Lattice Reduction Aided- Zero Forcing MIMO Detection using two Stage QR-Decomposition

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***Abstract*— Lattice Reduction Aided (LRA) MIMO Detection algorithms has been widely used in Multiple Input Multiple Output (MIMO) system for achieving better Bit Error Rate (BER) performance. QR- Decomposition (QRD) is a method applied to solve the performance-complexity trade-off issue of the channel matrix in mostly all the MIMO detection algorithms. In this paper, with the proposed two stage QRD method based on Recursive Block QRD (RBQRD), the performance enhancement of the LRA-Zero Forcing (ZF) MIMO detection algorithm in both the AWGN (Additive White Gaussian Noise) and Rician channels, has been investigated for 16- Quadrature Amplitude Modulation (QAM), comparing it with the traditional QRD algorithms.**

Index Terms*— MIMO, Maximum likelihood detection , QAM, AWGN channels, Rician channels

I. INTRODUCTION

MIMO systems has been deployed in a variety of wireless communication fields such as 4G (LTE, LTE-A), Next Generation WLAN networks (IEEE 802.11n/ac), etc. because it has high spectral efficiency. Recently Large MIMO and Massive MIMO systems have gained importance in the existing MIMO technologies. Due to the high density of networks, detection of MIMO systems face serious issues. The trade-off between performance and complexity still exist in the present day networks. Lattice reduction is a method applied in MIMO systems which was originally introduced by Lenstra Lenstra Lovasz (LLL) in 1982. It is a method for factorizing polynomials with rational coefficients [1]. LLL algorithm is well explained in [2]. Complexity and diagonal variants of LLL algorithm was also proposed and the performance improvement was made [2, 3]. Various other reduction techniques such as Hermite-Korkine-Zolotareff (HKZ) and Minkowski reduction algorithms are also followed for Lattice reduction [4]. In MIMO the channel is usually faced with the least squares problem before Lattice reduction. Normal equations, QRD and Singular Value Decomposition (SVD) are the methods adopted to overcome least squares problem [5]. QRD methods has been followed to avoid least squares problem in a full-rank matrix. Most of the techniques used in the literature are based on Gram-Schmidt (GS), Modified GS (MGS), Householder QRD (House QRD) and Givens Rotation-QRD (GR-QRD). In this paper, LRA MIMO detection based on different two stage QRD is performed. The first stage considers MGS, House QRD, GR-QRD, RBQRD before lattice reduction and the second stage takes into account GR-QRD inside LLL algorithm of lattice reduction. Their performance are compared in AWGN and indoor Rician channel models, Task Group ac (TGac -channels B, C and D) using 16-QAM.

The remainder of the paper is organized as follows. In Section
II the system model for MIMO in AWGN and indoor channel is introduced. Section III gives a review of several QR decomposition algorithms. Simulation results are given in Section IV. Conclusion will be drawn in Section V.

II. SYSTEM MODEL

In this section system model for two separate channel under flat fading and indoor fading is considered for the performance evaluation of LRA MIMO detection.

A. System with Channel Model under flat fading

In this model AWGN channel is considered in which a symbol synchronized MIMO system with *M* transmit and *N* receive antennas is considered. This means *M* independent signals are multiplexed at the transmitter end and demultiplexed at *N* receivers, this, is known as spatial multiplexing. If we group all signals into vectors then the system can be viewed as transmitting an $M\times 1$ vectors through an $N\times M$ channel matrix *H*. Also, each receiver will have its own noise source (assumed Gaussian). Thus, the overall baseband system model can be mathematically represented as

$$y = Hs + n \qquad (1)$$

where s is $N\times 1$ received vector and n is $N\times 1$ noise vector. The (i,j)th element, h_{ij}, of the matrix H denotes the complex channel response from jth transmit antenna to ith receive antenna, n is zero mean gaussian vector with a covariance matrix of $R_n = E\{nn^*\} = \sigma^2 n\, I$. The objective of the MIMO detection algorithm is to compute an estimate such that

$$\hat{s} = \arg\min_{s\in\Lambda^C} \|y - Hs\|^2 \qquad (2)$$

where Λ is the set of complex entries from the QAM constellation and C is the size of QAM constellation.

