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Liquid Level Control of Coupled-Tank System Using Fuzzy-Pid Controller

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Abstract: **Liquid level control of coupled-tank is widely used in the chemical industry - the environment is often affected by noise. The article deals with the fuzzy-PID controller applied to the nonlinear dynamic model of the liquid level of the coupled-tank system, taking into account the effects of noise. Fuzzy-PID controller is designed based on PID initial parameters (determined based on the linear model) and fuzzy logic calculator for tunning PID parameters (suitable for nonlinear models and noise). The study results are caried out throught simulation model on Matlab using the coupled-tank nonlinear model with noise, applying the fuzzy-PID proposed controller, PID based on Ziegler Nichols.**

Keywords: PID, Fuzzy, Level control, Coupled-tank

I. INTRODUCTION

Liquid level control is always in great demand in the chemical industry, petrochemical refining, water treatment, power generation and construction material production. In these technological processes, the fluid is pumped, stored in a tank, and then pumped to another tank. Over the liquid is processed by chemical reaction and/or agitation in the tank, where the liquid level in the tank is controlled [1,2,14]. The coupled tank systems are commonly used in industries and the master of controlling the level of liquid in the tank, the flow control between the tanks is an important of all technological process control systems. Today's chemicals - the field has a tremendous impact on our economy [1,13,14]. Improving the quality of control and increasing the efficiency of the processing/production process is always required in this field, in order to reduce production/processing cost and lower production cost.

Nonlinearity, associated kinetic and uncertainty are the major challenges posed by controlling the liquid level in the coupled-tank. Most of the coupled-tank object in published studies use a linear mathematical model when designing the controller, such as PID controlller [10,12], fuzzy controlller [9], fuzzy-PID [8], LQR, state feedback controller, model reference adaptive control [4,13].

A number of recent studies have also addressed the nonlinear model of coupled tank using nonlinear control strategies such as sliding mode control [5,7,11], backstepping control [3], passivity based control [6], fuzzy logic controller [1], neuro-fuzzy-sliding mode controller [2]. It can be seen that the quality of the liquid level control system of the coupled-tank in the recently published works is good, but the implementation of these controllers is complex, the disturbance factor is not really considered.

This article proposes a control approach: combining between the fuzzy logic calculator and traditional PID controller for a nonlinear model with noise of the liquid level coupled-tank control system. Firstly, the article presents a nonlinear model of the liquid level coupled-tank control system. Then, the PID controller is designed based on a linear model of the coupled-tank according to the method Ziegler-Nichols; designing the fuzzy logic calculator for tunning PID parameters applied in the nonlinear model with noise of the coupled-tank system. Finally, the study results is caried out throught simulation model on Matlab, showing the efiectiveness of the proposed control strategy.

II. DYNAMIC MODEL OF COUPLED-TANK SYSTEM

This article deals with the coupled tanks with the two separate vertical tanks (see Fig. 1). Both tanks are interconnected by a flow channel where a rotary valve will be used to vary the sectional area of the channel by changing the discharge coefficient of the valve B. The liquid is fed into the first tank through the DC-motor controlled electric valve. Then the liquid flows to the second tank through the manual valve B, the liquid flows out of the tanks through the manual valve A or/and the manual valve C by adjusting the discharge coefficient of the valve A, C. The liquid level in the second tank is measured by the liquid level untrasonic sensor that converts the real physical level in the second tank l_2 [cm] to an electrical voltage signal y [V].

