

# Kinematic and Isotropic Properties of Excavator Mechanism

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**Abstract**—This paper presents an account of forward kinematics of an excavator mechanism. A manipulator Jacobian is derived and kinematic analysis of the excavator is done. The necessary conditions and constraint equations which are required for the velocity and force isotropy are discussed. Instantaneous properties of mechanisms of general and isotropic configurations are illustrated.

**Keywords**— Hydraulic Excavator, kinematic synthesis, Isotropy, Jacobian, Singularity

## I. INTRODUCTION

An excavator is a typical hydraulic, human-operated machine used in construction operations, such as digging, ground leveling, carrying loads, dumping loads etc mainly powered by hydraulic system [1]. Fig. 1 shows a hydraulic excavator.

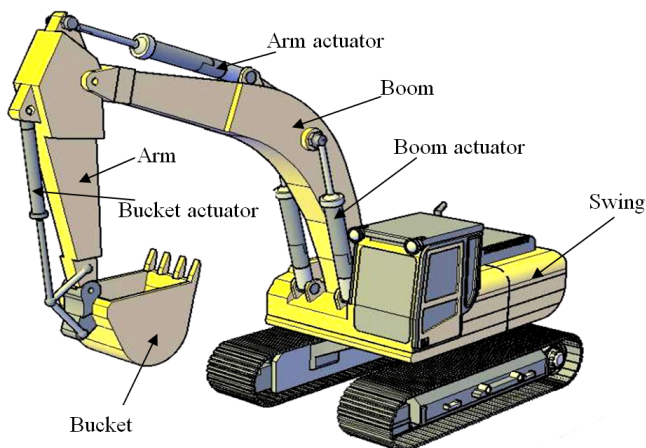


Fig. 1. A typical Hydraulic Excavator with its parts

The upper-body mainly consists of a boom, arm and bucket. An excavator consists of four revolute joints. These joints are found at the swing, boom, arm, and bucket. A coordinated movement of boom, arm and bucket is required to control the bucket tip position and trajectory. The excavator linkage is controlled by hydraulic cylinders and actuators. So, the joint angles are the functions of displacement of the actuator.

In this paper, forward kinematics of the hydraulic excavator is presented. A review of literatures related to excavators in the field of backhoe's kinematics is presented. The basic concepts of kinematic isotropy and force isotropy of excavator mechanism are discussed. In addition the overview

of literatures related to the kinematic isotropy and force isotropy of the mechanism are described.

## II. REVIEW OF LITERATURE

Kinematic modeling of the backhoe attachment is studied by many researchers to understand the relations between the position and orientation of the bucket and spatial positions of joint-links. A review of research work related to kinematics of excavators is as follows.

Kinematic modeling is helpful to follow the defined trajectory as well as digging operation. It can be carried out successfully at required location of the terrain using proper position and orientation of the bucket which will help ultimately the digging. Kinematic modeling of the backhoe attachment are discussed by many authors [2-4]. Fuad Mrad [5] developed simulation package using MATLAB with several embedded design and analysis tools. The package offered an integrated environment for trajectory design and analysis for an excavator while addressing the constraints related to the excavator structure, safety and stability, and mode of application. A complete pitch/plane model of a backhoe was developed by Donald Margolis [6]. It includes the hydraulics, dynamics and kinematics of the control linkage. Emil Assenov et al [7] carried out a study on kinematics of working mechanism of hydraulic excavator. The mechanism of this manipulator is a planar multi-linkage, which consists of hydraulic cylinders. Boris and Svetoslav [8] have developed two heuristic approaches for inverse kinematics of a real redundant excavator. They have presented a priority approach and alternating approach. In simulation, the method gives a very smooth overall motion. Daqing [9] derived a full kinematic model of excavator arm. He considered the excavator as planar manipulator with three degrees (boom, dipper and bucket) of freedom and attempted to control excavators arm to realize autonomous excavation.