B. System with Channel Model under Indoor fading

This model considers a MIMO indoor channel. For this the traditional kronecker MIMO model which deals with the indoor Rician fading is taken into account. The kronecker MIMO channel is given as [6],

$$H_k = {R_{tx}}^{1/2} H_w {R_{rx}}^{1/2} \quad (3)$$

where the elements of H_w are independent and identically distributed as circular symmetric complex Gaussian with zero-mean and unit variance. R_{tx} and R_{rx} are the transmitter and receiver correlation matrices given by the kronecker matrix

$$R = {R_{tx}}^{1/2} \otimes {R_{rx}}^{1/2} \quad (4)$$

For a 2×2 channel matrix the matrices R, R_{tx} and R_{rx} is given by

$$R = \begin{bmatrix} 1 & r_1 & t_1 & s_1 \\ r_1^* & 1 & s_2 & t_2 \\ t_1^* & s_2^* & 1 & r_2 \\ s_1^* & t_2^* & r_2^* & 1 \end{bmatrix} \quad (5)$$

$$R_{tx} = \begin{bmatrix} 1 & t \\ t^* & 1 \end{bmatrix}, \quad R_{rx} = \begin{bmatrix} 1 & r \\ r^* & 1 \end{bmatrix} \quad (6)$$

where the elements *r*,*t* and *s* represents receive, transmit and cross correlation respectively. The estimate $\hat{s}$ is now computed using the kronecker model of the Equation (3). So using Equation (2) the estimate $\hat{s}$ is given as

$$\hat{s} = \arg\min_{s \in \Lambda^C} \|y - H_k s\|^2 \quad (7)$$

Using the kronecker channel models, the indoor channel models for IEEE 802.11n/ac were obtained under the TGac amendment [7]. Channels (B, C and D) were defined [8] for different environment using Rician fading and are shown in Table I.

III. QR DECOMPOSITION

Nowadays most of the MIMO detection methods are combined with QRD [9]. In this paper also we apply various algorithms of QRD to SD.

TABLE I. TGAC CHANNELS

Parameters	B	C	D
Avg. 1st wall dist.(m)	5	5	10
RMS delay spread (ns)	15	30	50
Maximum delay (ns)	80	200	390
Number of taps	9	14	18
Number of clusters	2	2	3
K (dB) LOS/NLOS	0/-∞	0/-∞	3/-∞

QRD is a technique applied to computationally solve the channel matrix rather than the traditional methods, which has huge errors if the matrix involved is the ill conditioned. The standard solution is to make the QR factorization or QRD.

$$H = QR \quad (8)$$

where Q is the orthogonal matrix and R is the upper triangular matrix. Using Equation (7), the received signal can be modified as,

$$\begin{aligned} \hat{y} = [y_1, y_2, \ldots\ldots y_N] &= Q^H y \\ &= Q^H QRs + Q^H n \\ \hat{y} &= Rs + Q^H n \end{aligned} \quad (9)$$

$$= \begin{bmatrix} r_{1,1} & \cdots & r_{1,N-1} & r_{1,N} \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & r_{N,N} \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_{N-1} \\ s_N \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_{N-1} \\ n_N \end{bmatrix} \quad (10)$$

The main idea of QRD is not only to keep the channel orthogonal but also can simplify the procedure of signal processing. The basic algorithms used in QRD are GS, MGS, Householder transformation and GR. We also use some of the second variant of the existing algorithms, RBQRD and compare the performance of LRA-ZF MIMO detection both in AWGN and indoor channel models (channel B, C and D).

