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Literature Review of Waiting Lines Theory and its Applications in Queuing Model

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***Abstract:-* The purpose of this paper is to know the importance of Waiting Lines Theory, how it comes in existence , the history behind the Waiting Lines Theory. The use of Probability Distributions. How to get optimum level in Queuing model. Basic features of the Waiting Lines System. Queuing characteristics and different Queuing models used in Waiting Lines System.**

Keywords-* *Arriving customers, Service discipline, Nature of customer, Service mechanism, Waiting lines models.

I. INTRODUCTION

Waiting Lines Theory (Queuing Theory) is the Mathematical approach of queue. It is basically considered as branch of Operations Research. The results of Queuing models are often used in the Business decisions. Queues are one of the unpleasant parts of everyday human's life. As we know that there is increase in demand of facilities from customer side and if the service facilities is not in a position to satisfy the customer in a specific time, customer requires too much time to get service from service mechanism, result in the formation of queue. Under such conditions there is increase in the cost of customers waiting time. Where as in some other cases if the service facilities stand in idle condition waiting for the customers and there is too less demand from customer side will increase the cost of service facilities. In both the cases we get imperfect matching between the customers waiting cost and cost of service facility. It is just because of one cannot predicts the inter-arrival time of customers and service time of server. To get optimum level we have to minimize the sum of cost of customers waiting time and cost of service facilities. The expected total cost (TC) is the sum of the expected waiting cost for the arrivals per period (WC) and the expected facility cost (FC) of the service personnel per period. This can be written as(m=mean): $TC_m = WC_m + FC_m$ The expected waiting cost per period (WC_m) is the product of unit waiting cost (C_w) for an arrival per period and the average number of units in the system $E(n)$ during the period. $WC_m = C_w E(n)$

II. BASIC FEATURES OF WAITING LINES SYSTEM

A. *Arrival of customers*

It is a process of arrival for customers into the Waiting Lines System. Classification of arrival of customers as:

1. Single line or multiple lines,
2. Finite or infinite
3. Single customer or customers comes in bulk,
4. Arriving customers are totally under control or partially or no control,
5. Deterministic or Probabilistic process,
6. Empirical or a Theoretical Probability Distribution,
7. Independent or conditionally dependent variables,
8. Some times arrivals of customers is stationary.

B. *Service Discipline*

It works on the rule by which customers are selected from the queue for service.

Rules are classified as:

i. First-In First-Out (FIFO)
ii. Last-In First Out (LIFO)
iii. Service For Random Order (SRO)
iv. Priority Service. (PS)

C. *Nature of Customer*

As usual it is depending on the nature of arriving customers whether he is willingly accepted a waiting line or refuses it. If the system is filled up to its capacity, then the arriving customer is naturally rejected. In some other cases if there is a rejection of the primary system, the customer accepted secondary system and 'queue up' in an informal waiting line to enter in to the system. We note that there are mainly four items which must be specified for any given Queuing System.

i. Balking: If the customer experiences that waiting time are very large as the queue is moving very slowly, the customers might balk and refuse to join the queue.

ii. Reneging: After joining the queue customer experience that it will take too much time to enter the system which is worthless then he customer reneges i.e. leaves the queue.

iii. Collusion: Several customers may cooperate and only one of them may stand in the queue to reduce the waiting time and buy the required service.

iv. Jockeying: If there are more number of queues, there is a way for customers to change the queue which gives fast service than the other. In this process the customer scans the lines for the purpose of changing it.

4. Service mechanism: The service mechanism is worked on the policy decided for the service facility for the customers who are serviced and leave the service system. Service mechanism follows single channel-single phase, single channel-multiphase, multichannel-single phase, and multichannel-multiphase.

III. HISTORY OF WAITING LINES THEORY

In Waiting Lines Theory a model is constructed and record the inter-arrival time of customers and time required for service mechanism to complete the service. In 1909, Agner Krarup Erlang, father of Waiting Lines Theory which is also called Queuing Theory had its beginning in the research on the Waiting Line Theory. The first developers of Queuing Theory as applicable to the telephone industry were Tore Olaus and Erlang. Erlang experimented with fluctuating demand in telephone, traffic, later he published a report addressing the delays in automatic dialing equipment and its cost. Further he was extended to more general problems and to business applications of the waiting lines. Engset's formulations were not known until later because of the delay in publishing them and Erlang's model was first used by traffic engineers to develop better systems. Engset's main work was not in Queuing Theory and traffic engineering and his contributions are not as well known. The Danish mathematician, Erlang developed models that accounted for callers that dropped due to frustration from waiting for an operator and those that were patient enough to wait for their call to be connected. Erlang (M/D/1) Queuing model in 1917 and (M/D/K) Queuing model in 1920.

Queuing Characteristics of Queuing System:

There are six items which must be specified for any given Queuing System.

1. Mean arrival time of customer, λ
2. Mean service time of server, μ
3. Customer's behavior in the system
4. Capacity of the system
5. Number of service counters
6. The service rate is faster than arrival rate.

Classification of Probabilistic Queuing Models

i. Poisson-Exponential, Single server-Infinite population model (M/M/1:∞/FCFS)

ii. Poisson-Exponential, Single server-Finite population model (M/M/1: N/FCFS)

iii. Poisson-Exponential, Multiple server-Infinite population models (M/M/S: ∞/FCFS)

iv. Poisson-Exponential, Multiple server-Finite population model (M/M/S:N/FCFS)

Other Queuing models are

(i) Poisson Arrivals and Erlang Service Distribution $(M/E_k/1)$

(ii) Poisson Arrivals and General Service Time Distribution $(M/G/1)$

(iii) Poisson Arrivals and Regular Service Time Distribution $(M/D/1)$

(iv) Constant Arrival Rate and Constant Service Rate $(D/D/1)$

Equations for Poisson-Exponential, Single server-Infinite population model:

According to the Poisson probability distribution, the probability that n customers will arrive in system during a given interval t is given by

$$P_n(t) \to P_n\ ,\ as\ \ t \to \infty$$

i. e. $P_n = \dfrac{e^{-\lambda}(\lambda)^n}{n!}\ ;n = 0,1,2,......$

These can be solved by the successive substitution technique to yield

1. Probability of having exactly n customers in the system $P_n = (\rho)^n P_0$, for any value of n

where $\rho = \dfrac{\lambda}{\mu}$ is utilization factor, $P_0 = \left(1 - \dfrac{\lambda}{\mu}\right)$ is the probability of no units in the system ,

2. Percentage of idle workstation $= (1-\rho)100\%$

3. Expected number of units in the system $L_s = \sum_{n=0}^{\infty} nP_n = \dfrac{\lambda}{\mu - \lambda}$

4. Expected number of units in the queue waiting for service $L_q = \sum_{n=1}^{\infty}(n-1)P_n = \dfrac{\lambda^2}{\mu(\mu-\lambda)}$

5. Expected waiting time a unit spends in the queue $W_q = \dfrac{L_q}{\lambda} = \dfrac{\rho}{\mu - \lambda}$

6. Expected waiting time in system (time in queue plus service time) the queue

$$W_s = W_q + \frac{1}{\mu} = \frac{1}{\mu - \lambda}$$

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