The recurrent neural network was implemented by Hyongju et al [10] for better kinematics control of the excavator with obstacle avoidance capability. Michael [11] have described a simple framework for assessing different shovel designs, including kinematic performance of face shovels for surface mining excavation. A novel concept of applying tele-operated device has been developed by Dongnam [12] for the remote control of excavator-like dismantling equipment. Hongnian et al [13] have described modeling of excavator to carry out the kinematic which gives the trajectory of the excavator bucket based on the trajectory of

the excavator arm joints and the inverse kinematics which gives the desired joint variables corresponding to the desired bucket trajectory. Review of a work carried out by researchers in the field of kinematic modeling of the backhoe attachment can be found in [14]. The isotropic manipulators are obtained by solving a system of nonlinear equations developed from the Condition Number (CN) of the Jacobian or isotropy conditions. The obtained isotropic design, in general, cannot reach maximum number of isotropic positions. For 3R planar manipulators, Kircanski [15] employed the condition number to obtain two highly nonlinear equations with link parameters as design variables. A study of instantaneous kinematic planar two degrees of freedom mechanisms can be found in [16, 17]. Analytical and graphical representations of the properties have been used to study the relative influence of input velocities and accelerations on the end point. In isotropic configurations, the manipulator performs very well with regard to the force and motion transmission. The CN of a matrix indicates how far the said matrix is from a singularity. When CN is closer to unity, the matrix becomes easily invertible; and farther the CN is from unity, less invertible the matrix is. As CN becomes infinity, the matrix become singular and is not invertible. For  $CN = 1$ , the singular values ( $\sigma_i$ ) of  $[J]$  are identical; hence the Eigen values ( $\lambda_i$ ) of  $[J]$  must be identical [17]. Global Isotropy Index (GII) is proposed by Leo J [18] to quantify the configuration independent isotropy of a robot's Jacobian or mass matrix. A new three-degree-of-freedom fully isotropic parallel orientation mechanism is presented by Chin et al [19]. Isotropic design of a hybrid mechanism for three-axis machining applications is presented by Chablat [20]. Machine-tool mechanism is compared with a hybrid serial parallel structure of the table of the machine tool. The isotropic design of two types of spatial parallel manipulators namely a three dof manipulator and the Stewart-Gough platform are presented by Fattah [21]. The significance of the classical definition of manipulability ellipsoid highlighting its lack of significance in some circumstances is investigated by Legnani et al [22]. A new concept of Point of isotropy for both serial and parallel manipulators is introduced. This concept may be used to design new manipulators or to make isotropic the already existing manipulators just modifying the shape or dimension of the last link.

### III. KINEMATICS OF EXCAVATORS

Kinematics of excavators deals with the study of motion of bucket with respect to a fixed reference coordinate system without regard to the forces/moments that cause the motion. It gives an analytical description of the spatial displacement of the excavator as a function of time. It also enables to relate the joint-variable space to the position and orientation of the bucket of the excavator. There are two fundamental problems in excavator kinematics. The first problem is usually referred to as the direct (or forward) kinematics problem, while the second problem is the inverse kinematics problem [23].

#### A. Forward Kinematics

An excavator can be modeled as an open-loop articulated chain with boom, arm and bucket connected in series by revolute joint driven by actuators. One end of the chain is attached to a supporting base while the other end is free. The

relative motion of the joints results in the motion of the links that positions the bucket in a desired orientation. In forward kinematics, for a given excavator, given the joint angle vector  $[\Theta] = [\theta_1, \theta_2, \theta_3, \theta_4]^T$  and the link parameters ( $l_1, l_2, l_3, l_4$ ), the position and orientation of the bucket tip can be determined. In order to obtain the forward kinematic model of a typical hydraulic excavator, DH approach is used as discussed in Art 3.2.