A. *QRD algorithms*

There are several QRD algorithms available for channel matrix factorization. Algorithms which are fast and numerically stable perform better compared to other algorithms. Table II. gives some comparison of QRD algorithms in terms of improvements, drawbacks and complexity.

Taking into consideration the large deployment of MIMO networks and sacrificing the complexity aspects we would consider analyzing the performance of our proposed two stage QRD namely RBQRD with GR-QRD in a MIMO scenario (2×2) with 16-QAM constellation, comparing it with the other traditional QRD algorithms. Hence some insights of RBQRD are discussed in the next section.

TABLE II. IMPROVEMENT AND DRAWBACKS OF VARIOUS QRD ALGORITHMS

Algorithm	Improvement and drawbacks	Complexity
Modified Gram Schmidt [10]	Performs well but not numerically stable	$2mn^2$
House Holder transformation [10]	Requires fewer operations than GS and more stable than GS	$n^3/3$
Givens Rotation [11]	Numerical properties are closer to Householder but not efficient than Householder transformation	$3mn^2 - n^3$
Recursive Block QRD [12]	Numerically stable for large block size and more number of streams especially in dense networks. Additional complexity is introduced when the number of antennas/streams increases	$2nlog_{\frac{N}{k}}n + 2n^3 + 19$

B. *Recursive Block QRD (RBQRD) algorithm*

RBQRD is the most flexible approach of QRD which uses recursion method of decomposing the block of channels H_1, H_2 especially in heterogeneous networks [12]. According to the block QRD, if $H \in \mathbb{R}^{m\times n}$ the thin QR factorization of the channel matrix H can be given as

$$[H_1 |\, H_2] = [Q_1 |\, Q_2] \begin{bmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{bmatrix} \quad (11)$$

From the Equation (10), $Q_1 R_{11} = H_1, R_{12} = Q_1^T H_2, Q_2 R_{22} = H_2 - Q_1 R_{12}$ the Algorithm 1 for the RBQRD is obtained.

Algorithm 1: RBQRD algorithm

Initialization:
- Initialize n_b the blocking parameter
- Get the size of H and obtain n

Thin QR decomposition:
if $n < n_b$
- Perform thin QR factorization

else
Recursive Block QRD:
- Split H in half
- $n_1 = n/2$, round off n_1
- Find QR of left half of the H

$[Q_1, R_{11}] = BlockQR(H(:,1{:}n_1), n_b)$
- Find QR of the modified right half of the H

$R_{12} = Q_1^T * H(:, n_1 + 1{:}n)$
$H(:, n_1 + 1{:}n) = H(:, n_1 + 1{:}n) - Q_1 * R_{12}$
$[Q_2], R_{22}] = BlockQR(H(:, n_1 + 1{:}n), nb)$
- Synthesize Q and R from the results of H_1 and H_2

$Q = [Q_1\ Q_2]$
$R = [R_{11}\ R_{12};\ zeros(n - n_1, n_1)\ R_{22}]$
end

Using the Algorithm 1 we perform RBQRD in first stage before LRA-ZF MIMO detection and compare it with other QRD algorithms both in AWGN and TGac channels.

IV. Proposed LRA - ZF MIMO Detection

A brief overview of lattice reduction and the algorithms used in LRA MIMO detectors are presented in this section. Consider a lattice L as

$$L \triangleq \{s : s_i \in \mathcal{A}, 1 \le i \le 2n_t\} \quad (12)$$

and

$$\mathcal{Z} \triangleq \{z : z = Hs, s_i \in \mathcal{A}, 1 \le i \le 2n_t\} \quad (13)$$

