International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 04, April-2016

Linear Control of Air-Breathing Hypersonic Vehicle

Breeza Paulose, Jis Jose, Neena Skaria
Electrical Engineering Department
Viswajyothi College of Engineering and Technology
Vazhakulam,Ernakulam,Kerala,India

Abstract- **Air-Breathing Hypersonic vehicle is the next generation launch vehicle which is fully reusable. It is a single stage to orbit spacecraft. Scramjet Engine is used for propulsion. Advantages of air-breathing hypersonic vehicle are it reduces flight time, reduces the cost of vehicle and maximizes payload to Low Earth Orbit. This paper proposes the flight control design for an air-breathing hypersonic vehicle. The determination of dynamic characteristics of hypersonic vehicle requires an integrated approach since the propulsion system and airframe are highly coupled. The nonlinear model of the vehicle is linearized by small perturbation approach at trim conditions. Then the stability of the system is checked. Linear control design using Linear Quadratic Regulator is used for the control of the air-breathing hypersonic vehicle Robustness analysis is done for linear controller and the system is found to be moderately robust.**

I. INTRODUCTION

As mankind reaches out into space and develops the need for rapid transportation over long distances through the atmosphere of the Earth, the desire for hypersonic flight increases. Requirements for air-breathing hypersonic flight include rapid, on-demand access to space with a payload and quick access to distant point on earth. For commercial and military applications, these vehicle concepts seek aircraft-like operations, such as horizontal takeoff and landing and rapid turnaround capability to facilitate ground handling operations. Such vehicles must be reusable, reliable, safe and cost effective to operate and must have minimal negative environmental impacts on atmosphere.

Air-breathing refers to the ability of an engine to use the ambient environment (air) as a means for propulsion. For a standard commercial airplane, fuel is stored on board of the aircraft; the air necessary for combustion is pulled through the engine by the turbine. So this engine requires that the vehicle carry only fuel whereas in rocket engines both fuel and oxidizer needed to be carried. So in air-breathing engines, the volume that is dedicated to carrying an oxidizer can be used for payload. So it provides an opportunity for increased payloads.

In air-breathing hypersonic vehicle the engine used is scramjet engine (supersonic combustion ramjet). In scramjet engine, since the entire air flow is supersonic fuel get less time to mix with air. Thus combustor needed to be long. In fact, the entire lower surface of the aircraft can be considered as the engine. So the vehicle has highly integrated airframe and propulsion system.

From above discussions it is very clear that in Air-Breathing Hypersonic vehicle nonlinear coupling between aerodynamics and propulsion systems exists. The design of guidance and control systems for air-breathing hypersonic vehicles is a challenging task, due to the unique characteristics of the vehicle dynamics. This paper describes how to design a linear control system in the presence of these nonlinearities.

The system model is given in section II. The linearization of the system model and its stability analysis is described in III. Linear control design is presented in section IV. Robustness analysis is given in section V.

II. SYSTEM MODEL

The design of control systems for air-breathing hypersonic vehicles is a challenging task, due to the high nonlinearity of the equations of motion, stemming from strong couplings between propulsive and aerodynamic effects. A nonlinear model of the longitudinal dynamics for the air-breathing hypersonic vehicle is developed. The model derived includes the nonlinear interaction between propulsion system and aerodynamics.

The equations of motion of the longitudinal dynamics of air-breathing hypersonic vehicle is given by,

$$\begin{aligned}
\dot{h} &= V\sin(\theta-\alpha) \\
\dot{V} &= \frac{1}{m}(-T\cos\alpha - D) - g\sin(\theta-\alpha) \\
\dot{\alpha} &= \frac{1}{mV}(-T\sin\alpha - L) + Q + \frac{g}{V}\cos(\theta-\alpha) \\
\dot{\theta} &= Q \\
I_{yy}\dot{Q} &= M
\end{aligned} \qquad (1)$$

Where h is the altitude, V is the velocity, α is the angle of attack, θ is the pitch angle, Q is the pitch rate, T is the thrust, D is the drag, L is the lift and M is the moment.

The equations of motion capture inertial coupling effects between the pitch and normal accelerations of the vehicle and the aerodynamics and propulsion.

