Laser Pulse in Micropolar Thermoelastic Medium with Three Phase Lag Model

Arvind Kumar
Department of Mathematics
Punjab Technical University,
Kapurthala (India)

Abstract- **The present investigation deals with the deformation in micropolar generalized thermoelastic medium with three phase lag subjected to thermomechanical loading due to thermal laser pulse. Normal mode analysis technique is used to solve the problem. Concentrated normal force and thermal source are taken to illustrate the utility of approach. The closed form expressions of normal stress, tangential stress, couple-stress, and temperature distribution are obtained.. Some particular cases of interest are deduced from the present investigation.**

Keywords— Micropolar Thermoelastic, Three phase lag, Pulse Laser, concentrated normal force and concentrated thermal source.

I. INTRODUCTION

Modern engineering structures are often made up of materials possessing internal structure. Polycrystalline materials, materials with fibrous or coarse grain structure come in this category. Classical theory of elasticity is inadequate to represent the behavior of such materials. The analysis of such materials requires incorporating the theory of oriented media. The linear theory of micropolar elasticity was developed by Eringen [1]. A micropolar continuum is a collection of interconnected particles in the form of small rigid bodies undergoing both translational and rotational motions. Typical examples of such materials are granular media and multi-molecular bodies whose microstructures act as an evident part in their macroscopic responses. Rigid chopped fibers, elastic solids with rigid granular inclusions and other industrial materials such as liquid crystals are examples of such materials.

The generalized theory of thermoelasticity is one of the modified versions of classical uncoupled and coupled theory of thermoelasticity and has been developed in order to remove the paradox of physical impossible phenomena of infinite velocity of thermal signals in the classical coupled thermoelasticity. Hetnarski and Ignaczak [4] examined five generalizations of the coupled theory of thermoelasticity. The first generalization is due to Lord and Shulman [2] who formulated the generalized thermoelasticity theory involving one thermal relaxation time. Green and Lindsay [3] developed a temperature rate-dependent thermoelasticity that includes two thermal relaxation times. One can refer to Hetnarski and Ignaczak [4] for a review and presentation of generalized theories of thermoelasticity.

The third generalization of the coupled theory of thermoelasticity is developed by Hetnarski and Ignaczak and is known as low temperature thermoelasticity. The fourth generalization to the coupled theory of thermoelasticity introduced by Green and Nagdhi [5] and this theory is concerned with the thermoelasticity theory without energy dissipation. The fifth generalization to the coupled theory of thermoelasticity is developed by Tzou [6] and Chanderashekhariah [7] and is referred to dual phase-lag thermoelasticity. He introduced two phase lags to both the heat flux vector and the temperature gradient and considered constitutive equations to describe the lagging behavior in the heat conduction in solids. Roychoudhuri [8] has recently introduced the three-phase-lag heat conduction equation in which the Fourier law of heat conduction is replaced by an approximation to a modification of the Fourier law with the introduction of three different phase-lags for the heat flux vector, the temperature gradient and the thermal displacement gradient. The stability of the three-phase-lag heat conduction equation is discussed by Quintanilla and Racke [9]. Quintanilla has studied the spatial behavior of solutions of the three-phase-lag heat conduction equation.

Laser technology has a vital application in nondestructive materials testing and evaluation. When a solid is heated with a laser pulse, it absorbs some energy which results in an increase in localized temperature. This cause thermal expansion and generation of the ultrasonic waves in the material. The irradiation of the surface of a solid by pulsed laser light generates wave motion in the solid material. There are generally two mechanisms for such wave generation, depending on the energy density deposited by the laser pulse. At high energy density, a thin surface layer of the solid material melts, followed by an ablation process whereby particles fly off the surface, thus giving rise to forces that generates ultrasonic waves. At low energy density, the surface material does not melt, but it expands at a high rate and wave and wave motion is generated due to thermoelastic processes.

