rho} = \rho_{11} - \left(\frac{\rho_{12}^2}{\rho_{22}}\right),\quad \Omega = \frac{\rho\omega}{b_{11}},\ d = \frac{c_1}{\gamma_{22}}$$

This can be drawn that

as $f \to 0$ $i.e.\ d \to 1$, *the porous layer becomes non − porous solid*

as $f \to 1$ $i.e.\ d \to 0$, *the porous layer becomes fluid*

as $0 < d < 1$, *the layer is poro − elastic*

The solution of (8) is $u_2 = (A_1 e^{ik\xi} + B_1 e^{-ik\xi})e^{i(kx-\omega t)}$ (10)

4. Dynamic of Fiber reinforced material:

According to Spencer A.J.M. (1972), the Cauchy stress tensor for self reinforced elastic medium is given by

$$\begin{aligned}\tau_{ij}^* = \lambda\sum_{k=1}^{3} e_{kk}^*\delta_{ij} + 2\mu_T e_{ij}^* + \alpha\left(\sum_{k=1}^{3}\sum_{m=1}^{3} a_k a_m e_{km}^*\delta_{ij} + \sum_{k=1}^{3} a_i a_j e_{kk}^*\right) \\ + 2(\mu_L - \mu_T)\left(a_i\sum_{k=1}^{3} a_k e_{kj}^* + a_j\sum_{k=1}^{3} a_k e_{ki}^*\right) + \beta\sum_{k=1}^{3}\sum_{m=1}^{3} a_k a_m e_{km}^* a_i a_j\end{aligned} \qquad (11)$$

where $e_{ij}^* = (u_{i,j}^* + u_{j,i}^*)/2$, u_j^* is the components of displacement, $a = (a_i)^T$ is the preferred fiber direction

($i, j = 1,2,3$ which corresponds to x, y, z). The comma before an index represents (partial) space differentiation. Here τ_{ij}^{*} are the components of stress, e_{ij}^{*} components of infinitesimal strain, a_i the components of a and all refer to rectangular Cartesian coordinates. The coefficients $\lambda, \mu_T, \mu_L, \alpha, \beta$ are elastic constant with the dimension of stress. μ_T and μ_L are the transverse shear and longitudinal shear modulus in the preferred direction respectively. α and β are the specific stress components to take into account different layers for concrete part of the composite material.

The equation of motion without body force are given by

$$P_1 \frac{\partial^2 v}{\partial x^2} + 2iR_1 \frac{\partial^2 v}{\partial x \partial z} + Q_1 \frac{\partial^2 v}{\partial z^2} = \frac{\rho'}{\mu_T} \frac{\partial^2 v}{\partial t^2} \tag{12}$$

Where $P_1 = 1 + \left(\frac{\mu_L}{\mu_T} - 1\right) a_1^2$, $R_1 = \left(\frac{\mu_L}{\mu_T} - 1\right) a_1 a_3$, $Q_1 = 1 + (\frac{\mu_L}{\mu_T} - 1) a_3^2$

The solution of (12) is $v = A_2\, e^{-km_1 z}\, e^{i(kx - \omega t)}$, (13)

where $m_1 = \frac{i}{Q_1}(R_1 + \sqrt{R_1^2 + Q_1\left(P_1 - \frac{c^2}{c_2^2}\right)}\ )$, $c_2^2 = \frac{\mu_T}{\rho'}$, ρ' is the density of self- reinforced medium

5. Boundary Conditions are

$\sigma_{23} = 0 \quad at\ z = -h$

$\sigma_{23} = \tau_{23} \quad at\ z = 0$ (14)

$u_2 = v \quad at\ z = 0$

Using these boundary conditions, finally we obtain the dispersion equation

$\tan(k\xi h) = \frac{m_1 D}{G\xi}$, where $D = \mu_L + (\mu_L - \mu_T) a_3^2$ (15)

Now taking $k = k_1 + i\alpha$, where α is the attenuation coefficient, we obtain the real part of the dispersion equation(Using Mathematica software) in the form

$$k_1 h \left[\left(\frac{c^2}{c_G^2}F - \frac{N}{G}\right)^2 + \frac{c^2}{c_G^2}R^2\right]^{\frac{1}{4}} \cos\left[\frac{1}{2} Arg\left(\frac{c^2}{c_G^2}F - \frac{N}{G} + i\,\frac{c^2}{c_G^2}R\right)\right] - \alpha h \left[\left(\frac{c^2}{c_G^2}F - \frac{N}{G}\right)^2 + \frac{c^2}{c_G^2}R^2\right]^{\frac{1}{4}} \sin\left[\frac{1}{2} Arg\left(\frac{c^2}{c_G^2}F - \frac{N}{G} + i\,\frac{c^2}{c_G^2}R\right)\right] = \frac{1}{2}\left[Arg\left(1 - \frac{D1}{\sqrt{\frac{c^2}{c_G^2}F - \frac{N}{G} + i\frac{c^2}{c_G^2}R}\ G}\right) - Arg\left(1 + \frac{D1}{\sqrt{\frac{c^2}{c_G^2}F - \frac{N}{G} + i\frac{c^2}{c_G^2}R}\ G}\right)\right] \tag{16}$$

Where $D_1 = m_1 D$.