The Velocity V and the Altitude h are the desired outputs of the system which are the states itself. The control inputs elevator deflection(δ_e) and fuel air ratio(Φ) do not appear directly in the equations of motion. Instead, they enter

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 04, April-2016

through the nonlinear expressions of the forces and moments *T, M, L* and *D*. For the controller design, a simplified model was developed in the reference [3]. The simplified model, referred to as the curve-fitted model (CFM), approximates the behaviour of the model by replacing the aerodynamic forces and moments with curve-fitted approximations. The resulting non-linear model offers the advantage of being analytically tractable (though still complex) and more suitable for control design, while retaining the important dynamical features of the first-principle model. The approximations of the forces and moments used in the CFM are given as follows

$$\begin{aligned} L &= \frac{1}{2}\rho V^2 S C_L(\alpha,\delta_e) \\ D &= \frac{1}{2}\rho V^2 S C_D(\alpha,\delta_e) \\ M &= z_T T + \frac{1}{2}\rho V^2 S \bar{c}\left[C_{M,\alpha}(\alpha) + C_{M,\delta_e}(\delta_e)\right] \\ T &= C_T^{\alpha^3}\alpha^3 + C_T^{\alpha^2}\alpha^2 + C_T^{\alpha}\alpha + C_T^{0} \end{aligned} \tag{2}$$

The thrust *T* is strongly dependent on angle of attack α, because the angle of attack determines the airflow into the scramjet engine. In addition, the under slung nature of the scramjet engine produces a pitching moment directly proportional to thrust. This causes loop interconnection between the propulsion and aerodynamics and it should be carefully accounted in the control design.

$$\begin{aligned} C_{M,\alpha} &= C_{M,\alpha}^{\alpha^2}\alpha^2 + C_{M,\alpha}^{\alpha}\alpha + C_{M,\alpha}^{0} \\ C_{M,\delta_e} &= c_e\delta_e \\ C_L &= C_L^{\alpha}\alpha + C_L^{\delta_e}\delta_e + C_L^{0} \\ C_D &= C_D^{\alpha^2}\alpha^2 + C_D^{\alpha}\alpha + C_D^{\delta_e^2}\delta_e^2 + C_D^{\delta_e}\delta_e + C_D^{0} \\ C_T^{\alpha^3} &= \beta_1(h,q)\phi + \beta_2(h,q) \\ C_T^{\alpha^2} &= \beta_3(h,q)\phi + \beta_4(h,q) \\ C_T^{0} &= \beta_7(h,q)\phi + \beta_8(h,q) \\ C_T^{\alpha} &= \beta_5(h,q)\phi + \beta_6(h,q) \end{aligned} \tag{3}$$

Where *q* represents the dynamic pressure, and an expression of the form $\rho=\rho_o \exp[-(h-h_o)/h_s]$ is chosen for the air density. The scramjet engine included in the model produces a thrust force *T*, which depend strongly on states *h*, *V* and *α*, along with the input *Φ*. The mapping is approximately cubic in angle of attack, where as each coefficient in the polynomial is a linear function of *Φ*. The eight $\beta_i(h,q)$ coefficients change with dynamic pressure and altitude. Because these values vary on a much slower rate than *Φ* and *α*, the values of β_i are assumed to be constant for control design.

Nonlinear simulation of the air-breathing hypersonic vehicle is done in MATLAB. The inputs given are the air fuel ratio (*Φ*) and the elevator deflection angle (δ_e). The *Φ* and δ_e are constant trim values.

The initial values of the system states are the trim values. They are given in the table I.

TABLE I . TRIM CONDITIONS FOR SYSTEM MODEL

States & Inputs	*Value*	*Unit*
Altitude(h)	85,000	ft
Velocity(V)	7,702.0808	Ft/sec
Angle of attack(α)	1.5153	deg
Pitch angle(θ)	1.5153	deg
Pitch rate (Q)	0	Deg/sec
Fuel-air ratio(φ)	.2514	-
Elevator deflection(δ_e)	11.4635	deg

