

# Longitudinal Parameter Estimation of Aerospace Vehicle with Different Noise Levels

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**Abstract**— This paper aims to estimate the longitudinal aerodynamic parameters of the aerospace vehicle from the available measurements with different noise levels. In this current work Filter Error Method (FEM) which accounts for both process and measurement noise, is used to estimate the aerodynamic parameters from vehicle data. FEM algorithm is developed in MATLAB environment. With the developed algorithm studies are carried out at different instance of the vehicle and the estimated parameters are compared with wind tunnel predictions.

**Keywords**— Aerodynamic parameter; Filter Error Method; Gauss Newton Method; Kalman Filter.

## I. INTRODUCTION

Parameter estimation, is the subfield of system identification, which determines the best estimates of the parameters occurring in the model. Aerospace vehicle parameter estimation is the best example of system identification. An aircraft motion is described by equations of motion derived from the Newtonian mechanics, considering the vehicle as a rigid body. The aerodynamic forces and moments cannot be measured directly. However, aerodynamic modeling followed by parameter estimation helps to determine the aerodynamic parameters. Conventional methods of parameter estimation are (i) Equation Error Method[2] (ii) Output Error Methods (OEM)[3,4] and (iii) Filter Error Methods (FEM). Out of these methods Filter Error Method (FEM), is the most general stochastic approach for aircraft parameter estimation, which accounts for both process and measurement noise. In this current work Filter Error Method (FEM), is used to estimate the aerodynamic parameters. FEM algorithm is developed in MATLAB environment. State estimation is carried out with Kalman Filter and the parameters are updated using Gauss Newton Method. With the developed algorithm studies are carried out at different instance of the vehicle and the estimated parameters are compared with wind tunnel predictions.

## II. LONGITUDINAL DYNAMICS

Aerospace vehicle experiences aerodynamic forces and moments during their motion in atmosphere.

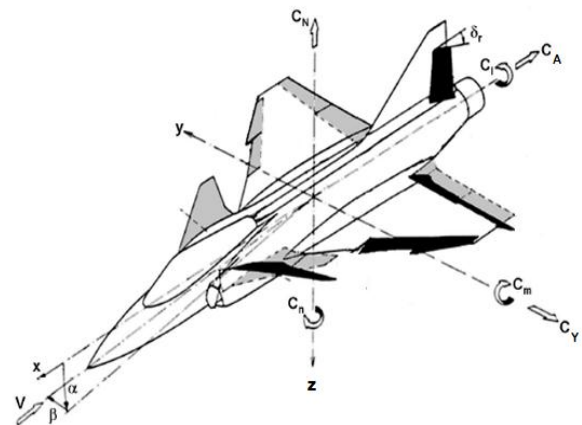


Fig. 1. Aerodynamic forces and moments

Fig.1 shows the different forces and moments acting on the vehicle along the three axis. Aerodynamic forces acting on the vehicle with respect to longitudinal plane are given from (1)-(2). [1]

$$F_x = -C_A \bar{q} S \quad (1)$$

$$F_z = -C_N \bar{q} S \quad (2)$$

Aerodynamic moment is given by (3).

$$M = C_m \bar{q} S c \quad (3)$$

Where  $\bar{q}$  is the dynamic pressure, S is the reference area, c is

the longitudinal reference length,  $C_A, C_N, C_m$  are the total axial force coefficient, total normal force coefficients and total pitching moment coefficients. The state equations are given from (4)-(7)

$$\dot{V} = \frac{u\dot{u} + v\dot{v} + w\dot{w}}{\sqrt{u^2 + v^2 + w^2}} \quad (4)$$

$$\dot{\alpha} = \frac{u\dot{w} - w\dot{u}}{u^2 + w^2} \quad (5)$$

$$\dot{\theta} = \frac{1}{\cos \psi} (q \cos \phi - r \sin \phi) \quad (6)$$

$$\dot{q} = \frac{C_m \bar{q} S c + I_{xz} (r^2 - p^2) + r p (I_{zz} - I_{xx}) + I_{xy} (\dot{p} + q r) + I_{yz} (\dot{r} - p q)}{I_{yy}} \quad (7)$$