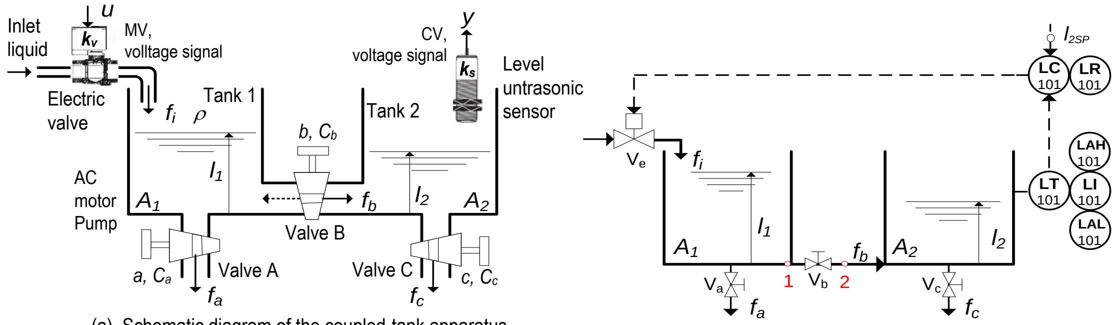
$$y = k_s l_2 \tag{1}$$

where k_s [V/cm] is gain of level untrasonic sensor.

The control objective is to control the height of the liquid level in the second tank by manipulating the flow rate of the liquid into the first tank by means of the electric valve voltage. Assume that the valve's output volume flow rate f_i [cm^3/s] is proportional to the manipulating voltage applied to electric valve u [V] as below equation:

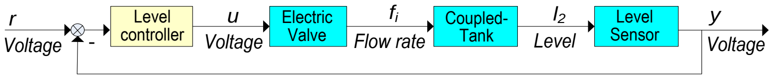
$$f_i = k_v u \tag{2}$$

where k_v is gain of the electric valve [cm^3/s/V].



(a). Schematic diagram of the coupled-tank apparatus

(b). P&ID of the coupled-tank liquid level control system



c). The block diagram of the coupled-tank control system

Fig. 1. Description of the coupled tank liquid level control system

The liquid used in the coupled tank is assumed to be steady, non-viscous, incompressible type of liquid. Applying Bernoulli's principle for the liquid at point 1 (before valve B) and point 2 (after valve B) with corresponding pressure p_1 and p_2 (Fig. 1b), we have 2 cases:

Case 1: when the liquid level in tank 1 is higher or equal the liquid level in tank 2, $l_1 \geq l_2$, the liquid flows from tank 1 into tank 2, we obtain the balance equation:

$$\frac{p_1}{\rho}=\frac{v_2^2}{2}+\frac{p_2}{\rho}\Leftrightarrow\frac{\rho g(l_1-l_2)}{\rho}=\frac{\left(f_b/bC_b\right)^2}{2}\Leftrightarrow f_b=bC_b\sqrt{2g}\sqrt{l_1-l_2}\tag{3a}$$

where $g=981$[cm^2/s] is acceleration of gravity; l_1 [cm] is level in the first tank; ρ [g/cm^3] is liquid density; f_b [cm^3/s] is volume flow rate through valve B, v_2 [cm/s] is liquid velocity at point 2; b [cm^2] is section area of valve B, C_b [%] is percentage of opening valve B

Case 2: when the height of the liquid level in tank 1 is less than in tank 2, such as $l_1<l_2$, the liquid flows from tank 2 into tank 1, we obtain the balance equation:

$$f_b=-bC_b\sqrt{2g}\sqrt{l_2-l_1}\tag{3b}$$

Combining the above equations, we have flow-rate equation through the valve B as follows

$$f_b=sign(l_1-l_2)bC_b\sqrt{2g}\sqrt{|l_1-l_2|}\tag{3c}$$

Similarly, we obtain the volume flow-rate equations through the valve A, C as:

$$f_a=aC_a\sqrt{2g}\sqrt{l_1}\tag{4}$$

$$f_c=cC_c\sqrt{2g}\sqrt{l_2}\tag{5}$$

where a, c [cm^2] are respectively section area of valve A, C; and C_a, C_c [%] are percentage of opening valve A, C respectively.