#### B. DH Representation of Excavators

An excavator consists of a sequence of rigid bodies, called links, connected by revolute joints. Each joint-link pair constitutes one degree of freedom. Link '0' is usually attached to a supporting base link '0', and the last link is attached with a bucket. The joints and links are numbered outwardly from the base; thus, joint '1' is the point of connection between links '1' and the supporting base link '0'. A joint axis (for joint  $i$ ) is established at the connection of two links. This joint axis will have two normals connected to it, one for each of the links. The relative position of two such connected links (link  $i-1$  and link  $i$ ) is given by  $d_i$  which is the distance measured along the joint axis between the normals. The joint angle  $\theta_i$  between the normals is measured in a plane normal to the joint axis. Hence,  $d_i$  and  $\theta_i$  may be called the distance and the angle between the adjacent links, respectively. They determine the relative position of neighboring links. A link  $i$  is connected to, at most, two other links (e.g., link  $i-1$  and link  $i+1$ ); thus, two joint axes are established at both ends of the connection. The significance of links, from a kinematic perspective, is that they maintain a fixed configuration between their joints which can be characterized by two parameters:  $l_i$  and  $\alpha_i$ . The parameter  $l_i$  is the shortest distance measured along the common normal between the joint axes (i.e., the  $z_{i-1}$  and  $z_i$  axes for joint  $i$  and joint  $i+1$ , respectively), and  $\alpha_i$  is the angle between the joint axes measured in a plane perpendicular to  $l_i$ . Thus,  $l_i$  and  $\alpha_i$  may be called the length and the twist angle of the link  $i$ , respectively. They determine the structure of link  $i$  [23]. Every coordinate frame is determined and established on the basis of DH convention. In the first coordinate system  $\{O_0\}$ , axis  $Y_0$  is perpendicular to the plane of paper and away from the reader. In the remaining coordinate frames,  $X_i$  and  $Y_i$  axes assigned are as shown in Fig.2.  $Z_i$  ( $i=1, 2, 3$  and  $4$ ) axes are perpendicular to the plane of paper.

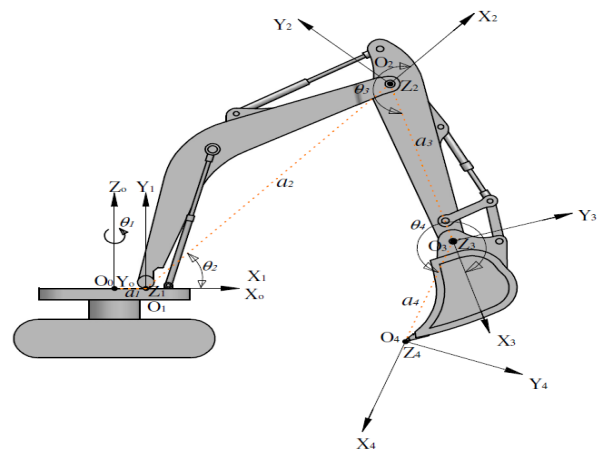


Fig. 2. Coordinate frames assignment and the Joint angles

DH parameters are identified and tabulated in Table.1.

Joint <i>i</i>	$\theta_i$	$\alpha_i$	$l_i$	$d_i$
1	$\theta_1$	$90^\circ$	$l_1$	0
2	$\theta_2$	0	$l_2$	0
2	$\theta_3$	0	$l_3$	0
2	$\theta_4$	0	$l_4$	0

$\theta_i$ = joint angle from the  $x_{i-1}$  axis to the  $x_i$  axis about  $z_{i-1}$  axis  
 $d_i$ = distance from  $x_{i-1}$  axis to  $x_i$  axis measured along  $z_i$  axis  
 $l_i$ = distance between the  $z_{i-1}$  and  $z_i$  axes measured along  $x_i$  axis  
 $\alpha_i$ = offset angle from  $z_{i-1}$  axis to the  $z_i$  axis about  $x_i$  axis