Where  $u, v, w$  are the translational velocity,  $p, q, r$  are the body rates,  $V$  is the vehicle velocity,  $\alpha$  is the angle of attack,  $I_{xx}, I_{yy}, I_{zz}$  are the moment of inertia and  $I_{xy}, I_{yz}, I_{zx}$  are the product of inertia,  $\theta, \phi, \psi$  are the euler angles. The aerodynamic coefficients are modelled in linear form as:

$$C_A = C_{A0} + C_{A\alpha} \alpha + C_{A\delta e1} \delta_{e1} + C_{A\delta e2} \delta_{e2} + C_{A\delta r1} \delta_{r1} + C_{A\delta r2} \delta_{r2} \quad (8)$$

$$C_N = C_{N0} + C_{N\alpha} \alpha + C_{N\delta e1} \delta_{e1} + C_{N\delta e2} \delta_{e2} + C_{N\delta r1} \delta_{r1} + C_{N\delta r2} \delta_{r2} \quad (9)$$

$$C_m = C_{m0} + C_{m\alpha} \alpha + C_{m\delta e1} \delta_{e1} + C_{m\delta e2} \delta_{e2} + C_{m\delta r1} \delta_{r1} + C_{m\delta r2} \delta_{r2} \quad (10)$$

where  $C_{A0}, C_{N0}, C_{m0}, C_{A\alpha}, C_{N\alpha}, C_{m\alpha}$  are the basic coefficients and  $C_{A\delta e1}, C_{A\delta e2}, C_{A\delta r1}, C_{A\delta r2},$

$C_{N\delta e1}, C_{N\delta e2}, C_{N\delta r1}, C_{N\delta r2}, C_{m\delta e1}, C_{m\delta e2}, C_{m\delta r1}, C_{m\delta r2}$  are the incremental coefficients due to control surface deflections. Thus the unknown parameters to be estimated are given as:

$$\theta^T = [C_{A0} C_{A\alpha} C_{A\delta e1} C_{A\delta e2} C_{A\delta r1} C_{A\delta r2} C_{N0} C_{N\alpha} C_{N\delta e1} C_{N\delta e2} C_{N\delta r1} C_{N\delta r2} C_{m0} C_{m\alpha} C_{m\delta e1} C_{m\delta e2} C_{m\delta r1} C_{m\delta r2}] \quad (11)$$

Observation equations are given by (12)-(18)

$$V_m = V \quad (12)$$

$$\alpha_m = \alpha \quad (13)$$

$$\theta_m = \theta \quad (14)$$

$$q_m = q \quad (15)$$

$$\dot{q}_m = \frac{C_m \bar{q} S c + I_{xz} (r^2 - p^2) + r p (I_{zz} - I_{xx}) + I_{xy} (\dot{p} + q r) + I_{yz} (\dot{r} - p q)}{I_{yy}} \quad (16)$$

$$a_{xm} = \frac{-C_A \bar{q} S}{m} \quad (17)$$

$$a_{zm} = \frac{-C_N \bar{q} S}{m} \quad (18)$$

Where,  $m$  is the mass of the vehicle,  $V_m, \alpha_m, \theta_m, q_m$  are the measured velocity, angle of attack, euler angle, and body rate,  $\dot{q}_m$  is the measured angular accelerations and  $a_{xm}, a_{zm}$  are the measured accelerations.

### III. EFFECT OF NOISE

The real aerospace vehicle measurements are affected by both measurement noise and process noise. To study the effect of the noise on the estimated co-efficients, noise is added in  $q, a_x, a_z$  by specifying the signal to noise ratio. The noise added is shown in table 1.

Table 1: Noise added in the variable

Variable	Noise added (signal to noise ratio)			
	Case1	Case 2	Case3	Case 4
$q$	-	40	50	80
$a_x$	-	40	50	80
$a_z$	-	40	50	80

Due to the presence of process noise, Equation Error Method, Output Error Method estimation techniques yield poor estimation results. Therefore, Filter Error Method [5] is used in this work which account for process and measurement noise. Since the system is stochastic in nature a steady state Kalman filter is used for obtaining the true state variables from the noisy measurements.

### IV. RESULTS AND DISCUSSION

Estimation is carried out for a period of 10s with 500 samples. Region considered for the estimation process is for the vehicle time 310-320s. The initial values of the estimates are taken from the wind tunnel data. The variation of angle of attack, dynamic pressure, Mach number, relative velocity and control surface deflections during this region are shown in Fig.3 and 4. The estimated longitudinal aerodynamic coefficients are shown in Fig 5,6 and 7.

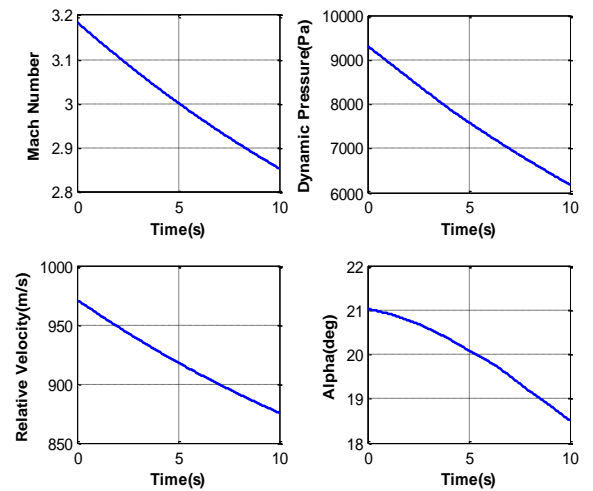


Fig. 3. Mach number, relative velocity, dynamic pressure and alpha during the estimation region.