The response of open loop plant dynamics to initial condition is given below

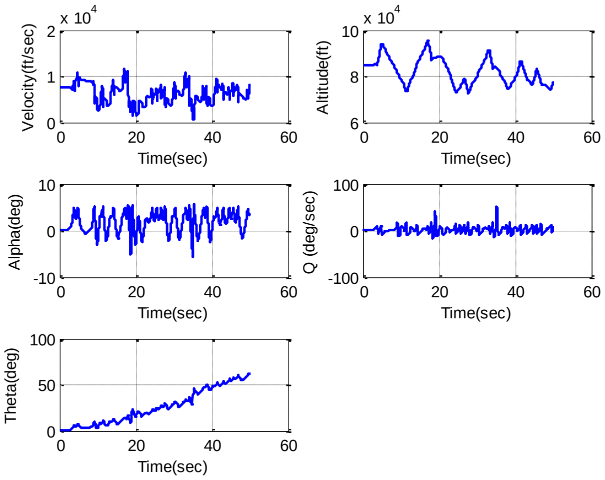

Fig. 1. Nonlinear Open loop response

From the attitude response it is clear that up to four seconds the system is showing some diverging response. After two seconds the nonlinearity comes into effect and the system responses are varying in a random pattern. It shows short period instability. To validate it plant dynamics is linearized.

III.LINEARIZATION

Linearization of the air-breathing hypersonic vehicle equations of motion begins with consideration of perturbed flight. Perturbed flight is defined relative to a steady-state (trimmed) flight condition using a combination of steady-state and perturbed variables for hypersonic vehicle motion parameters and for forces and moments. Simply stated, each motion variable, force and moment in equations of motion is redefined as the summation of steady state value and perturbed value. The assumption of small perturbations allows linearization of the equations of motion [6].

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 04, April-2016

The following four-step approach summarizes the linearization technique:

Step1: Recast each variable in terms of trimmed (steady-state) value and a perturbed value. Assume small perturbations. Use trigonometric identities.
Step2: Apply small angle assumptions to trigonometric functions of perturbed angles. [$sin\Delta\theta=\Delta\theta$, $cos\Delta\theta=1$]
Step3: Assume that the products of small perturbations are negligible.
Step4: Remove steady state equations from perturbed equation. The Remaining perturbed equations are linearized differential equations with perturbed variables as unknowns.

A. Example of linearization

Consider the altitude equation of Air-breathing Hypersonic vehicle. The flight path angle $\gamma = \theta - \alpha$

$$\dot{h} = V \sin(\theta - \alpha) \quad (5)$$

$$\dot{h} = V \sin\gamma \quad (6)$$

Assume small perturbations Δh and ΔV and $\Delta\gamma$. Each variables are redefined as the summation of steady state value and perturbed value.

$$\dot{h} + \Delta\dot{h} = (V + \Delta V)\sin(\gamma + \Delta\gamma)$$

Expand the equation

$$\dot{h} + \Delta\dot{h} = (V + \Delta V)[\sin\gamma\cos\Delta\gamma + \cos\gamma\sin\Delta\gamma]$$

Apply small angle assumptions

$$\dot{h} + \Delta\dot{h} = (V + \Delta V)[\sin\gamma + \cos\gamma\Delta\gamma]$$

$$\dot{h} + \Delta\dot{h} = V\sin\gamma + V\cos\gamma\Delta\gamma + \Delta V\sin\gamma + \Delta V\Delta\gamma\cos\gamma$$

$$\dot{h} + \Delta\dot{h} = V\sin\gamma + V\cos\gamma\Delta\gamma + \Delta V\sin\gamma$$

$$\Delta\dot{h} = V\cos\gamma\Delta\gamma + \Delta V\sin\gamma$$

Then apply the trim values. The linearized altitude equation is

$$\Delta\dot{h} = 7702.0808\Delta\theta - 7702.0808\Delta\alpha \quad (7)$$

Like the altitude equation, all other state equations, moment equations, lift, drag equations and the coefficient equations are linearized about the trim conditions. Then the state equations can be written as

$$\dot{X} = AX + BU \quad (8)$$

Where $X = [\Delta h \quad \Delta V \quad \Delta\alpha \quad \Delta\theta \quad \Delta Q]^T$ and $U = \begin{bmatrix}\Delta\delta_e \\ \Delta\phi\end{bmatrix}$ are the new state variables and new inputs.

$$A = \begin{bmatrix} 0 & 0 & -7702.0808 & 7702.0808 & 0 \\ 0 & -.001346 & 21.284 & -32.2 & 0 \\ 0 & -9.919\times10^{-7} & -.0696 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -2.08\times10^{-6} & 2.945 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ -37.19 & 24.43 \\ -0.01121 & -8.385\times10^{-5} \\ 0 & 0 \\ -1.49 & .123 \end{bmatrix}$$

From the *A* (state matrix) matrix the eigen values of the system can be found, and the open loop poles are at 0, 1.6817, -1.7513and -0.0007±0.0058i. One of the eigen values are positive. So the system is unstable.