Very rapid thermal processes (e.g., the thermal shock due to exposure to an ultra-short laser pulse) are interesting from the stand point of thermoelasticity, since they require a coupled analysis of the temperature and deformation fields. A thermal shock induces very rapid movement in the structural elements, giving the rise to very significant inertial forces, and thereby, an increase in vibration. Rapidly oscillating contraction and expansion generates temperature changes in materials susceptible to diffusion of heat by conduction [10]. This mechanism has attracted considerable attention due to the extensive use of pulsed laser technologies in material processing and non-destructive testing and characterization [11, 12]. The so-called ultra short lasers are those with pulse durations ranging from nanoseconds to femto seconds. In the case of ultra short pulsed laser heating, the high intensity energy flux and ultra short duration lead to a very large thermal gradients or ultra-high heating may exist at the boundaries. In such cases, as pointed out by many investigators, the classical Fourier model, which leads to an infinite propagation speed of the thermal energy, is no longer valid [13]. Researchers have proposed several models to describe the mechanism of heat conduction during short-pulse laser heating, such as the parabolic one-step model [14], the hyperbolic one-step model [15], and the parabolic two-step and hyperbolic two-step models [16, 17]. It has been found that usually the microscopic two-step models, i.e., parabolic and hyperbolic two-step models, are useful for thin films. Simulation on laser ultrasound wave form in non-metallic materials was discussed by Wang et al [18].

Scruby et al. [19] considered the point source model to study the ultrasonic generation by lasers. He studied the heated surface by laser pulse irradiation in the thermoelastic system as a surface center of expansion (SCOE). He also discussed the applications of laser technology in flaw detection and acoustic microscopy. Rose [20] later presented a more exact mathematical basis. Point source model explain main features of laser-generated ultrasound waves but this model fails to explain precursor in epicenter waves. Later introducing the thermal diffusion McDonald [21] and Spicer [22] proposed a new model known as laser-generated ultrasound model. This model reported excellent agreement between theory and experiment for metal materials. But due to the optical penetration effect, this model cannot be applied to the study of laser-generated ultrasound in non-metallic material directly. The optical absorption occurs at the surface layer in metallic materials, and the heat penetration is resulted due to heat diffusion. In non-metallic materials, the laser beam can penetrate the specimen to some finite depth and induced a buried bulk- thermal source, so the features of the laser-generated ultrasound will be significantly different from that in metallic materials.

Dubois [23] experimentally demonstrated that penetration depth play a very important role in the laser-ultrasound generation process. Ezzat et al. [24] discussed the thermo-elastic behavior in metal films by fractional ultrafast laser. Al-Huniti and Al-Nimr [25] investigated the thermoelastic behavior of a composite slab under a rapid dual-phase lag heating. The comparison of one-dimensional and two-dimensional axisymmetric approaches to the thermomechanical response caused by ultrashort laser heating was studied by Chen et al. [26]. Kim et al. [27] studied thermoelastic stresses in a bonded layer due to pulsed laser radiation. Thermoelastic material response due to laser pulse heating in context of four theorems of thermoelasticity was discussed by Youssef and Al-Bary [28]. Theoretical study of the effect of enamel parameters on laser induced surface acoustic waves in human incisor was studied by Yuan et al [29]. A two- dimensional generalized thermoelastic diffusion problem for a thick plate under the effect of laser pulse thermal heating was studied by Elhagary [30].

In this research, taking into account the radiation of ultra short laser, we have established a model for micropolar thermoelastic medium with three phase lag model. The stress components and temperature distribution have been computed numerically. The resulting expressions are then applied to the problem of a micropolar thermoelastic three phase lag medium whose boundary is subjected to two types of loads i.e. mechanical load and thermal load. The resulting quantities are shown graphically to show the effect of laser irradiation

II. BASIC EQUATION

Following Roychoudhuri [8], the basic equations in a homogeneous, isotropic micropolar generalized thermoelastic medium with three phase lag model in the absence of body forces and body couples are given by:

$$(\lambda+\mu)\nabla(\nabla.\boldsymbol{u})+(\mu+K)\nabla^2\boldsymbol{u}+K\nabla\times\boldsymbol{\phi}-\beta_1\nabla T=\rho\ddot{\boldsymbol{u}} \quad , \quad (1.1)$$

$$(\gamma\nabla^2-2K)\boldsymbol{\phi}+(\alpha+\beta)\nabla(\nabla.\boldsymbol{\phi})+K\nabla\times\boldsymbol{u}=\rho j\ddot{\boldsymbol{\phi}}, \quad (1.2)$$

$$\left[K^*\left(1+\tau_v\frac{\partial}{\partial t}\right)+K_1\frac{\partial}{\partial T}\left(1+\tau_t\frac{\partial}{\partial t}\right)\right]\nabla^2 T=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q{}^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\rho C_E\ddot{T}+\gamma T_0\ddot{e}_{kk}-\rho\dot{Q}\right), \quad (1.3)$$

$$t_{ij}=\left(\lambda_0\phi^*+\lambda u_{r,r}\right)\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right)+K\left(u_{j,i}-\epsilon_{ijk}\phi_k\right)-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\delta_{ij}T, \quad (1.4)$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}+b_0\epsilon_{mji}\phi^*_{,m}, \quad (1.5)$$

