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KAMPÉ DE FÉRIET'S FUNCTION AND LAURICELLA FUNCTION OF MATRIX ARGUMENTS IN COMPLEX CASE

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ABSTRACT

In this paper, we have studied the Mathai's definitions of the Kampé de Fériet's functions, the Lauricella functions of matrix arguments in complex case.

1. INTRODUCTION

Techniques of Mathai [1,2] for positive symmetric matrix, we define Kampé de Fériet's, Lauricella and other functions of matrix argument in complex case. All matrices used in this paper are hermitian positive definite. All the matrices appearing in this paper are p x p real Hermitian positive definite and meanings of all the other symbols used are the same as in the work of Mathai [1,2]

FUNCTION OF MATRIX ARGUMENT IN THE COMPLEX CASE :

We consider real valued scalar function of a single matrix argument of the type $\tilde{Z} = \tilde{X} + i\tilde{Y}$ where $\tilde{X}$ and $\tilde{Y}$ are p x p matrices with real elements and $i = \sqrt{-1}$ as well as scaler functions of many matrices $\tilde{Z}_j$, j = 1, 2, …….K where each $\tilde{Z}_j$ is of the type $\tilde{Z}$ above in the real case. We confined our discussion to the situation where the argument matrix was real symmetric positive definite. This was done so that the fractional power of matrices and functions of such matrices could be uniquely defined. Corresponding properties are of we restrict to the class of Hermitian positive definite matrices.

Definition : Hermitian positive definite matrix due to Mathai [3], We will denote the conjugate of $\tilde{Z}$ by $\tilde{\tilde{Z}}$ if $\tilde{Z}$ hermitian, then $\tilde{Z} = \tilde{Z}^*$, that is

$$\tilde{Z} = \tilde{Z}^* \quad \Rightarrow \quad \tilde{X} + i\tilde{Y} = (\tilde{X} + i\tilde{Y})^* = \tilde{X}' + i\tilde{Y}'$$

$$\Rightarrow \quad \tilde{X} = \tilde{X}' \text{ and } \tilde{Y} = \tilde{Y}'$$

Thus $\tilde{X}$ is the symmetric and $\tilde{Y}$ is skew symmetric. Further if $\tilde{Z}$ is hermitian positive definite, then all the eigen values of $\tilde{Z}$ are real and positive.

Further, matrix variate gamma in the complex case is

$$\tilde{\Gamma}_p(\alpha) = \pi^{\frac{p(p-1)}{2}} \Gamma(\alpha)\Gamma(\alpha-1)\ldots\ldots\Gamma(\alpha-\pi+1)$$

We will use the notation $\tilde{Z} > 0$ to indicate that $\tilde{Z}$ is hermitian positive definite. Constant matrices will be written without a tilde whether the elements are real or complex unless it has to be emphasized that the matrix involved has complex elements. Then in that case a constant matrix will also be written with a tilde.

2. DEFINITIONS

2.1 The Kampé de Fériet's function

$$F^{r:q;k}_{s:m;n} = F^{r:q;k}_{s:m;n}\left[\begin{matrix}(a_r): & (b_q); & (c_k); \\ (\alpha_s): & (\beta_m); & (\gamma_n);\end{matrix} -\tilde{X}, -\tilde{Y}\right]$$

of matrix arguments is defined as that class of functions for which the M-transform is the following:

$$M\left(F^{r:q;k}_{s:m;n}\right) = \int_{\tilde{X}>0}\int_{\tilde{Y}>0} \left|\tilde{X}\right|^{\rho_1 - p}\left|\tilde{Y}\right|^{\rho_2 - p} \times$$

$$F^{r:q;k}_{s:m;n}\left[\begin{matrix}(a_r): & (b_q); & (c_k); \\ (\alpha_s): & (\beta_m); & (\gamma_n);\end{matrix} -\tilde{X}, -\tilde{Y}\right] d\tilde{X}d\tilde{Y}$$