After coordinate frames and link parameters are established for each link, a homogeneous transformation matrix relating the  $i^{th}$  coordinate frame to the  $(i-1)^{th}$  coordinate frame can be obtained and is given by  ${}^{i-1}T_i$

$${}^{i-1}T_i = \begin{bmatrix} \cos \theta_i & -\cos \alpha_i \sin \theta_i & \sin \alpha_i \sin \theta_i & a_i \cos \theta_i \\ \sin \theta_i & \cos \alpha_i \sin \theta_i & \sin \alpha_i \sin \theta_i & a_i \sin \theta_i \\ 0 & \sin \alpha_i & \cos \alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

By substituting the link parameters of Table.1 for the link  $i$  in the composite homogeneous transformation matrix (1), individual link transformation matrices  ${}^{i-1}T_i$  (where  $i=1, 2, 3, 4$ ) can be obtained as

$${}^0T_1 = \begin{bmatrix} C_1 & 0 & S_1 & l_1 C_1 \\ S_1 & 0 & -C_1 & l_1 S_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1T_2 = \begin{bmatrix} C_2 & -S_2 & 0 & l_2 C_2 \\ S_2 & C_2 & 0 & l_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2T_3 = \begin{bmatrix} C_3 & -S_3 & 0 & l_3 C_3 \\ S_3 & C_3 & 0 & l_3 S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^3T_4 = \begin{bmatrix} C_4 & -S_4 & 0 & l_4 C_4 \\ S_4 & C_4 & 0 & l_4 S_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

where  $C_i = \cos \theta_i$ ,  $S_i = \sin \theta_i$ ,  $i=1,2,3,4$  and  ${}^0T_1$  represents the link transformation matrix for the link 1 in the  $\{0\}$  coordinate system. Similarly, other link transformation matrices,  ${}^1T_2, {}^2T_3, {}^3T_4$  can be defined.

If the position of a point  $p_i$  of a link  $i$  in the  $i^{th}$  coordinate system is known, then using this  ${}^{i-1}T_i$  link transformation matrix, one can determine position of the point ( $p_i$ ) of a link  $i$  in the  $(i-1)^{th}$  coordinate system, (i.e.,  $p_{i-1}$ ) using the relation (3),

$$P_{i-1} = {}^{i-1}T_i p_i \quad (3)$$

If the position and orientation of the tip of the bucket with reference to the  $\{O_3\}$  coordinate frame is known, then using successive matrix transformations, position and orientation of the tip of the bucket with reference to the base  $\{O_0\}$  coordinate system can be determined using the relation (3).

### C. Position of the Bucket

If the joint variables,  $[\Theta] = [\theta_1, \theta_2, \theta_3, \theta_4]^T$  are known, the coordinates of the tip of the bucket  $\{O_4\}$  can be determined in

the base coordinatesystem  $\{O_0\}$  by using equation (3) successively as

$${}^0T_4 = {}^0T_1 {}^1T_2 {}^2T_3 {}^3T_4 \quad (4)$$

using chain rule.

${}^0T_4$  is a homogeneous transformation matrix that relates the vector of the fourth coordinate frame to a vector in the base coordinate system. Substituting (2) in (4),  ${}^0T_4$  can be obtained.

$$\begin{bmatrix} C_1 C_{234} & -C_1 S_{234} & S_1 & C_1 (l_4 C_{234} + l_3 C_{23} + l_2 C_2 + l_1) \\ S_1 C_{234} & -S_1 S_{234} & -C_1 & S_1 (l_4 C_{234} + l_3 C_{23} + l_2 C_2 + l_1) \\ S_{234} & C_{234} & 0 & l_4 S_{234} + l_3 S_{23} + l_2 S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} {}^0R_4 & {}^0P_4 \\ 0 & 1 \end{bmatrix} \quad (5)$$

In the above homogeneous transformation matrix,  ${}^0R_4$  represents the orientation of the  $\{O_4\}$  coordinate frame with reference to the  $\{O_0\}$  coordinate frame.  ${}^0P_4$  represents position vector which points from the origin of the  $\{O_0\}$  coordinate system to the origin of the  $\{O_4\}$  coordinate system.