So a proper control design must be attempted to achieve stability and tracking.

IV.LINEAR CONTROL DESIGN

In this section a design method commonly called Linear Quadratic Regulator (LQR) technique will be used for the control of nonlinear Air-breathing hypersonic vehicle model. We assume that all state variables are measurable and are available for feedback.

The formulation of the problem is as follows. Consider linear system

$$\dot{x} = Ax + Bu$$
$$y = Cx \quad (9)$$

find a control function *u(t)* that will minimize cost function given by

$$J = \frac{1}{2}\int_0^\infty (x'Qx + u'Ru)dt \quad (10)$$

The function inside the integral is a quadratic form and the matrices *Q* and *R* are symmetric. It is assumed that *R* is positive definite (i.e., it is symmetric and has positive eigen values) and *Q* is positive semi definite (i.e., it is symmetric and has nonnegative eigen values). These assumptions imply that the cost is nonnegative, so its minimum value is zero.

Several procedures are available to solve LQR problem. One approach to finding a controller that minimizes the LQR cost function is based on finding the positive-definite solution of the following algebraic Riccati equation(ARE).

$$A'P + PA + Q - PBR^{-1}B'P = 0 \quad (11)$$

The solution of ARE is *P* matrix, and the state feedback gain matrix can be calculated from the following expression.

$$K = R^{-1}B'P \quad (12)$$

It turns out that under the conditions stated shortly, the positive-definite solution of the ARE results in an

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 04, April-2016

asymptotically stable closed-loop system. The conditions are the following. The system is controllable, R is positive definite (this ensures that the inverse exists), and Q can be factored as $Q = C_q^{'} C_q$, where C_q is any matrix such that (C_q, A) is observable. These conditions are necessary and sufficient for the existence and uniqueness of the optimal controller that will asymptotically stabilize the system.

Manually solving the Riccati equation is tedious and almost impossible for third or higher order system. Since the model considered for this work is 5th order, the problem is solved by using MATLAB.

The linear LQR controller is applied to nonlinear model of the air-breathing hypersonic vehicle. The Q and R matrices are selected so as to obtain the optimum response. The Q and R matrices are chosen as

$$Q = \begin{bmatrix} 26.2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{bmatrix}, R = \begin{bmatrix} 500 & 0 \\ 0 & 1000 \end{bmatrix}$$

The initial conditions of the states are the trim values. The commands given to velocity and altitude states are the trim values itself. It is because the linear model can be used to calculate the control law for nonlinear model only at trim point or around the trim point. So the controller is designed to force the system to remain in the initial values. The altitude and velocity responses are as shown below.

