Laminar MHD Convection In A Vertical Channel

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ABSTRACT

The fully developed laminar and MHD convection in a vertical channel with uniform and asymmetric temperatures has been studied.

The governing equations (momentum and energy balanced equations) has been written in a dimensionless form. A perturbation method has been employed to obtain the solution of the dimensionless velocity u and temperature θ in terms of the perturbation parameter$E$.

It has been pointed out that the effect of magnetic parameter enhances the effect of flow in the downward direction while it lowers this effect in the upward flow.

Keywords: MHD, Laminar, Convection.

1. INTRODUCTION

The study of magneto hydrodynamic (MHD) flows with viscous dissipation and buoyancy effects has many importance in industrial, technological and geothermal applications such as temperature plasmas, cooling of nuclear reactors, liquid metals etc.

A comprehensive review of the study of MHD flows in relations to the applications to the above areas has been made by several authors.

Mixed convection in a vertical parallel – plate channel with uniform wall temperatures has been studied by Tao (1960). Javerci (1975) dealt with the effects of viscous dissipation and Joule heating on the fully developed MHD flow with heat transfer in a channel. The exact solution of the energy equation was derived for constant heat flux with small magnetic Reynolds number.

Sparrow et al (1984) observed experimentally flow reversal in pure convection in a one – sided heated vertical channel. The mixed convection in a vertical parallel – plate channel with asymmetric heating, where one plate is heated and the other is adiabatic has been studied by Habchi and Acharya (1986).

Gav et al (1992) showed experimentally for mixed convection that the increasing buoyancy parameter will make the reversal flow wider and deeper taking into consideration the effects of viscous dissipation.

Barletta (1998) demonstrated that in the case of upward flow reversal decreases with viscous dissipation. However, in the case of downward flow, the flow reversal increases with viscous dissipation. He also calculated criteria for the onset of flow reversal for both directions of flow, but without taking viscous dissipation into account.

Barletta (1999) investigated the fully developed and laminar convection in a parallel – plate channel by taking into account both viscous dissipation and buoyancy. Uniform and asymmetric temperatures are prescribed at the channel walls. The velocity field is considered as parallel. A perturbation method is employed to solve the momentum balance equation and energy balance equation. A comparison between the velocity and temperature profiles in the case of laminar forced convection with viscous dissipation is performed in order to point out the effect of buoyancy. The case of convective boundary conditions is also discussed.

Attia (2006) studied the effect of variable viscosity in the transient couette flow of dusty fluid with heat transfer between parallel plates. The fluid is acted upon by a constant pressure gradient and external uniform magnetic field is applied perpendicular to the plates.

Alim, et al (2007) investigated the pressure work and viscous dissipation effects on MHD natural convection along a sphere. The laminar natural convection flow from a sphere immersed in a viscous incompressible fluid in the presence of magnetic field has been considered in this investigation.

Ranuka et al (2009) studied the MHD effects of unsteady heat convective mass transfer flow past an infinite vertical porous plate with variable suction, where the plate's temperature oscillates with the same frequency as that of variable suction velocity with soret effects. The governing equations are solved numerically by using implicit finite difference method.

Saleh and Hashim (2009) analysed flow reversal phenomena of the fully – developed laminar combined free and forced MHD convection in a vertical plate – channel. The effect of viscous dissipation is taken into account.

Tamia and Samad (2010) studied and analysed the radiation and viscous dissipation effects on a steady two – dimensional magneto hydrodynamics free convection flow along a stretching sheet with heat generation. The non – linear partial differential equations governing the flow field under consideration have been transformed by a similarity transformation into a system of non – linear ordinary differential equations and then solved numerically by applying Nachtsheim – Swigert shooting iteration technique together with sixth order Runge – Kutta integration scheme. Resulting in non – dimensional velocity, temperature and concentration profiles are then presented graphically for different values of the parameters of physical engineering interest.