Therefore, the position of the tip of the bucket (origin of the  $\{O_4\}$  coordinate system) can be written from the relation (5) as given by the relation (6).

$${}^0P_4 = \begin{bmatrix} C_1 (l_4 C_{234} + l_3 C_{23} + l_2 C_2 + l_1) \\ S_1 (l_4 C_{234} + l_3 C_{23} + l_2 C_2 + l_1) \\ l_4 S_{234} + l_3 S_{23} + l_2 S_2 \end{bmatrix} \quad (6)$$

where  $C_{234} = \cos(\theta_2 + \theta_3 + \theta_4)$  and  $S_{234} = \sin(\theta_2 + \theta_3 + \theta_4)$

The joint variable  $\theta_i$  is usually constant during the execution of adigging task, so the excavator arm moves in a vertical plane only. Therefore an excavator can be assumed as a planar mechanism of three ( $\theta_2, \theta_3, \theta_4$ ) degrees of freedom (dof) which manipulates in vertical plane only. The position and orientation of the tip of the bucket  $\{O_4\}$  can be calculated considering the  $\{O_1\}$  frame as the reference using

$${}^1T_4 = {}^1T_2 {}^2T_3 {}^3T_4 \quad (7)$$

$${}^1T_4 = \begin{bmatrix} C_{234} & -S_{234} & 0 & l_2 C_2 + l_3 C_{23} + l_4 C_{234} \\ S_{234} & C_{234} & 0 & l_2 S_2 + l_3 S_{23} + l_4 S_{234} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} {}^1R_4 & {}^1P_4 \\ 0 & 1 \end{bmatrix}$$

Therefore, the position of the tip of the bucket (origin of the  $\{O_4\}$  coordinate system) with reference to the  $\{O_1\}$  frame can be written as

$${}^1P_4 = [l_2 C_2 + l_3 C_{23} + l_4 C_{234} \quad l_2 S_2 + l_3 S_{23} + l_4 S_{234} \quad 0]^T \quad (8)$$

### D. Jacobian of Excavator

The matrix which relates changes in joint velocities to Cartesian velocities is called the Jacobian Matrix. This is a time-varying, position dependent linear transformation matrix.

It has a number of columns equal to the number of degrees of freedom in joint space, and a number of rows equal to the number of degrees of freedom in the Cartesian space. The Jacobian that relates joint velocities to Cartesian velocity of the tip of the bucket  $\{O_4\}$  is given by

$$[\mathbf{v}] = [\mathbf{J}] [\dot{\boldsymbol{\theta}}] \tag{9}$$

where  $\dot{\boldsymbol{\theta}} = \dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4$  is the vector of joint rates of the excavator and  $[\mathbf{v}]$  is a vector of Cartesian velocities and  $[\mathbf{J}]$  is excavator Jacobian. As discussed in the previous paragraph, excavator can be considered as a planar mechanism of 3 dof. Therefore, Jacobian of an excavator is a 2X3 matrix. Since the Jacobian matrix  $[\mathbf{J}]$  of the excavator is of size 2X3, it is not possible to determine determinant of the matrix. So, usual method of determining the inverse of the Jacobian matrix will not work. Jacobian matrix for the 3 dof excavator linkage is derived as below.

$$[\mathbf{J}] = \begin{bmatrix} \frac{\partial X}{\partial \theta_2} & \frac{\partial X}{\partial \theta_3} & \frac{\partial X}{\partial \theta_4} \\ \frac{\partial Y}{\partial \theta_2} & \frac{\partial Y}{\partial \theta_3} & \frac{\partial Y}{\partial \theta_4} \end{bmatrix} \tag{10}$$

From the relation (8),

$${}^1P_4 = [l_2C_2 + l_3C_{23} + l_4C_{234} \quad l_2S_2 + l_3S_{23} + l_4S_{234}]^T$$

$$\begin{aligned} \text{Let,} \quad X &= l_2C_2 + l_3C_{23} + l_4C_{234} \\ Y &= l_2S_2 + l_3S_{23} + l_4S_{234} \end{aligned} \tag{11}$$

where X and Y gives the position of the  $\{O_4\}$  frame with reference to the  $\{O_1\}$  frame.