6. Particular Cases:

Case-1: In case the resistivity of fluid $b_{11} = 0$ i.e. F=1 and R=0 , then the dispersion equation takes the form

$$\tan\left(k_1 h\sqrt{\frac{c^2}{c_G^2}-\frac{N}{G}}\right) = \frac{(\mu_L+(\mu_L-\mu_T)a_3^2)\left(R_1+\sqrt{R_1^2+Q_1\left(P_1-\frac{c^2}{c_2^2}\right)}\right)}{Q_1 G\sqrt{\frac{c^2}{c_G^2}-\frac{N}{G}}} \tag{17}$$

The term containing the attenuation term in equation (16) becomes zero and this is the dispersion equation when love wave propagates through a fluid saturated porous layer without fluid resistance rest upon a reinforced half-space. When reinforcement parameter is zero i.e. $\mu_L = \mu_T = \mu$, $R_1 = 0, P_1 = Q_1 = 1$, then Half space becomes an isotropic medium. The dispersion equation becomes

$$\tan\left(k_1 h\sqrt{\frac{c^2}{c_G^2}-\frac{N}{G}}\right) = \frac{\mu\sqrt{1-\frac{c^2}{c_2^2}}}{G\sqrt{\frac{c^2}{c_G^2}-\frac{N}{G}}} \tag{18}$$

This is similar to the dispersion equation obtained for the prorogation of Love wave in a fluid saturated porous layer lying over an isotropic half-space.

Case-2: If the upper layer becomes isotropic i.e. N=G=μ and $c_1 = c_G = \sqrt{\mu/\rho}$, then dispersion equation takes the standard form as

$$\tan\left(k_1 h\sqrt{\frac{c^2}{c_1^2}-1}\right) = \frac{\sqrt{1-\frac{c^2}{c_2^2}}}{\sqrt{\frac{c^2}{c_1^2}-1}} \tag{19}$$

This is the dispersion equation of Love waves in a isotropic layer lying over a isotropic half-space.

7. Numerical Results

To see the effect of porosity, resistivity of fluid and reinforcement parameters we have taken a particular model. The model consists of a fiber reinforced half space lying below a kerosene sandstone layer.
For this model we calculate the phase velocity ratio i.e. c/c_G which is a function of the real part of non-dimensional wave number h .
The material constants for a reinforced medium has been considered as

$$\mu_L = 5.66 \times 10^9 N/m^2, \mu_T = 2.46 \times 10^9 N/m^2, \rho' = 7800\ kg/m^2$$

Moreover, we take $\varphi = \pi/3$ so that $a_1 = \sin\left(\frac{\pi}{3}\right), a_2 = \cos\left(\frac{\pi}{3}\right)$ and other data are specified on the body of the graphs.
The elastic parameters for kerosene saturated porous sandstone are taken as Yew and Jogi (1976)

$$\frac{N}{G} = 2, \quad \rho_{11} = 1.926137 \times 10^3\ kg/m^2, \rho_{12} = -0.002137 \times 10^3 kg/m^2,$$
$$\rho_{13} = 0.215337 \times 10^3\ kg/m^2$$

Figure 2 shows the attenuation of Love waves decreases as the frequency increases. As 'd ' increases, the attenuation also increases. The difference is quite distinct for smallest frequency.

Figure 3 shows the variation of phase velocity with respect to wave number of different frequencies with attenuation $\alpha h = 0.$ It is observed that as the wave number increases the magnitude phase velocity increases. After a certain wave number the phase velocity becomes constant.

Figure 4 also depicts the variation of phase velocity with respect to wave number for different frequencies with attenuation $h = 0.2$. It is observed that the magnitude of phase velocity is more in comparison to figure 3. But as the wave number increases the phase velocity decreases like as figure 1.

Figure 5 shows a variation of phase velocity with respect to porosity d for attenuation factor $\alpha h = 1,2,3$. But it is interesting to note that the phase velocity of Love waves remains same when d=1 i.e. the porous layer becomes solid.

Figure 6 presents the phase velocity variation with respect to wave number for the elastic constant ratio $\frac{N}{G} = 2,3,4$ of porous medium. As $\frac{N}{G}$ increases the phase velocity increases. But the increase in wave numbers also increases phase velocity.

8. Conclusions

There is a significant effect of attenuation, porosity and anisotropy simultaneously in the propagation of Love waves in a transverse isotropic fluid saturated porous layer rest upon a self-reinforced half space. The attenuation is more for low frequency and less for high frequency. The porosity factor d also affects the attenuation. As d increases the magnitude of attenuation also increases. The increase in wave number decreases the phase velocity. It is also seen that when the layer becomes a perfect solid the phase velocity remains same for all attenuation factors. The phase velocity increases as the elastic constant ratio of porous medium increases.

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