The plate surface is illuminated by laser pulse given by the heat input

$$Q=I_0 f(t)g(x_1)h(x_3) \quad (1.6)$$

Where, I_0 is the energy absorbed. The temporal profile $f(t)$ is represented as,

$$f(t)=\frac{t}{t_0^2}e^{-\left(\frac{t}{t_0}\right)} \quad (1.7)$$

Here t_0 is the pulse rise time. The pulse is also assumed to have a Gaussian spatial profile in x_1

$$g(x)=\frac{1}{2\pi r^2}e^{-\left(\frac{x_1^2}{r^2}\right)} \quad (1.8)$$

where r is the beam radius, and as a function of the depth x_3 the heat deposition due to the laser pulse is assumed to decay exponentially within the solid,

$$h(x_3)=\gamma^* e^{-\gamma^* x_3} \quad (1.9)$$

Equation (1.7a) with the aid of (1.7b, 1.7c and 1.7d) takes the form

$$Q=\frac{I_0\gamma^*}{2\pi r^2 t_0^2}te^{-\left(\frac{t}{t_0}\right)}e^{-\left(\frac{x_1^2}{r^2}\right)}e^{-\gamma^* x_3} \quad , \quad (1.10)$$

Here λ, μ, α, β, γ, K, are material constants, ρ is mass density, $\boldsymbol{u}=(u_1,u_2,u_3)$ is the displacement vector and $\boldsymbol{\phi}=(\phi_1,\phi_2,\phi_3)$ is the microrotation vector, T is temperature and T_0 is the reference temperature of the body chosen, t_{ij} are components of stress, m_{ij} are components of couple stress, e_{ij} are components of strain, e_{kk} is the dilatation, δ_{ij} is Kroneker delta function, τ^0, τ^1 are the diffusion relaxation times and τ_0, $\tau_1$are thermal relaxation times with $\tau_0 \geq \tau_1 \geq 0$. Here $\tau^0 = \tau^1 = \tau_0 = \tau_1 = \gamma_1 = 0$ for Coupled Thermoelastic theory (CT) model. $\tau_1 = \tau^1 = 0, \epsilon = 1,\ \gamma_1 = \tau_0$ For Lord-Shulman (LS) model and $\varepsilon = 0$, $\gamma_1 = \tau^0$ where $\tau^0 > 0$ for Green-Lindsay (GL) model.

In the above equations symbol (",") followed by a suffix denotes differentiation with respect to spatial coordinates and a superposed dot ("˙") denotes the derivative with respect to time respectively.

III. FORMULATION OF THE PROBLEM

We consider a micropolar generalized thermoelastic solid with rectangular Cartesian coordinate system $OX_1X_2X_3$ having origin on x_3-axis with x_3-axis pointing vertically downward the medium. A normal force/thermal source is assumed to acting on the origin of the rectangular Cartesian co-ordinate system.

If we restrict our problem for plane strain parallel to x_1x_3-plane with

$$\boldsymbol{u} = (u_1, 0, u_3), \boldsymbol{\phi} = (0, \phi_2, 0), \qquad (2.1)$$

Then the field equations in micropolar generalized thermoelastic solid in the absence of body forces and body couples the equations of motion can be written as:

$$(\lambda+\mu)\frac{\partial e}{\partial x_1} + (\mu+K)\nabla^2 u_1 - K\frac{\partial \phi_2}{\partial x_3} - \beta_1\frac{\partial T}{\partial x_1} = \rho\ddot{u}_1 \qquad (2.2)$$

$$(\lambda+\mu)\frac{\partial e}{\partial x_3} + (\mu+K)\nabla^2 u_3 + K\frac{\partial \phi_2}{\partial x_1} - \beta_1\frac{\partial T}{\partial x_3} = \rho\ddot{u}_3 \qquad (2.3)$$