Therefore,

$$[\mathbf{J}] = \begin{bmatrix} -l_2S_2 - l_3S_{23} - l_4S_{234} & -l_3S_{23} - l_4S_{234} & -l_4S_{234} \\ l_2C_2 + l_3C_{23} + l_4C_{234} & l_3C_{23} + l_4C_{234} & l_4C_{234} \end{bmatrix} \tag{12}$$

Equation (12) gives the Jacobian of the excavator which resembles, in its simplest form, 3R planar mechanism. It can be noticed here that Jacobian of the 3R planar manipulator is same as that of equation (12). As Jacobian matrix is not square, it can't be inverted using usual procedure. So the method of inversion of rectangular matrices will be discussed in the following paragraph along with the concept of redundancy in planar manipulator.

*E. Redundant manipulators*

In this section, the topic of redundant manipulators is briefly addressed. A redundant manipulator is one that is equipped with more degrees of freedom than are required to perform a specified task. This provides the manipulator with an increased level of dexterity. Further, the Jacobian matrix for a redundant manipulator is not square, and thus cannot be inverted to solve the inverse velocity problem.

To address this problem, the concept of a pseudo inverse and the Singular Value Decomposition (SVD) are introduced here.

In previous paragraph, we have dealt primarily with positioning tasks. In these cases, the task was determined by specifying the position, orientation or both for the tip of the bucket. For these kinds of positioning tasks, the number of degrees of freedom for the task is equal to the number of parameters required to specify the position and orientation

information. A manipulator is said to be redundant when its number of internal degrees of freedom (or joints) is greater than the dimension of the task space.

Consider a 3 dof planar excavator linkage performing the task of positioning the tip of the bucket in the plane. Here, the task can be specified by  $(x, y) \in \mathbb{R}^2$ , and therefore the task space is two-dimensional. The forward kinematic equations for 3 dof planar excavator linkage are given by equation (9). Clearly, since there are three variables  $(\theta_2, \theta_3, \theta_4)$  and only two equations, it is not possible to solve uniquely for  $\theta_2, \theta_3, \theta_4$  given a specific  $(x, y)$ . The Jacobian for this manipulator is given by equation (10). When using the relationship (10) for  $\dot{\boldsymbol{\theta}}$ , we have a system of two linear equations in three unknowns. Thus there are also infinitely many solutions to this system, and the inverse velocity problem cannot be solved uniquely. The inverse velocity problem is easily solved when the Jacobian is square with nonzero determinant. However, when the Jacobian is not square, as is the case for redundant manipulators, the equation (13) cannot be used, since a non-square matrix cannot be inverted.

$$\dot{\boldsymbol{\theta}} = \mathbf{J}^{-1} \mathbf{v} \tag{13}$$

To deal with the case when  $m < n$ , we use the following result from linear algebra.

For  $\mathbf{J} \in \mathbb{R}^{m \times n}$ , if  $m < n$  and rank of  $[\mathbf{J}] = m$ , then  $(\mathbf{J}\mathbf{J}^T)$  exists. In this case  $(\mathbf{J}\mathbf{J}^T) \in \mathbb{R}^{m \times m}$ , and has rank  $m$ . Using this result, we can regroup terms to obtain

$$\mathbf{J}^+ = \mathbf{J}^T (\mathbf{J}\mathbf{J}^T)^{-1} \tag{14}$$

is called a right pseudo-inverse of  $\mathbf{J}$ , since  $\mathbf{J}\mathbf{J}^+ = \mathbf{I}$ .