The coupled tank dynamics are based on the principle of mass balance which states that the rate of change of liquid mass in each tank equals the net of liquid mass flows into the tank. Here it assumes that the liquid density and cross area of tanks are constant

$$\frac{dw_1}{dt}=w_i-w_a-w_b\Leftrightarrow\rho A_1\frac{dl_1}{dt}=\rho f_i-\rho f_a-\rho f_b\Leftrightarrow A_1\frac{dl_1}{dt}=f_i-f_a-f_b\tag{6}$$

$$\frac{dw_2}{dt}=w_b-w_c\Leftrightarrow\rho A_2\frac{dl_2}{dt}=\rho f_b-\rho f_c\Leftrightarrow A_2\frac{dl_2}{dt}=f_b-f_c\tag{7}$$

where w_i, w_a, w_b, w_c are mass flow-rate; A_1, A_2 [cm^2] are respectively section area of the first tank and second tank.

Using above equations (2), (3c), (4), (5), thus we obtain

$$\frac{dl_1}{dt}=\frac{1}{A_1}(k_bu-aC_a\sqrt{2g}\sqrt{l_1}-sign(l_1-l_2)bC_b\sqrt{2g}\sqrt{|l_1-l_2|})\tag{8}$$

$$\frac{dl_2}{dt}=\frac{1}{A_2}(sign(l_1-l_2)bC_b\sqrt{2g}\sqrt{|l_1-l_2|}-cC_c\sqrt{2g}\sqrt{l_2})\tag{9}$$

The equations (8) and (9) represent a non-linear dynamic relationship of the liquid level (l_1 and l_2) in the two tanks with the ideal equations for the valves. In general applications, the square root law is only an approximation by solving directly the no-linear equations (8) & (9). But if the operating point is known and does not change quite often then it is convenient to linearize the system obtained by first principles around the desired operating point. This makes the process significantly simpler and the model works well in a region around the chosen operating point. This allows us to easily use linear control theory to design linear controller for the linear model of the coupled-tank, such as PID controller.

The linear model of the coupled-tank: At the desired operating point of the fluid level in the second tank L_{2s}, the control system is at steady state, so on:

$$\frac{dL_{1s}}{dt}=0=\frac{1}{A_1}(k_bU_s-aC_a\sqrt{2g}\sqrt{L_{1s}}-sign(L_{1s}-L_{2s})bC_b\sqrt{2g}\sqrt{|L_{1s}-L_{2s}|})\tag{10}$$

$$\frac{dL_{2s}}{dt}=0=\frac{1}{A_2}(sign(L_{1s}-L_{2s})bC_b\sqrt{2g}\sqrt{|L_{1s}-L_{2s}|})-cC_c\sqrt{2g}\sqrt{L_{2s}})\tag{11}$$

where L_{1s} is height at steady state, U_s is pump voltage at at steady state.

Considering a small incremental change in the control input, Δu in U_s, which subsequently cause an incremental change in height in the two tanks, Δl_1 in L_{1s} and Δl_2 in L_{2s}. Hence, equations (8) and (9), assuming that the fluid always flows from tank 1 to tank 2, can be re-written as:

$$\frac{d(\Delta l_1 + L_{1s})}{dt} = \frac{1}{A_1}[k_b(\Delta u + U_s) - aC_a\sqrt{2g}\sqrt{\Delta l_1 + L_{1s}} - bC_b\sqrt{2g}\sqrt{\Delta l_1 - \Delta l_2 + L_{1s} - L_{2s}}] \qquad (12)$$

$$\frac{d(\Delta l_2 + L_{2s})}{dt} = \frac{1}{A_2}(bC_b\sqrt{2g}\sqrt{\Delta l_1 - \Delta l_2 + L_{1s} - L_{2s}} - cC_c\sqrt{2g}\sqrt{\Delta l_2 + L_{2s}}) \qquad (13)$$

