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Leveraging Artificial Intelligence for the Solution of Complex Differential Equations: A Systematic Review, Critical Analysis, and Roadmap for Future Research

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Abstract

In recent years, artificial intelligence (AI), particularly deep learning, has become a strong alternative or complement to classical numerical solvers for partial differential equations (PDEs). This paper presents a systematic review of AI-based techniques like Physics-Informed Neural Networks (PINNs), Deep Operator Networks (DeepONets), and Fourier Neural Operators (FNOs). We take a close look at their theoretical underpinnings, algorithmic implementations, and practical performance on a benchmark problem from fluid dynamics. In contrast to previous reviews, this one emphasises reproducibility, failure modes, costs, and uncertainty quantification. By using comparison, we found that AI solvers have speedups of about 1000× over classical ones but suffer from spectral bias, training instability, and limited generalization across parameter regimes. We synthesize recent advances in neural operators, geometry-aware deep networks, and hybrid physics-AI frameworks. The paper ends with a roadmap for building scientific computing foundation models and reproducible benchmarking standards.

Keywords – Physics-informed neural networks; neural operators; Fourier neural operator; DeepONet; partial differential equations; scientific machine learning; uncertainty quantification.

1. Introduction

PDE models describe conservation laws, transport phenomena, and mechanical responses across different scales. The conventional numerical techniques such as finite difference, finite element (FEM), finite volume (FVM), spectral methods, are equipped with clear convergence proof and error control. Nonetheless, their application to complex, high dimensional or inverse problems has three key limitations:

(i) the curse of dimensionality (exponential growth in degrees of freedom),

(ii) prohibitive computational cost for many-query tasks (e.g., uncertainty propagation, optimization, real-time control).

(iii) difficulty in assimilating noisy or sparse measurement data.

A new class of methods has appeared in the last five years that treats PDE solving as a machine learning problem. Artificial Intelligence-based solvers can be broadly categorized into 2 paradigms

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• Physics informed learning (exemplified by PINNs [1]), where the PDE residual is embedded as a soft constraint in the loss function, enabling mesh free, data efficient solutions.
• Operator learning, as exemplified by DeepONet [3] and FNO [2], aims at learning the mapping from input functions to output solution fields, e.g., initial conditions and boundary shapes/forces/material parameters, which allows us to produce output fields almost instantly for new input fields.
Since 2019, many hundreds of papers have expanded on these ideas. However, the field suffers from a reproducibility crisis, overblown claims, and an absence of standard benchmark [5]. The current assessment aims to be a balanced, critical and comprehensive one.

Contributions of this paper:

- A systematic literature search and screening protocol (PRISMA informed)
- Comprehensive mathematical formulations, including practical implementation choices, e.g., activation functions, optimizers, normalisation.
- A quantitative meta-analysis of performance on standard benchmarks like Burgers' equation, Darcy flow, and Navier-Stokes
- An important point of discussion regarding failure modes spectral bias, stiff PDEs, non convex loss landscapes, and out of distribution generalisation.
- An overview of new developments: uncertainty quantification, geometric deep learning for irregular domains, and foundation models.
- A reproducibility checklist and recommendations for future research.

2. Systematic Review Methodology

We followed a structured approach inspired by PRISMA guidelines.

Search strategy:

• Databases: Scopus, Web of Science, arXiv (cs.LG, math.NA, physics.comp ph).
• Between January 2017 and March 2025:

We will search for the keywords: ("physics informed neural network*" OR "PINN" OR "operator learning" OR "DeepONet" OR "Fourier neural operator" OR "neural PDE solver") AND ("partial differential equation" OR "PDE").

Inclusion criteria:

• Suitable sources include peer-reviewed journal articles or conference proceedings which allow full-text published. Preprints are only accepted if they are a method or benchmark.
• Any research paper that reports quantitative error metrics (relative L_2 error, maximum pointwise error) on at least one standard PDE benchmark.
• Investigations using a conventional solver for accuracy or computing time benchmarks.
• Criteria for exclusion
• Writings exclusively on ODEs without any extension to PDEs.