Now the solution to the equation (14) can be obtained by

$$\dot{\boldsymbol{\theta}} = \mathbf{J}^+ \mathbf{v} + (\mathbf{I} - \mathbf{J}^+ \mathbf{J}) \mathbf{b} \tag{15}$$

in which  $\mathbf{b} \in \mathbb{R}^n$  is an arbitrary vector.

IV. ISOTROPY OF PLANAR 3R MANIPULATOR

The conditions for kinematic isotropy, force isotropy and Jacobian in force isotropy domain are discussed in this section. The isotropic configuration of excavator mechanism is presented.

*A. Kinematic isotropy*

Excavator during digging operation can be considered as 3R manipulator. Isotropic conditions for redundant manipulators are slightly different from non-redundant manipulators. For non-redundant manipulators such as 2R manipulator, the condition (16) holds good when  $\mathbf{J}$  is orthogonal matrix.

$$[\mathbf{J}]^T [\mathbf{J}] = [\mathbf{J}] \cdot [\mathbf{J}]^T = [\mathbf{I}] \tag{16}$$

Where  $[\mathbf{I}]$  is an identity matrix.

But, for redundant manipulators such as 3R manipulators, condition defined by equation (16) doesn't hold good. Since, If  $\mathbf{J}$  is not a square matrix, then the conditions  $[\mathbf{J}]^T [\mathbf{J}] = \mathbf{I}$  and  $[\mathbf{J}] \cdot [\mathbf{J}]^T = \mathbf{I}$  are not equivalent. The condition  $[\mathbf{J}]^T [\mathbf{J}] = \mathbf{I}$  says that the columns of  $\mathbf{J}$  are orthogonal. This can only happen if  $\mathbf{J}$  is an  $m \times n$  matrix with  $n \leq m$ . Similarly,  $[\mathbf{J}] \cdot [\mathbf{J}]^T = \mathbf{I}$  says that the rows of  $\mathbf{J}$  are orthogonal, which requires  $n \geq m$ .

Keeping this in mind, the conditions for isotropy of non-redundant manipulators are given as,

Condition of orthogonality of  $J$ , i.e.,  $J_1 J_2^T = 0$

Condition of equality of the magnitude of  $J$ , i.e.,  $\|J_1\| = \|J_2\|$ .

Where  $J_1$  and  $J_2$  are the rows of the Jacobian,  $J$ . Using the conditions of isotropy, we can obtain the following two linear equations in second and third absolute joint angles. Using the condition of orthogonality of rows of  $J$ ,

$$J_{11} \times J_{21} + J_{12} \times J_{22} + J_{13} \times J_{23} = 0 \tag{17}$$

On simplification, we get

$$-l_2 l_3 s_3 - l_2 l_4 s_4 - 2l_3^2 c_3 s_3 - 3l_4^2 c_4 s_4 - 2l_3 l_4 c_3 s_4 - 2l_3 l_4 c_4 s_3 = 0 \tag{18}$$

Using the condition of equality of the magnitude of rows of  $J$ ,

$$J_{11}^2 + J_{12}^2 + J_{13}^2 - J_{21}^2 - J_{22}^2 - J_{23}^2 = 0 \tag{19}$$

On simplification, we get

$$2l_3^2 s_3^2 - 3l_4^2 c_4^2 - 2l_3^2 c_3^2 + 3l_4^2 s_4^2 - l_2^2 - 2l_2 l_3 c_3 - 2l_2 l_4 c_4 - 4l_3 l_4 c_3 c_4 + 4l_3 l_4 s_3 s_4 = 0 \tag{20}$$

It can be noted here that  $\theta_2$  is ignored from both equation (18) and (20), since, isotropy of planar manipulator is independent of first joint angle in case of 3R manipulator.