In Equation (12) and (13) L is the transmit lattice, $\mathcal{Z}$ is the received lattice with noise and n_t is the number of transmit antennas. The channel matrix can be reduced in LRA MIMO detectors with the well known LLL algorithms [13]. Various LLL algorithms such as original LLL, complex LLL and MLLL had been used for LRA MIMO detection. We used an original LLL for the proposed LRA-ZF MIMO detection in this paper. In our proposed design shown in Fig 1. for a larger block size the channel is factorized using RBQRD and then fed to the lattice reduction phase which uses GR-QRD. From the algorithm, we obtain $\widetilde{H} = H\mathcal{U}$, where $\mathcal{U}$ is the uni-modular matrix. The system model of Equation (1) now changes to

$$y = \widetilde{H}x + n \quad (14)$$

where $x = \mathcal{U}^{-1}s$. the estimate of s obtained using LRA-ZF MIMO detection denoted as $\hat{s}_{LR-ZF}$ is given as

$$\hat{s}_{LR-ZF} = \mathcal{Q}\left(\mathcal{U}\,\mathcal{Q}\left([\widetilde{H}^T\widetilde{H}]^{-1}\,\widetilde{H}^T y\right)\right) \quad (15)$$

In Equation (15), $\mathcal{Q}\,(.)$ is the nearest neighbor quantization operation.

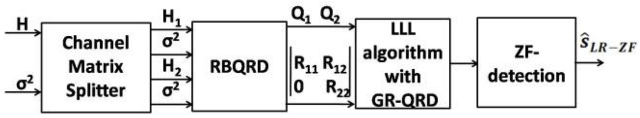

Fig .1. LRA-ZF MIMO detector with two stage-QRD

V. Simulation Results

A 16-QAM MIMO (2×2) is considered in our simulations. MATLAB was considered for the simulation purposes. The simulation parameters used are shown in Table III. The BER performance of LRA-ZF MIMO detection using MGS, GR and RBQRD in AWGN and TGac channels (B,C and D) are shown in Fig 2, 3 and 4. Clearly from the figures the BER performance for LRA-ZF MIMO detection is almost the same at low and medium SNR's. At high SNR (> 25dB), RBQRD has the best performance compared to other two algorithms. We can also

TABLE III. Simulation Parameters

Parameters	Value
Number of symbols	1000
Number of antennas (TX ×RX)	2×2
Number of users	1
Bandwidth	20 MHz
Channel Type	AWGN, TGac (B C and D)
Mapper schemes	16-QAM
MIMO detection	LRA-ZF
Blocking parameter for RBQRD	3

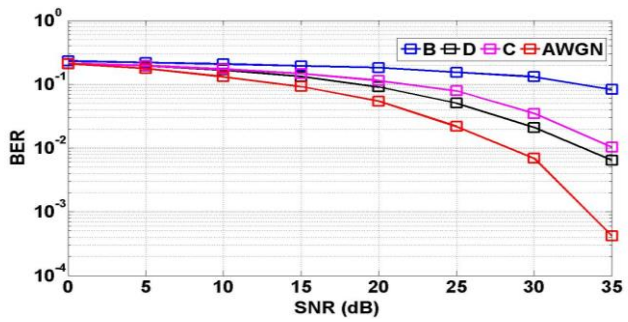

Fig .2. Performance of LRA-ZF MIMO detection for a 16-QAM MIMO (2×2) using MGS in first stage for an AWGN and TGac channels

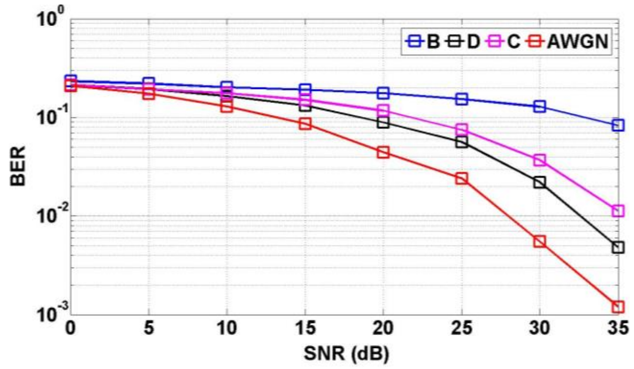

Fig 3: Performance of LRA-ZF MIMO detection for a 16-QAM MIMO (2×2) using GR-QRD in first stage for an AWGN and TGac channels