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• Studies that do not provide details necessary for reproducibility (e.g., hyperparameters, random seed, code link).
• Outcome of Screening. First search yielded 1,247 records. 312 full text articles were assessed after duplicate removal and abstract screening. In the end, there were 87 studies that met the inclusion criteria for this review. The appendix contains the full list.

3. Foundational AI Methods for PDEs

3.1 Physics-Informed Neural Networks (PINNs)

PINNs approximate the solution $u(x,t)$ by a neural network $u\hat{}(x,t;\theta)$ with parameters θ. The loss function is:
$\mathrm{L}(\theta)=\lambda r \mathrm{L}r+\lambda bc \mathrm{L}bc+\lambda ic \mathrm{L}ic$,

where

- $\mathrm{L}r=1/Nr\sum_{i}^{Nr}\|\mathrm{N}[u\hat{}](xi,ti)-f(xi,ti)\|^2$ (PDE residual),
- $\mathrm{L}bc$ and $\mathrm{L}ic$ enforce boundary/initial conditions,
- λ are weighting coefficients.

Automatic differentiation (AD) calculates derivatives reliably and without mesh production. But for high order derivatives, the AD becomes costly and the loss landscape is often ill conditioned. Recent enhancements now include.

• Adaptive activation functions that learn the slope of nonlinearities
• Reweighting using neural tangent kernel to balance various loss terms [7].
• Causal training that respects the temporal order of evolution equations.

Limitations:
PINNs faces difficulties in dealing with multi scale problems and sharp gradients. The error tends to plateau after a few thousand epochs and does not reach above 10−810−8 relative error.

3.2 Deep Operator Networks (DeepONet)

DeepONet [3] learns the operator G:U→V, where U and V are function spaces. It consists of two sub-networks:

• Branch net encodes the input function u at a fixed set of sensor points.
• The trunk net encodes the evaluation coordinates y.
The output is G(u)(y)≈ branch(u).trunk(y).

Variants:

• Incorporating Physical Knowledge to the DeepONet Using PDE Residual Loss Reduces Data Requirement
• Different branches of DeepONet manage parametric dependences (e.g., material properties).

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After training, it has very fast inference and can naturally handle irregular grids. Some weakness of learning approach of scientific workstation and AI based CDFML are as follows:
1. Data hungry.
2. Branch Network's discretisation dependence.
3. Resolution invariant breaking.

3.3 Fourier Neural Operator (FNO)

FNO [2] parameterises the integral kernel in Fourier space. For a PDE with solution v*v*, one iterative layer is:

$$vt+1(x)=\sigma(Wvt(x)+\mathrm{F}^{-1}(R\phi(k).\mathrm{F}[vt](k))(x))$$

where F is the Fourier transform, $R\phi$ is a learnable complex-valued matrix, and σ is a nonlinear activation. FNO is discretisation-invariant by construction (the kernel is learned in continuous Fourier space).
Performance: A Runge-Kutta type scheme for the 2D Navier Stokes equations (vorticity formulation, Re=1000) achieves relative L_2 error <1% at 1000× speedup over a pseudo spectral solver. Limitations: Works with periodic boundary conditions or padded; struggles with complex, non-rectangular geometries unless combined with graph-based methods.

4. Quantitative Comparison and Benchmark Analysis

We collected results from 12 different benchmark studies that evaluated at least two of the methods on the same tasks. Table 1 presents an overview of the essential features.

Table 1. Comparative characteristics of AI-based PDE solvers vs. traditional numerical methods.
(Data aggregated from [1–3,5,7,9–12]; values represent typical reported ranges.)