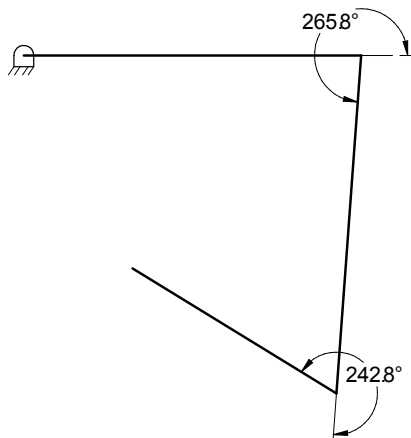


Fig. 3. Isotropic configuration of excavator mechanism

Equations (18) and (20) are the two simultaneous non-linear equations which are derived from the conditions of isotropy. In these equations, there are three link lengths and four trigonometric functions which are to be determined in order to get the isotropic configurations of planar 3R manipulator.

Upon solving the equations, (18) and (20) gives, eight isotropic configurations. Method of solving these equations is beyond the scope this paper. One of the solutions is adopted for excavator mechanism and is shown in figure.3. Keeping the link lengths in the proportion of  $l_2 = l_3 = \sqrt{2}$  and  $l_4 = 1$ , and joint angles as shown in figure. 3, isotropic configuration of excavator mechanism is achieved.

### B. Force Isotropy

A manipulator is isotropic with respect to the force, if it can exert the same force along all the directions. This property can be investigated by means of the force ellipsoid. In this section, the Jacobian in force domain, force ellipsoid and necessary conditions for Jacobian of manipulator for it to be force isotropic are discussed.

Let us consider a bucket of an excavator. The tip of the bucket is used for digging operation. The force exerted by the tip of the bucket on the ground to remove the soil depends on the orientation of the tip of the bucket and transmission angle of the linkage. This digging force varies with the orientation of the tip of the bucket with ground surface. Only at some orientation, digging force will be a maximum. Using the concept of force isotropy, one can design an excavator linkage which produces or exerts uniform force on the ground surface. i.e., at whatever the orientation the bucket is, the digging force should be uniform. This is one example how force isotropy can be used. There are many other applications where force isotropy is being used.

### C. Jacobian in Force Domain

The jacobian in force domain is derived in reference [24] and same may be given as

$$\tau = J^T F \tag{21}$$

The equation (21) leads to a conclusion that, the Jacobian transpose maps Cartesian forces acting at the tip of the bucket into equivalent joint torques. If the Jacobian is singular,  $F$  could be increased or decreased in certain directions without effect on the value calculated for  $\tau$ . This also means that, at near singular configurations, mechanical advantage tends toward infinity, such that, with small joint torques, large forces could be generated at the end effector. Thus, singularities manifest themselves in the force domain as well as in the velocity domain [24].

### V. CONCLUSIONS

The kinematics of a manipulator is among the most important factors affecting manipulator performance. In particular, the kinematics determines the motion and force transformations from the manipulator joints to the endpoint. By varying the kinematic parameters, the transformations can be optimized for a particular performance capability. Through these discussions, it was proved that, when a mechanism is synthesized such that it exhibits the properties of isotropic configuration, a best performance can be expected from it. The concept of force isotropy can be used while synthesizing the excavator mechanism. The tip of the bucket is used for digging operation. The force exerted by the tip of the bucket on the ground to remove the soil depends on the orientation of the tip of the bucket and transmission angle of the linkage. This digging force varies with the orientation of the tip of the bucket with ground surface. Only at some orientation, digging force will be maximum. Using the concept of force isotropy, one can design an excavator linkage which produces or exerts uniform force on the ground surface while digging. i.e., irrespective of the orientation of the bucket, the digging force should be uniform. The extensive amounts of forces are executed during the digging operation. These forces

sometimes adversely affected on the mechanical components of the excavator backhoe and may be damaged during the digging operation.

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