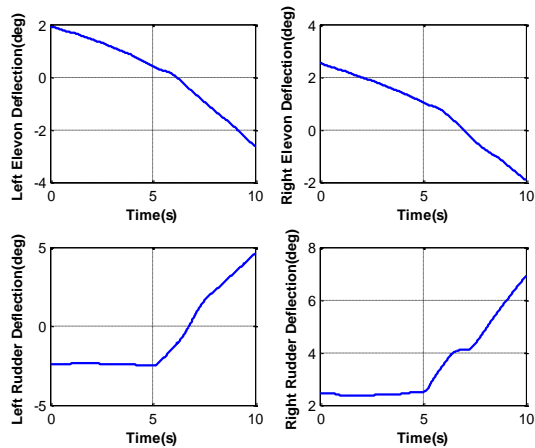
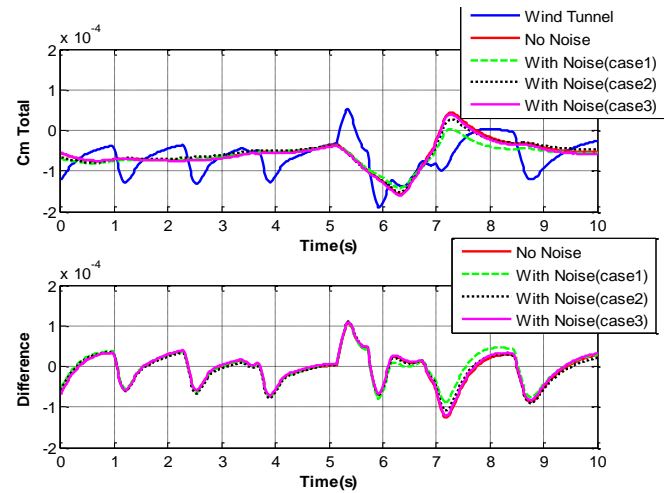
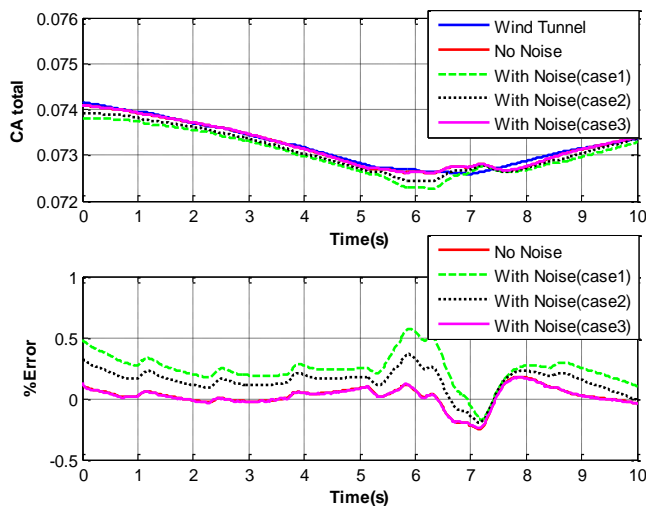
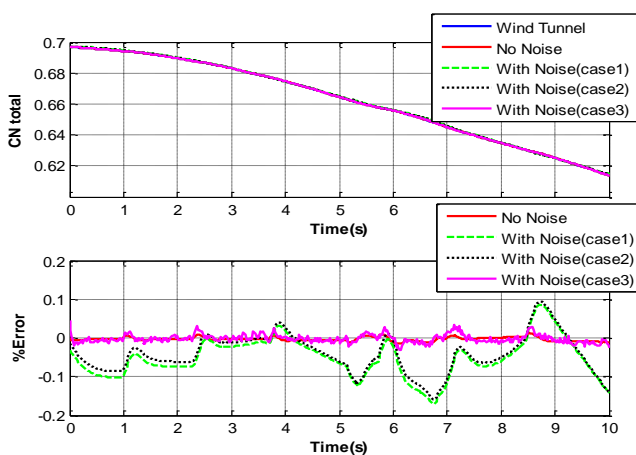


Fig.4.Control surface deflections during the estimation region

Fig.7.The estimated total aerodynamic moment coefficient  $C_m$ Fig. 5 The estimated total aerodynamic force coefficient  $C_A$ Fig. 6 The estimated total aerodynamic force coefficient  $C_N$ 

## CONCLUSION

Filter error algorithm is formulated and developed in MATLAB environment. With the developed algorithm longitudinal aerodynamic parameters are estimated with different noise levels. The estimated longitudinal aerodynamic coefficients are compared with wind tunnel prediction. Maximum of 0.55%, 0.13%, error, are observed in total aerodynamic force coefficients, 0.000012 difference are observed in total aerodynamic moment coefficient.

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