Following Newton's binomial generalized theorem, if x<<1 then we can approximate:

$$(1+x)^\alpha \approx 1 + \alpha x \qquad (14)$$

Applying the above approximation (14), we obtain the below equations:

$$\sqrt{\Delta l_1 + L_{1s}} = \sqrt{L_{1s}(1+\frac{\Delta l_1}{L_{1s}})} \approx \sqrt{L_{1s}}(1+\frac{\Delta l_1}{2L_{1s}}) = \sqrt{L_{1s}} + \frac{\Delta l_1}{2\sqrt{L_{1s}}} \qquad (15)$$

$$\sqrt{\Delta l_2 + L_{2s}} = \sqrt{L_{2s}(1+\frac{\Delta l_2}{L_{2s}})} \approx \sqrt{L_{2s}}(1+\frac{\Delta l_2}{2L_{2s}}) = \sqrt{L_{2s}} + \frac{\Delta l_2}{2\sqrt{L_{2s}}} \qquad (16)$$

$$\sqrt{\Delta l_1 - \Delta l_2 + L_{1s} - L_{2s}} = \sqrt{(L_{1s} - L_{2s})(1+\frac{\Delta l_1 - \Delta l_2}{L_{1s} - L_{2s}})} \approx \sqrt{L_{1s} - L_{2s}} + \frac{\Delta l_1 - \Delta l_2}{2\sqrt{L_{1s} - L_{2s}}} \qquad (17)$$

Substitute these approximation equations (15-17) into (12-13) and in combination with equations (10-11), we obtain:

$$\frac{d\Delta l_1}{dt} = -(k_1 + k_2)\Delta l_1 + k_2\Delta l_2 + \frac{k_b}{A_1}\Delta u \qquad (18)$$

$$\frac{d\Delta l_2}{dt} = k_3\Delta l_1 - (k_3 + k_4)\Delta l_2 \qquad (19)$$

$$k_1 = \frac{aC_a}{A_1}\sqrt{\frac{g}{2L_{1s}}};\ k_2 = \frac{bC_b}{A_1}\sqrt{\frac{g}{2(L_{1s} - L_{2s})}};\ k_3 = \frac{bC_b}{A_2}\sqrt{\frac{g}{2(L_{1s} - L_{2s})}};\ k_4 = \frac{cC_c}{A_2}\sqrt{\frac{g}{2L_{2s}}}$$

The equations (18) and (19) describe the linear model of the coupled-tank system, where input is the incremetal pump voltage $\Delta u(t)$, and output is the incremetal fuild level in the second tank $\Delta l_2(t)$. By taking the Laplace transform of equations (18-19) the following transfer function is obtained:

$$G_{L2}(s) = \frac{\Delta l_2(s)}{\Delta u(s)} = \frac{k_3 k_b / A_1}{s^2 + (k_1 + k_2 + k_3 + k_4)s + (k_1 k_3 + k_1 k_4 + k_2 k_4)} \qquad (20)$$

In this paper, we design a fuzzy-PID controller applied for coupled-tank system with following parameters [15].

Tab 1. *Constants involved in coupled-tank system of Fig. 1*

Parameter	Desctiption	Value	Unit
k_v	Gain of DC-motor electric valve	3.3	$cm^3/s/V$
k_s	Gain of level untrasonic sensor	6.1	cm/s
C_a	Percentage of opening valve A	60	%
C_b	Percentage of opening valve B	80	%
C_c	Percentage of opening valve C	60	%
D_1, D_2	Inner diameter of tank 1, 2	6	cm
D_A	Inner diameter of valve A	0.5	cm
D_B	Inner diameter of valve B	0.7	cm
D_C	Inner diameter of valve C	0.5	cm
H_{max}	Max. height of liquid level in tank 1, 2	30	cm

Assume that the desired height of fluid level in the second tank L_{2s}=15[cm], from equations (10), (11) and (20), we obtain the linear model of the liquid level process of the coupled-tank system as below:

$$G_{L2}(s) = \frac{\Delta l_2(s)}{\Delta u(s)} = \frac{0.0176}{s^2 + 0.362s + 0.007} \qquad (21)$$

III. THE FUZZY-PID CONTROLLER DESIGN FOR LIQUID LEVEL PROCESS OF THE COUPLED-TANKS SYSTEM

The structure of the fuzzy-PID controller for the liquid level process of the coupled-tank system is proposed as in Fig. 2. The fuzzy-PID controller is a combination of the basic PID and the fuzzy logic calculator. The initial parametters k_{P0}, k_{I0}, k_{D0} of the basic PID are definited based on the common methods, such as Ziegler Nichols (PID-ZN), Chien-Hrones- Reswick (PID-CHR). The k_{PF}, k_{IF}, k_{DF} are seft-tunning parameters of PID based on fuzzy logic calcutalor (FuzzyCal block in Fig.2) for the nonlinear model of coupled-tank with the noise.