Viscous and Joule dissipation effects are considered on MHD non – linear flow and heat transfer past a stretching porous surface embedded in a porous medium under a transverse magnetic field has been studied by Anjali Devi and Ganga (2010).

Patrulescu et al (2010) investigated the steady mixed convection flow in a vertical channel for laminar and fully developed flow regime. In the modeling of heat transfer the viscous dissipation term was also considered. Temperature on the right wall is assumed constant while a mixed boundary condition (Robin boundary condition) is considered on the left wall.

This work presents a solution of the problem of laminar MHD convection in a vertical channel with uniform and symmetric temperatures. The solution is found by using one term perturbation series solution.

2. Formulation of the Problem and Governing equations.

The problem of laminar and fully – developed flow of a Newtonian fluid in a vertical parallel – plate with effect of magnetic field and viscous dissipation and buoyancy effects were considered.

The momentum and energy balance equations (Saleh and Hashim (2009)) are;

$$g\beta(T-T_b)-\frac{1}{\rho_b}\frac{\partial P}{\partial X}+\nu\frac{d^2U}{dY^2}-\frac{\sigma_e B_0^2 U}{\rho_b}=0 \qquad (1)$$

$$\alpha\frac{d^2T}{dY^2}+\frac{\nu}{c_p}\left(\frac{dU}{dY}\right)^2+\frac{\sigma_e B_0^2 U^2}{\rho_0 c_p}=0 \qquad (2)$$

Where g is the acceleration due to gravity, ν is the kinematic viscosity which is defined by $\nu=\frac{\mu}{\rho_b}$, $P=p+\rho_b gX$ is the difference between the pressure p and the hydrostatic pressure P. α is thermal conductivity and $\mu=\rho_b\nu$ is the dynamic viscosity

Equations (2) and the boundary condition on T, shows that P depends on X; T depends only on Y; T_b is a constant; $\frac{\partial P}{\partial X}$ is a constant.

The bulk temperature T_b and the mean velocity U_m (Barletta, 1999) are given by

$$T_b=\frac{1}{LU_m}\int_0^L UT\,dY \quad \text{and} \quad U_m=\frac{1}{L}\int_0^L U\,dY \qquad (3)$$

The boundary conditions are:

$$U(0)=0\,, \qquad T(0)=T_0,$$
$$U(L)=0\,, \qquad T(L)=T_0 \qquad (4)$$

Moreover, due to the symmetry of the problem, both U and T are symmetric functions of Y, so that they must fulfill the condition

$$U'(0)=0, \quad T'(0)=0 \qquad (5)$$

Introducing the dimensionless quantities below

$$u=\frac{U}{U_m}, \quad \theta=\frac{T-T_b}{\Delta T}, \quad y=\frac{Y}{L}, \ \lambda=\frac{-L^2}{\mu U_m}\frac{dP}{dX}, \ Gr=\frac{64L^3 g\beta\,\Delta T}{\nu^2},$$

$$Re=\frac{4LU_m}{\nu}, \quad E=\frac{Gr}{Re}, \quad Br=\frac{\mu U_m^2}{\alpha\Delta T}$$

Where the temperature difference ΔT is given by

$$\Delta T=\frac{\mu U_m^2}{k}$$

While the Grashof number Gr is always positive, the Reynolds number Re and the parameter E can be either positive or negative. In particular, in the case of upward flow $(U_m>0)$, both

Re and E are positive, while, for downward flow($U_m < 0$), these dimensionless parameters are negative.