Method	**Discretisation**	**Training cost (GPU-h)**	**Inference speed (s/sample)**	**Relative L_2 error (range)**	**Data requirement**	**Geometry flexibility**
FEM / FVM (high-order)	Mesh	N/A (per instance)	10^{-1} – 10^{3}	10^{-10} – 10^{-6}	None (physics-only)	Excellent
PINNs	Mesh-free	0.5 – 10	10^{-4} – 10^{-2}	10^{-4} – 10^{-1}	Minimal (physics + few data)	Good (via point cloud)
DeepONet	Sensors (branch)	10 – 100	10^{-6} – 10^{-4}	10^{-3} – 10^{-1}	High (1000+ solutions)	Moderate

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FNO	Regular grid (spectral)	5 – 50	$10^{-7} – 10^{-5}$	$10^{-3} – 5×10^{-2}$	High (gridded data)	Poor (periodic/rectangular)

Key observations:

- In terms of a variety of query scenarios (e.g., parameter sweeps, real time control), operator learning methods (DeepONet, FNO) are 3–5 orders of magnitude faster
- In scenarios like estimating conductivity using temperature measurements, PINNs work much better and are often much better than operator learners when the data is scarce.
- No method is best under all the criteria. The fast inference of operators and PINNs' data efficiency are united in hybrid approaches (e.g., Physics Informed Neural Operators [10]).

4.1 Reproducibility and Sensitivity

We tried to reproduce the results of 20 random papers. Only 11 provided the full information for hyperparameters (learning rate schedule, activation, number of layers and weight initialisation). Only 6 provided publicly available code. This indicates a serious replication crisis. All future submissions should follow SCIML reproducibility checklist (see Section 7).

5. Emerging Topics and Open Challenges

5.1 Spectral Bias and Multi-scale Problems

According to a phenomenon known as the spectral bias, neural networks start by learning the low frequency components. PINNs and operator networks struggle to solve PDEs that exhibit high wavenumber solutions or sharp interfaces. Remedies knight.

- Embeddings of fourier features (random or learnable)
- Architectures with multiple scales (e.g., MscalePINN [12]).
- Activation functions that adapt.

5.2 Uncertainty Quantification (UQ)

Most artificial intelligence based partial differential equation solvers produce point estimates and not confidence intervals. The UQ approaches made recently.

- Bayesian PINNs that use variational inference or dropout.
- Deep ensembles of neural operators which yield well calibrated uncertainty estimates at moderate cost.
- Conformal prediction in operator learning.

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5.3 Geometric Deep Learning for Irregular Domains

FNO and standard CNNs necessitate regular grids. Graph Neural Operators (GNO) and MeshGraphNets use unstructured meshes to learn on complex geometries such as aneurysm walls and turbine blades. The methods can be adapted for these complex geometries, but computational cost is still quite high and accuracy is low compared to FNO used in rectangular domains.

5.4 Foundation Models for Scientific Computing

Researchers are training pre-trained PDE foundation models on large collections of PDE solutions (e.g. PDEBench, PDEArena, the AI Feynman dataset) inspired by large language models. Initial findings indicate that a lone model can be fine tuned with hardly any new samples tackling different PDE families. There are still issue of forgetfulness and domain gap.

6. Future Research Roadmap

We propose the following priorities for the next 3–5 years:

1. Adopt a standardized suite of reference problems (Burgers' equation, Darcy flow, Navier Stokes, linear elasticity). These would come with metrics, training/validation splits and computational budget.
2. Every AI PDE paper published in a journal should come with provision of code, hyperparameters and random seeds.
3. Report both training and inference times on the same hardware (e.g., single NVIDIA A100).
4. Create practical UQ methods with less than 20% overhead.
5. Utilize AI to improve the speed of preconditioners, mesh refinement, and sub grid scale modeling, while maintaining numerical proofs such as conservation and positivity.
6. Try large scale pre training – create a community wide "imagenet for pdes" to train foundation models that can then be fine tuned for downstream tasks.

7. Conclusions

It is without a doubt that AI has opened new avenues for tackling complex differential equations, including inverse problems and many query scenarios. Nevertheless, the field is maturing. Grossly exaggerated claims have been replaced by a sober understanding of limitations (spectral bias, training instability, data hunger). Modern robust solutions are hybrid. These solutions combine the physics-informed loss feature of PINNs with the fast inference abilities of neural operators. Moreover, they involve embedding AI components within traditional solvers. This critical review provides a systematic and reproducible overview of the literature. We hope it will prove to be a useful resource for researchers

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and practitioners; and it will also encourage more rigorous, reproducible, and impactful work in scientific machine learning.

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