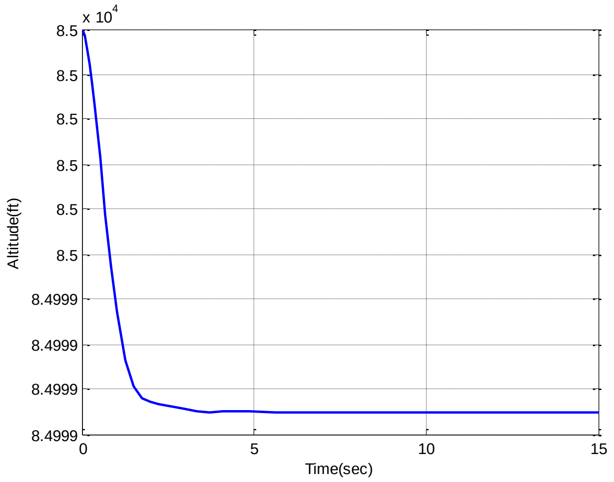

Fig 2:Closed loop Altitude response

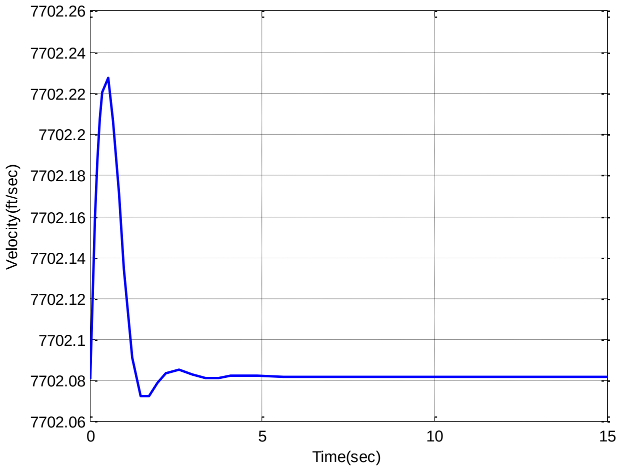

Fig 3:Closed loop Velocity response

The responses are satisfactory. The velocity response is tracking the commanded value perfectly. The altitude response has steady state error. Here, linear controller is used for the control of nonlinear model. During linearization, some of the nonlinear properties of the plant is lost. This is the reason for steady state error.

V. ROBUSTNESS ANALYSIS

The ultimate aim of a control system engineer is to build a system that will work in real environment. Since the real environment may change with time (as components age or their parameters vary with temperature or other environmental conditions) or operating conditions may vary (load changes, disturbances), the control system must be able to withstand these variations.

The particular property a control system must possess to operate properly in realistic situations is called robustness. The LQR design ensures a certain degree of robustness for the stability of closed loop system in the presence of parametric uncertainties.

To check the robustness of the LQR controller in air-breathing hypersonic vehicle, two cases are considered

1. Mass and moment of inertia are varied up to ± 20% .

2. Coefficient of moment, Drag coefficient and Lift coefficient varied by ± 20%

The stability of the air-breathing hypersonic vehicle is evaluated relative to a steady state trimmed flight condition. At high altitudes the vehicle is subjected to various aerodynamic forces. This may change the mass of the vehicle. The moment coefficient depends on angle of attack and elevator deflection.. The lift and drag coefficients depends on angle of attack and elevator deflection.. These parameters are varied for the robustness analysis.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 04, April-2016