3.1. Designing the basic PID controller

The basic PID is designed based on the linear model of the liquid level process of the coupled-tank system. Using the Ziegler Nichols method, we can determine the initial paramaters k_{P0}, k_{I0}, k_{D0}.

The transfer function of the level control object as:

$$G_{obj}(s) = k_s G_{L2}(s) = \frac{0.1074}{s^2 + 0.362s + 0.007} = \frac{K}{(T_1 s + 1)(T_2 s + 1)} \qquad (22)$$

where $K=15.372$, $T_1=2.93$, $T_2=48.78$

Arcoding to the Ziegler Nichols 1st method, the parameters k_{P0}, k_{I0}, k_{D0} can be determined as follows:

$$G_C(s) = k_{P0} + \frac{k_{I0}}{s} + k_{D0}s \qquad (23)$$

where:

$$k_{P0} = \frac{1.2T_2}{KT_1} = \frac{1.2 * 48.78}{15.372 * 2.93} = 1.31$$

$$k_{I0} = \frac{k_{P0}}{2T_1} = 0.22$$

$$k_{D0} = k_{P0}\frac{T_1}{2} = 1.92$$

Howerver with the nonlinear model of the coupled-tank, the acceptable pamameters are k_{P0} = 15, k_{I0} = 0.3, k_{D0} = 11.

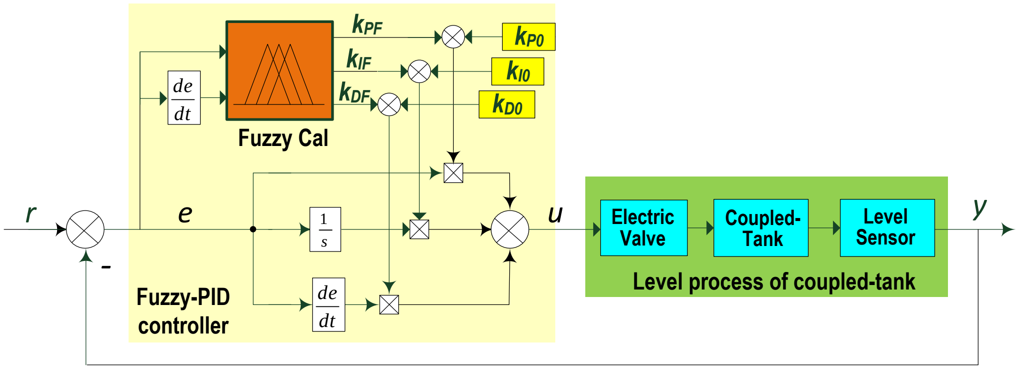


Fig. 2. Structure of fuzzy-PID for liquid level process of coupled-tank

3.2. Designing the fuzzy logic calculator

The fuzzy logic calculations block (FC) have: two inputs - level error in the second tank (EL), derivative of level error (DEL) corresponding to input voltage error signal $e=y-r$ (*r*- level setpoint, *y*- level in tank 2) and *de/dt*; three output is PL, IL, DL corresponding to the output value k_{PF}, k_{IF}, k_{DF}.

Using membership functions are shaped triangular for all variables, fuzzied for all input variables by 5 fuzzy sets {NL (Negative Large), NS (Negative Small), ZE (ZEro), PS (Positive Small), PL (Positive Large)}, fuzzied for all output variables by 5 fuzzy sets {SM (SMall), ME (MEdium), LA (LArge), QL (Quite Large), VL (Very Large)}. The physical domain of the input & output variables are determined as: EL$\in$[-20,20], DEL$\in$[-2,2], PL$\in$[0,20], IL$\in$[0,1], DL$\in$[0,15].