$$K\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) + \gamma\nabla^2\phi_2 - 2K\phi_2 = j\rho\ddot{\phi}_2 \qquad (2.4)$$

$$\left[K^*\left(1+\tau_v\frac{\partial}{\partial t}\right) + K_1\frac{\partial}{\partial T}\left(1+\tau_t\frac{\partial}{\partial t}\right)\right]\nabla^2 T = \left(1+\tau_q\frac{\partial}{\partial t} + \frac{\tau_q{}^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\rho C_E\ddot{T} + \gamma T_0\ddot{e}_{kk} - \rho\dot{Q}\right) \qquad (2.5)$$

For further consideration it is convenient to introduce in equations (1.1)-(1.5) the dimensionless quantities defined as:

$$(x_1', x_3') = \frac{\omega^*}{c_1}(x_1, x_3), (u_1', u_3') = \frac{\omega^*}{c_1}(u_1, u_3), (t', \tau_0', v') = \omega^*(t, \tau_0, v), T' = \frac{\beta_1 T}{\rho c_1^2}, \tau_{ij}' = \frac{\tau_{ij}}{\beta_1 T_0}, \phi_2' = \frac{\rho c_1^2}{\beta_1 T_0}\phi_2,$$
$$m_{ij}' = \frac{\omega^*}{\rho c_1^3}m_{ij}, Q' = \frac{\beta_1^2}{\rho c_1^2}Q, \ \omega^* = \frac{\rho C^* c_1^2}{K^*}, \gamma_1 = (3\lambda+2\mu+K)\alpha_t, c_1^2 = \frac{(\lambda+2\mu+K)}{\rho} \qquad (2.6)$$

Using (2.6), the equations (2.2)-(2.5) reduce to:

$$a_1\frac{\partial e}{\partial x_1} + a_2\nabla^2 u_1 - a_3\frac{\partial \phi_2}{\partial x_3} - \beta_1\frac{\partial T}{\partial x_1} = \rho\ddot{u}_1 \qquad (2.7)$$

$$a_1\frac{\partial e}{\partial x_3} + a_2\nabla^2 u_3 + a_3\frac{\partial \phi_2}{\partial x_1} - \beta_1\frac{\partial T}{\partial x_3} = \rho\ddot{u}_3 \qquad (2.8)$$

$$a_8\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) + a_6\nabla^2\phi_2 - 2a_3\phi_2 = \ddot{\phi}_2 \qquad (2.9)$$

$$\left[K^*\left(1+\tau_v\frac{\partial}{\partial t}\right) + K_1\omega^*\frac{\partial}{\partial T}\left(1+\tau_t\frac{\partial}{\partial t}\right)\right]\nabla^2 T = \left(1+\tau_q\frac{\partial}{\partial t} + \frac{\tau_q{}^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(a_{13}\ddot{T} + a_{14}\ddot{e}_{kk} - a_{15}\dot{Q}\right) \qquad (2.10)$$

Using the potential functions ϕ and ψ as:

$$u_1 = \frac{\partial\phi}{\partial x_1} + \frac{\partial\psi}{\partial x_3}, \quad u_3 = \frac{\partial\phi}{\partial x_3} - \frac{\partial\psi}{\partial x_1}, \qquad (2.11)$$

The equations (2.7)-(2.10) reduce to:

$$\nabla^2\phi - \ddot{\phi} - T = 0, \qquad (2.12)$$

$$\left[\left[K^*\left(1+\tau_v\frac{\partial}{\partial t}\right) + K_1\omega^*\frac{\partial}{\partial T}\left(1+\tau_t\frac{\partial}{\partial t}\right)\right]\nabla^2 - a_{13}\left(1+\tau_q\frac{\partial}{\partial t} + \frac{\tau_q{}^2}{2}\frac{\partial^2}{\partial t^2}\right)\right]T - a_{14}\frac{\partial^2}{\partial t^2}\left(1+\tau_q\frac{\partial}{\partial t} + \frac{\tau_q{}^2}{2}\frac{\partial^2}{\partial t^2}\right)\nabla^2\phi = a_{15}\left(\frac{\partial}{\partial t} + \tau_q\frac{\partial^2}{\partial t^2} + \frac{\tau_q{}^2}{2}\frac{\partial^3}{\partial t^3}\right)te^{-\left(\frac{t}{t_0}\right)}e^{-\left(\frac{x_1^2}{r^2}\right)}e^{-\gamma^* x_3}, \qquad (2.13)$$

$$a_2\nabla^2\psi - \ddot{\psi} + a_3\phi_2 = 0, \qquad (2.14)$$

$$a_6\nabla^2\phi_2 - 2a_3\phi_2 - a_8\nabla^2\psi = \ddot{\phi}_2, \qquad (2.15)$$