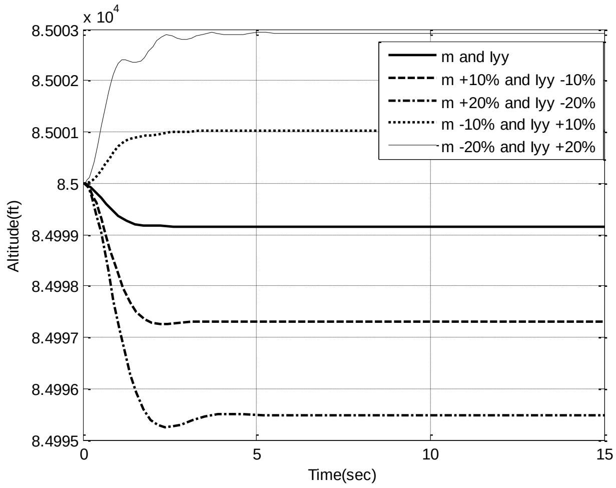

Fig 4. Altitude response with mass and moment of inertia varied by ± 20%

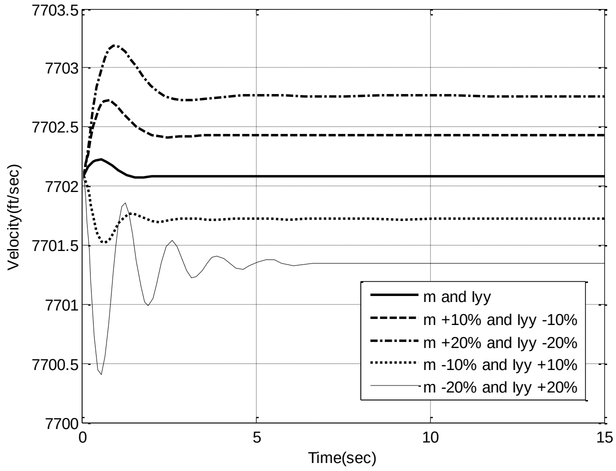

Fig 5. Velocity response with mass and moment of inertial varied by ± 20%

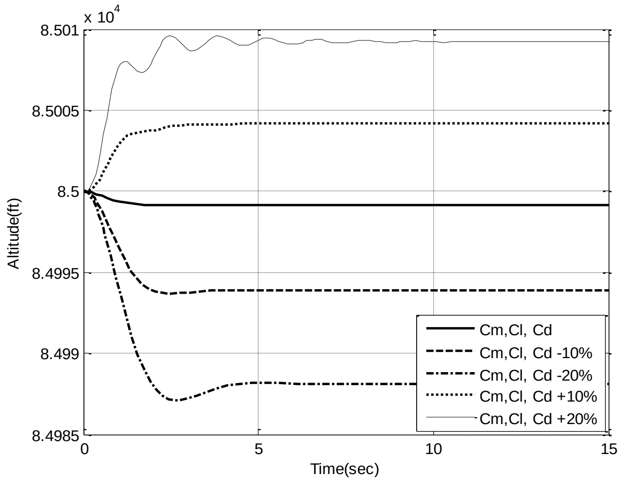

Fig 6. Altitude response Cm, Cd and Cl varied by ± 20%

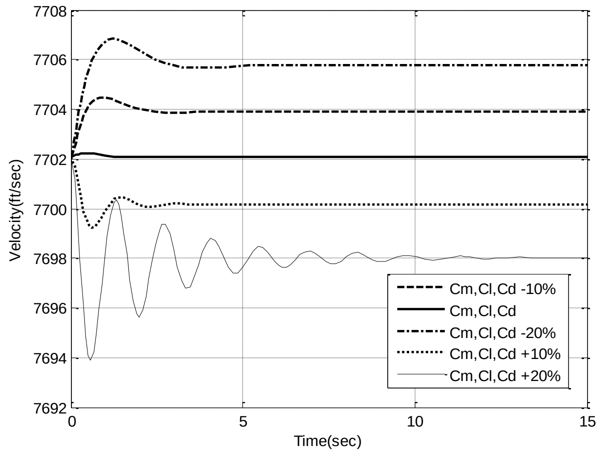

Fig 7. Velocity response Cm, Cd and Cl varied by ± 20%

By varying the parameters in the system model the robustness of the controller is evaluated. The tracking performance is still good and the controller is moderately robust.

VI. CONCLUSION

This paper describes the flight propulsion control design for an air-breathing hypersonic vehicle. The nonlinear model of the vehicle is linearized by small perturbation approach at trim conditions and the stability of the system is checked. The system is found to be unstable. Linear control design using Linear Quadratic Regulator is used for the control of the air-breathing hypersonic vehicle. The robustness of the controller is evaluated by parameter variation, and the LQR controller has moderate robustness.

REFERENCES

[1] Chavez, F. and Schmidt, D., "Analytical Aeropropulsive/Aeroelastic Hypersonic-Vehicle Model with Dynamic Analysis," *Journal of Guidance, Control, and Dynamics,* Vol. 17, No. 6, Nov./Dec. 1994, pp. 1308– 1319.

[2] Bolender, M. and Doman, D., "Non-Linear Longitudinal Dynamical Model of an Air- breathing Hypersonic Vehicle," *Journal of spacecrafts and rockets,* Vol.44, No.2,March-April 2007,pp.374-387

[3] Jason T.Parker, Bolender, M. and Doman, D., "Control-Oriented Modelling of an Air-Breathing Hypersonic Vehicle", *Journal of Guidance, Control, and Dynamics,* Vol. 30, No.3, May-June. 2007, pp. 856– 869

[4] K.P. Groves, D.O. Sigthorssony, A.Serraniz, S.Yurkovich," Reference Command Tracking for a Linearized Model of an Air-breathing Hypersonic Vehicle", *Proceedings of the 2005 AIAA Guidance, Navigation, and Control Conference,* Aug 2005.

[5] L. Fiorentini, A. Serrani, Michael A. Bolender, David B. Doman," Nonlinear Robust/Adaptive Controller Design for an Air-breathing Hypersonic Vehicle Model", *Proceedings of the 2005 AIAA Guidance, Navigation, and Control Conference,* Aug 2007.

[6] Thomas R. Yechout, Steven L. Morris, David E. Bossert, Wayne F. Hallgren, "Introduction to Aircraft Mechanics: Performance, Static Stability, Dynamic Stability, and Classical feedback control "*American Institute of Aeronautics and Astronautics,* Inc-2003