Depending on the characteristics of the level control proces of the coupled-tank and the PID control principle in order to improve quality control for this system (see Tab.2), we define the 25 basic fuzzy rules as Tab.3.

Tab.2. The effect of k_P, k_I, k_D tunning

Closed-loop respond	Rise time	Steady time	Over-shoot	Steady error	Stability
Increasing k_P	Decrease	Small change	Increase	Decrease	Degrade
Increasing k_I	Small decrease	Decrease	Increase	Eliminate	Degrade
Increasing k_D	Small decrease	Decrease	Small decrease	Small change	Increase

Tab.3. The basic fuzzy rule of k_{PL}, k_{IL}, k_{DL}

PL IL DL		EL				
		NL	NS	ZE	PS	PL
DEL	NL	SM	SM	SM	SM	SM
	NS	SM	ME	SM	SM	SM
	ZE	SM	SM	LA	LA	QL
	PS	SM	SM	LA	QL	VL
	PL	SM	SM	QL	VL	VL

Using the Max-Min composition rule and the cetroid defuzzification method, we can obtain the clear output value of FC: k_{PF}, k_{IF}, k_{DF} for level control loop. Thus, the fuzzy-PID controller can be calculated by equations:

$$k_P^* = k_{P0} + k_{PF}\,,\ k_I^* = k_{I0} + k_{IF}\,,\ k_D^* = k_{D0} + k_{DF} \qquad (24)$$

IV. SIMULATION RESULT

The simulated diagram of the fluid level process of coupled-tank system is described as Fig. 3. The fuzzy-PID controller is a combination of the FuzzyCal block with the VariablePID. The self-tunning parameters of fuzzy-PID is determine on equation (24), here k_{P0}, k_{I0}, k_{D0} are initial parmaters of PID and k_{PF}, k_{IF}, k_{DF} are the clear output value of the FuzzyCal block. The fluid level process of coupled-tank is used as nonlinear model, using equations (8) and (9).

The simulation is carried out with three controllers: Fuzzy-PID, PID-ZN1, PID-CHR. The quality control system is evaluated through four indexes (overshoot, rise time, steady time, steady error) in two circumstances: (a). varying setpoint level; (b). as impacted by the bound noise with small margin.

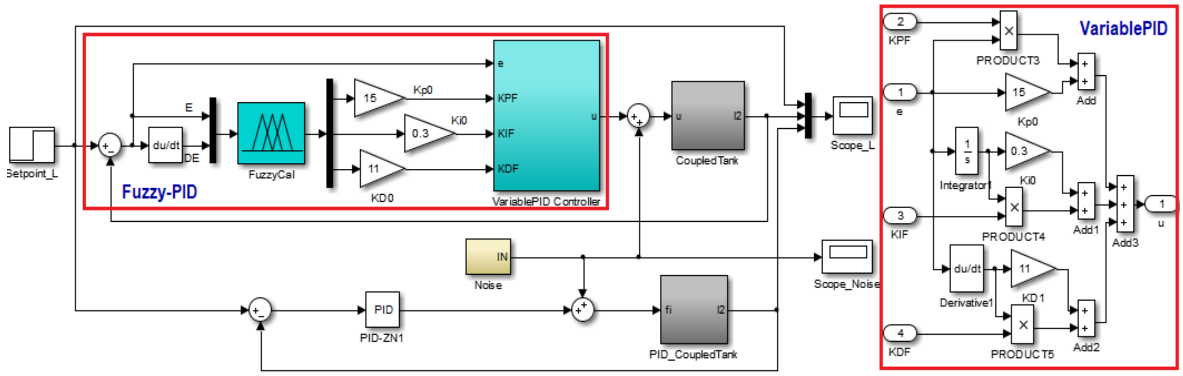


Fig. 3. Simulation of the fluid level coupled-tank control system using fuzzy-PID