Therefore, as a consequence of the dimensional quantities and together with the momentum balance equation (1), the energy balance equation (2) and equations (3) – (5) yields ;

$$\frac{d^2u}{dy^2} - Mu = -\frac{E\theta}{16} + \lambda \qquad \frac{d^2\theta}{dy^2} + BrMu^2 = -Br\left(\frac{du}{dy}\right)^2 \qquad (6)$$

Where $M = \frac{\sigma_0 \beta_0^2 L^2}{\rho_0 \nu}$ which is the magnetic parameter and $Br = \frac{1}{\rho_b c_p}$

$$u(1) = 0,\ u'(0) = 0, \theta(1) = -\varphi,\ \theta'(1) = 1 \qquad (7)$$

$$\int_0^1 u\theta dy = 0 \ ,\ \int_0^1 udy = 1 \qquad (8)$$

From equations (6) to (8) the functions $u(y)$ *and* $\theta(y)$ can be determined as well as the constants $\boldsymbol{\lambda}$ and R given the value of dimensionless parameter E. In particular, it is easily verified that the choice $E = 0$ correspond to the absence of buoyancy forces, i.e., to forced convection.

3. Perturbation Series Solution

The perturbation method which gives the solutions of equations (6) to (8) is described .Let us expand the functions $u(y), \theta(y)$ and the constants γ and φ as a power series in the parameter E , namely

$$u(y) = u_0(y) + u_1(y)E + u_2(y)E^2 + \cdots = \sum_{n=0}^{\infty} u_n(y) E^n \qquad (9)$$

$$\theta(y) = \theta_0(y) + \theta_1(y)E + \theta_2(y)E^2 + \cdots = \sum_{n=0}^{\infty} \theta_n(y) E^n \qquad (10)$$

$$\gamma = \gamma_0 + \gamma_1 E + \gamma_2 E^2 + \cdots = \sum_{n=0}^{\infty} \lambda_n E^n \qquad (11)$$

$$\varphi = \varphi_0 + \varphi_1 E + \varphi_2 E^2 + \cdots = \sum_{n=0}^{\infty} R_n E^n \qquad (12)$$

These power series expansions are substituted in equations (6) to (8).
Thus, the original boundary value problem is mapped into a sequence of boundary value problems which can be solved in succession, in order to obtain the coefficients of the power series expansion reported in equations (9) to (12). A detailed analysis on the application of perturbation methods in heat transfer problems can be found.

The boundary problem which corresponds to $n = 0$ in the series is the following:

$$\frac{d^2u_0}{dy^2} - Mu_0(y) = -\lambda_0 \, , u_0'(0) = 0 \, , u_0(1) = 0 \, , \ \int_0^1 u_0 dy = 1 \qquad (13)$$

$$\frac{d^2\theta_0}{dy^2} + BrM[u_0(y)]^2 = -Br\left(\frac{du_0}{dy}\right)^2 \, , \theta_0'(0) = 0 \, , \theta_0(1) = -\varphi_0 \, , \int_0^1 u_0\theta_0 dy = 0 \quad (14)$$

The boundary problem which corresponds to $n = 1$ is as follows:

$$\frac{d^2u_1}{dy^2} - Mu_1(y) = -\lambda_1 \,, u_1'(0) = 0 \,, u_1(1) = 0 \,, \int_0^1 u_1 dy = 0 \qquad (15)$$

$$\frac{d^2\theta_1}{dy^2} + 2BrM[u_0(y)u_1(y)] = -Br\frac{du_0}{dy}\frac{du_1}{dy} \,, \theta_1'(0) = 0 \,, \theta_1(1) = -\varphi_1 \,, \int_0^1 (u_0\theta_1 + u_1\theta_0) = 0$$
(16)

Equations (13) and (14) are easily solved, because the function $\theta_0(y)$ does not affect the function $u_0(y)$. The latter, together with the constant λ_0, is determined by solving equation (13), namely

$$u_0(y) = \frac{\sqrt{M}}{sinh\sqrt{M} - Mcosh\sqrt{M}}\left[cosh\sqrt{M}y + cosh\sqrt{M}\right], \;\; \lambda_0 = \frac{-M\sqrt{M}cosh\sqrt{M}}{sinh\sqrt{M} - Mcosh\sqrt{M}} \qquad (17)$$