Where,

$a_1 = \frac{\lambda+\mu}{\rho c_1^2}, a_2 = \frac{K+\mu}{\rho c_1^2}, a_3 = \frac{K}{\rho j\omega^{*2}}, a_6 = \frac{\gamma}{j\rho c_1^2}, a_1 = \frac{K}{\rho c_1^2}, a_{13} = \rho c_1^2 C_E, a_{14} = \beta_1 T_0\gamma, a_{15} = \frac{\rho c_1^2}{\beta_1\omega^*}$

IV. SOLUTION OF THE PROBLEM

The solution of the considered physical variables can be decomposed in terms of the normal modes as in the following form:

$$\{\phi, \psi, T, \phi_2, C\}(x_1, x_3, t) = \{\bar{\phi}, \bar{\psi}, \bar{T}, \overline{\phi_2}, \bar{C}\}(x_3)e^{i(kx_1-\omega t)} \qquad (3.1)$$

Here ω is the angular velocity and k is wave number. After some simplifications the general solution of the above system satisfying the radiation conditions that $(\hat{\phi}, \hat{T}, \widehat{\phi_2}, \hat{\psi}) \to 0$ as $x_3 \to \infty$ are given as following:

$$\hat{\phi} = B_1 e^{-m_1 x_3} + B_2 e^{-m_2 x_3} + B_3 e^{-m_3 x_3} + L_1 e^{-\gamma^* x_3} \qquad (3.9)$$

$$\hat{T} = d_1 B_1 e^{-m_1 x_3} + d_2 B_2 e^{-m_2 x_3} + d_3 B_3 e^{-m_3 x_3} + L_2 e^{-\gamma^* x_3} \qquad (3.10)$$

$$\hat{\psi} = B_4 e^{-m_4 x_3} + B_5 e^{-m_5 x_3} \qquad (3.11)$$

$$\widehat{\phi_2} = e_4 B_4 e^{-m_4 x_3} + e_5 B_5 e^{-m_5 x_3} \qquad (3.12)$$

V. BOUNDARY CONDITIONS

We consider concentrated normal force and concentrated thermal source at the boundary surface $x_3 = 0$, mathematically, these can be written as:

$$t_{33} = -F_1, t_{31} = 0 , m_{32} = 0, T = F_2 , \quad (4.1)$$

where F_1 is the magnitude of the applied force and F_2 is the constant temperature applied on the boundary.

Case 1: for the normal force: $F_2 = 0$

Case 2: for the thermal source: $F_1 = 0$

Substituting the values of $\hat{\phi}, \widehat{\phi^*}, \hat{T}, \hat{\psi}, \widehat{\phi_2}$ from the equations (3.9)-(3.12) in the boundary condition (4.1) and using (1.4)-(1.5), (2.1), (2.11), (3.1) and solving the resulting equations, we obtain:

$$\widehat{t_{33}} = \sum_{i=1}^{4} G_{1i} e^{-m_i x_3} + M_1 e^{-\gamma^* x_3} \quad (4.3)$$

$$\widehat{t_{31}} = \sum_{i=1}^{4} G_{2i} e^{-m_i x_3} + M_2 e^{-\gamma^* x_3} \quad (4.4)$$

$$\widehat{m_{32}} = \sum_{i=1}^{4} G_{3i} e^{-m_i x_3} + M_3 e^{-\gamma^* x_3} \quad (4.5)$$

$$\hat{T} = \sum_{i=1}^{4} G_{4i} e^{-m_i x_3} + M_4 e^{-\gamma^* x_3}, \quad (4.6)$$

VI. SPECIAL CASES

A. Micropolar Thermoelastic Solid

In absence of three phase lag effect in Equations (4.3) - (4.7), we obtain the corresponding expressions of stresses, displacements and temperature for micropolar generalized thermoelastic half space.

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