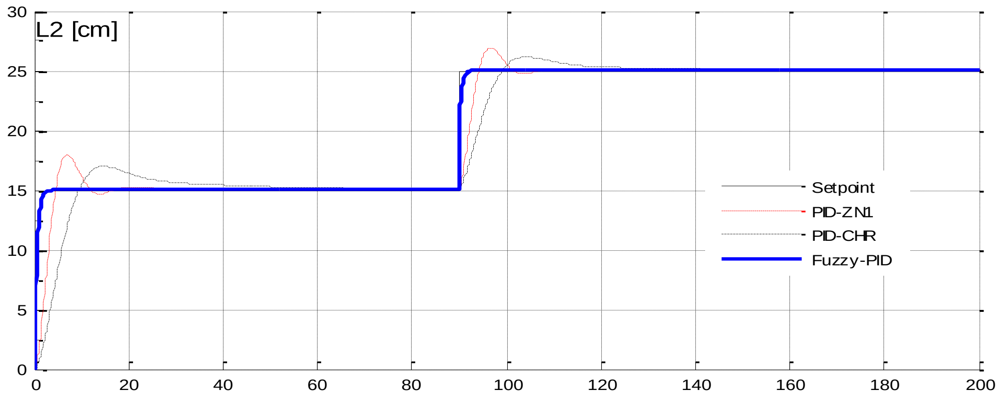


Fig. 4. Response curves of the level controllers as varying setpoint level

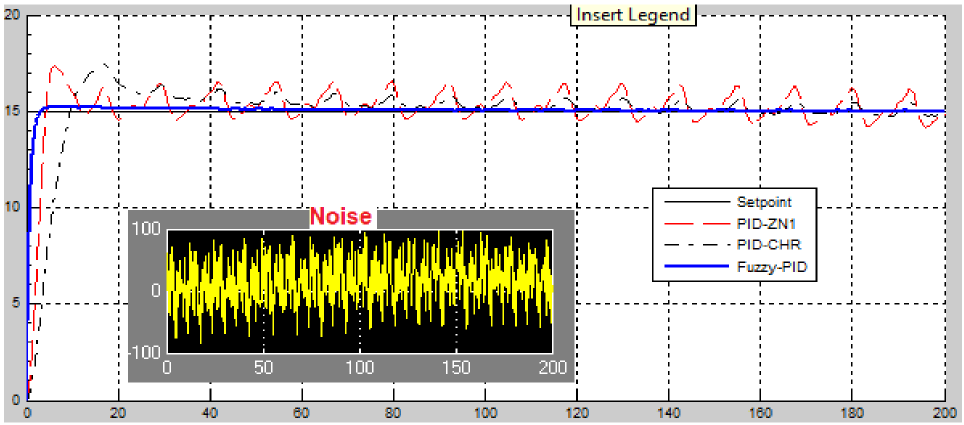


Fig. 5. Response curves of the level controllers as having noise with small margin

The simulation results, as using PID-ZN1, PID-CHR and Fuzzy-PID controller, is presented in Tab. 4.

Tab. 4. Performance of Fuzzy-PID controller & others

Controller / Index	Fuzzy-PID	PID-ZN1	PID-CHR
Rise time	Small, ~1.5s	Large, ~4.1s	Very large, ~8.4s
Steady time	Small, ~3.1s	Large, ~11.2s	Very large, ~8.4s
Overshoot	Not	Large, ~19.4%	Small, ~12.3%
Steady error	Eliminate, or very small with noise	Very small, but large swing with noise	Very small, but swing with noise

The simulating results show that fuzzy-PID has the best control quality: not overshoot, eliminating steady error, the smallest steady time and eliminating neraly the effect of the disturbaces, when it was compared to traditional PID controllers.

V. CONCLUSION

This paper has presented a case study where the basic PID controller is combined with the fuzzy logic calculator for the nonlinear model of the liquid level of coupled-tank system. The simulation results suggest that the fuzzy-PID proposed controller can be applied to the liquid level control process in the chemical industry, where noise is always presented. The fuzzy-PID controller can improve quality of the liquid level coupled-tank control system, increase the process efficiency and bring economic benefit to end-user. However, we need to study in more detail about dynamics of actuator & sensor, according to the actual device to obtain a more realistic control object model, which helps to control the fluid level in coupled-tank better.

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