By substituting equation (17) in equation (14), one obtains the function $\theta_0(y)$ and the constant φ_0, which are given by

$$\theta_0(y) = -BrA^2\left\{4cosh2\sqrt{M}y + 16cosh(y+1)\sqrt{M} + 16\cosh(y-1)\sqrt{M} + 4My^2cosh2\sqrt{M} + 4My^2\right\} + B + [1 + BrA^3C + BD]E \;\; , \varphi_0 = [-1 - BrA^3C + BD]E \qquad (18)$$

Where;

$$A = \frac{\sqrt{M}}{2sinh\sqrt{M} - Mcosh\sqrt{M}}$$

$$B = BrA^2\left[20cosh2\sqrt{M} + 4Mcosh2\sqrt{M} + 4M + 16\right]$$

$$C = \frac{106}{3\sqrt{M}}sinh3\sqrt{M} + \frac{38}{\sqrt{M}}sinh\sqrt{M} + 8cosh\sqrt{M} - 8cosh3\sqrt{M} + \frac{4M}{\sqrt{M}}sinh\sqrt{M} + \frac{4M}{3}cosh3\sqrt{M} + 4cosh\sqrt{M} + \frac{2M}{\sqrt{M}}exp\left(3\sqrt{M}\right) - 4Mexp\left(-3\sqrt{M}\right) + \frac{8}{\sqrt{M}}exp\left(-\sqrt{M}\right) - \frac{8}{\sqrt{M}}$$

$$D = \frac{2}{\sqrt{M}}sinh\sqrt{M} + 2cosh\sqrt{M}$$

$$E = \frac{1}{\frac{2}{\sqrt{M}}sinh\sqrt{M} + 2cosh\sqrt{M}}$$

4. Discussion of Results

In this section, 1 – term perturbation series is employed to evaluate the dimensionless velocity profile u and temperature profileθ since the other terms resulted in a trivial solutions. The resulted equations were solved with MATLAB and the results were plotted graphically.
The effect of magneto hydrodynamic in a vertical channel, the velocity profile and the temperature profile are depicted graphically against y for different values of the magnetic parameter$M$. Figures 1 and 2 demonstrate the variations of velocity u and temperature profile for θ for different values of the magnetic parameter.
It is observed from figure 1 that the velocity decreases with increase in the magnetic parameter. The mean velocity U_m occurs at$M = -0.15$. While figure 2 shows that, the temperature increases with increase in the magnetic parameter.

It is clear that the effect of the magnetic parameter on the velocity and temperature profile, pointed out that the effect of the MHD in vertical channel enhances the effect of flow in the downward direction and it lowers this effect on the upward flow.