infer that LRA- ZF MIMO detection performs best in AWGN mode when compared to TGac channel. Among the TGac channels, performance of LRA-ZF MIMO detection using QRD is better in channel D rather than channel B and C. Hence using various other QRD algorithms the BER performance of LRA-ZF MIMO detection is compared for the same 16-QAM MIMO (2×2) scenario in AWGN and channel D (Fig. 5 and 6). Due to indoor fading of channel D, LRA-ZF MIMO detection performed well in AWGN channel. We can also view that the other QRD algorithms except the Householder QRD method (sub-optimal), performed even better than the optimal ML detection approach in medium and high SNR regime.

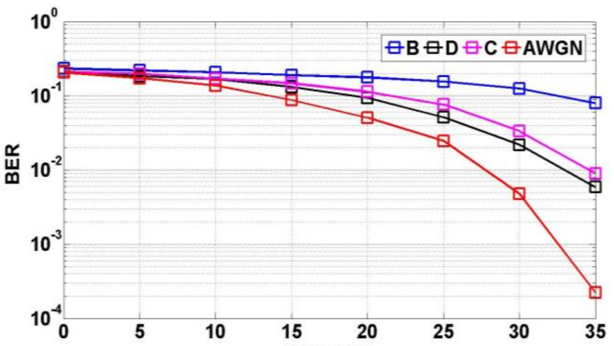

Fig 4. Performance of LRA-ZF MIMO detection for a 16-QAM MIMO (2×2) using RBQRD in first stage for an AWGN and TGac channels

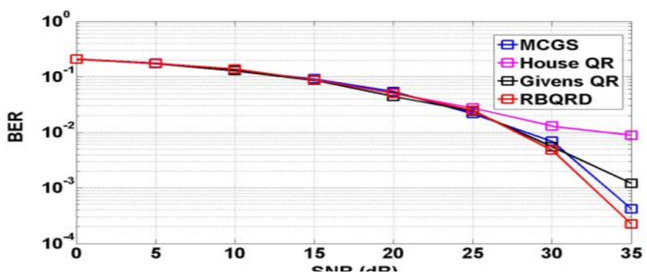

Fig 5. Performance of LRA-ZF MIMO detection for a 16-QAM MIMO (2×2) in AWGN channel

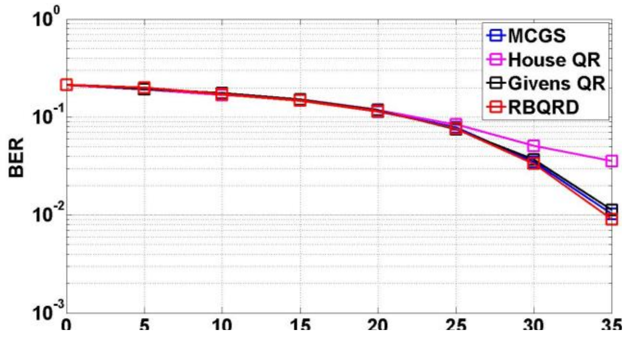

Fig 6: Performance of LRA-ZF MIMO detection for a 16-QAM MIMO (2×2) in Tgac Channel D

VI. CONCLUSION

LRA-ZF MIMO detection with the variants of two-stage QRD algorithms were presented. Performance in terms of BER at low, medium and high SNR regime were considered with the AWGN and indoor channel variation. Results have shown that the first stage CGS, Householder transformation and RBQRD algorithms with a second stage of GR-QRD provide better results with the AWGN. In addition with the indoor channel variation we have shown that the performance of LRA-ZF MIMO detection using two-stage QRD with the first stage as RBQRD is better in channel D next to AWGN rather than B and C. As a future work the complexity of the QRD algorithms with the indoor channel variation in LRA-ZF MIMO detection will be studied.

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