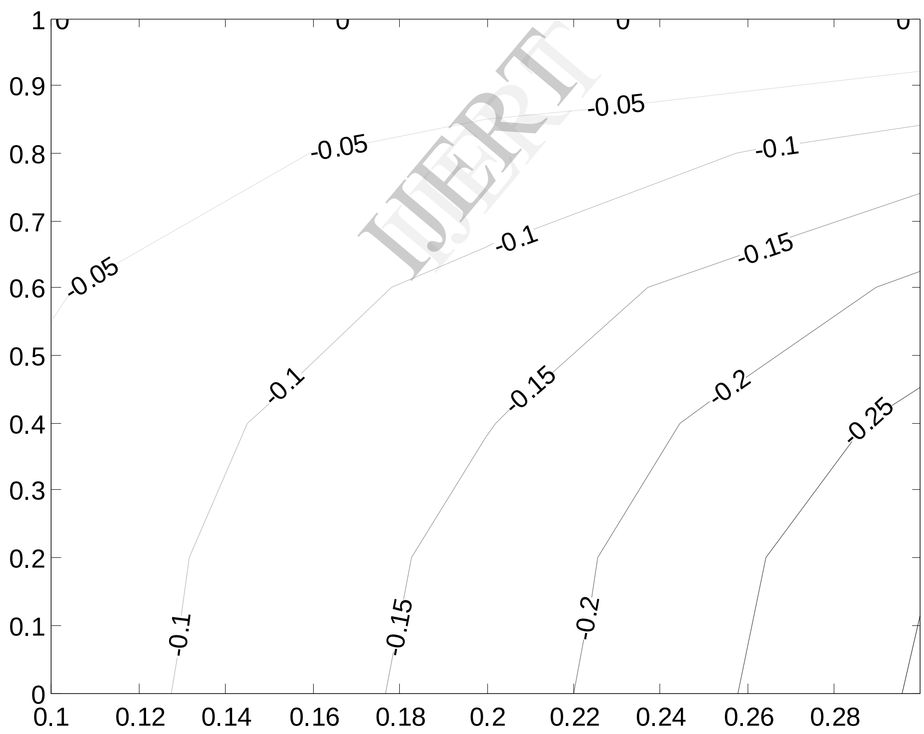

Figure 1: Effect of Mon Velocity profile

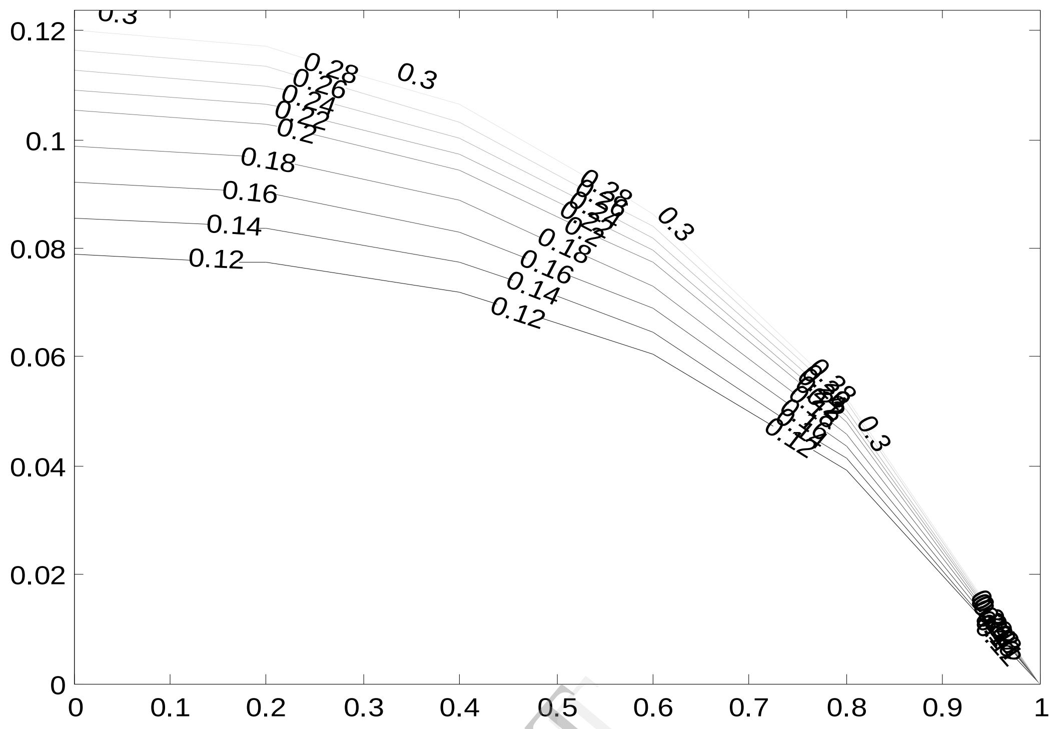

Figure 2. Effect of M on temperature profile

5. Summary and Conclusion

The fully developed laminar and MHD convection in a vertical channel with uniform and asymmetric temperatures has been studied.

The governing equations (momentum and energy balanced equations) has been written in a dimensionless form. The solution of the dimensionless velocity u and the dimensionless temperature θ has been determined in terms of the perturbation parameter$E$.

A perturbation method has been employed to evaluate the dimensionless velocity u and temperatureθ. It has been pointed out that the effect of magnetic parameter enhances the effect of flow in the downward direction while it lowers this effect in the upward flow.

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