$$= \frac{\left\{\prod_{j=1}^{s}\tilde{\Gamma}_p(\alpha_j)\right\}\left\{\prod_{j=1}^{m}\tilde{\Gamma}_p(\beta_j)\right\}\left\{\prod_{j=1}^{n}\tilde{\Gamma}_p(\gamma_j)\right\}\left\{\prod_{j=1}^{r}\tilde{\Gamma}_p(a_j-\rho_1-\rho_2)\right\}}{\left\{\prod_{j=1}^{r}\tilde{\Gamma}_p(a_j)\right\}\left\{\prod_{j=1}^{q}\tilde{\Gamma}_p(b_j)\right\}\left\{\prod_{j=1}^{k}\tilde{\Gamma}_p(c_j)\right\}\left\{\prod_{j=1}^{s}\tilde{\Gamma}_p(\alpha_j-\rho_1-\rho_2)\right\}}$$

$$\times \frac{\left\{\prod_{j=1}^{q}\tilde{\Gamma}_p(b_j-\rho_1)\right\}\left\{\prod_{j=1}^{k}\tilde{\Gamma}_p(c_j-\rho_2)\right\}\tilde{\Gamma}_p(\rho_1)\tilde{\Gamma}_p(\rho_2)}{\left\{\prod_{j=1}^{m}\tilde{\Gamma}_p(\beta_j-\rho_1)\right\}\left\{\prod_{j=1}^{n}\tilde{\Gamma}_p(\gamma_j-\rho_2)\right\}} \qquad \ldots(2.1)$$

for $\text{Re}\left(\rho_1, \rho_2, a_j-\rho_1-\rho_2, j=1,\ldots,r; \alpha_j-\rho_1-\rho_2, j=1,\ldots,s; b_j-\rho_1, j=1,\ldots,q;\right.$

$$\left.\beta_j-\rho_1, j=1,\ldots,m; c_j-\rho_2, j=1,\ldots,k; \gamma_j-\rho_2, j=1,\ldots,n\right) > p-1$$

2.2 The Lauricella function

$$F_A^{(n)} = F_A^{(n)}\left(a, b_1, \ldots, b_n; c_1, \ldots, c_n; -\tilde{X}_1, \ldots, -\tilde{X}_n\right)$$

of matrix arguments is defined as that class of functions which has the following matrix transform:

$$M\left(F_A^{(n)}\right) = \int_{\tilde{X}_1 > 0} \cdots \int_{\tilde{X}_n > 0} \left|\tilde{X}_1\right|^{\rho_1 - p} \cdots \left|\tilde{X}_n\right|^{\rho_n - p} \times F_A^{(n)}\left(a, b_1, \ldots, b_n; c_1, \ldots, c_n; -\tilde{X}_1, \ldots, -\tilde{X}_n\right) d\tilde{X}_1 \ldots d\tilde{X}_n$$

$$= \frac{\left\{\prod_{j=1}^{n} \tilde{\Gamma}_p(c_j)\right\}\left\{\prod_{j=1}^{n} \tilde{\Gamma}_p(b_j - \rho_j)\right\} \tilde{\Gamma}_p(a - \rho_1 - \ldots - \rho_n)\left\{\prod_{j=1}^{n} \tilde{\Gamma}_p(\rho_j)\right\}}{\tilde{\Gamma}_p(a)\left\{\prod_{j=1}^{n} \tilde{\Gamma}_p(b_j)\right\}\left\{\prod_{j=1}^{n} \tilde{\Gamma}_p(c_j - \rho_j)\right\}} \qquad \ldots(2.2)$$

for $\operatorname{Re}\left(b_j - \rho_j, c_j - \rho_j, \rho_j, a - \rho_1 - \ldots - \rho_n\right) > p - 1;\ j = 1, \ldots, n.$

2.3 $F_B^{(n)} = F_B^{(n)}\left(a_1, \ldots, a_n, b_1, \ldots, b_n; c; -\tilde{X}_1, \ldots, -\tilde{X}_n\right)$

$$M\left(F_B^{(n)}\right) = \int_{\tilde{X}_1 > 0} \cdots \int_{\tilde{X}_n > 0} \left|\tilde{X}_1\right|^{\rho_1 - p} \cdots \left|\tilde{X}_n\right|^{\rho_n - p} \times F_B^{(n)}\left(a_1, \ldots, a_n, b_1, \ldots, b_n; c; -\tilde{X}_1, \ldots, -\tilde{X}_n\right) d\tilde{X}_1 \ldots d\tilde{X}_n$$

$$= \frac{\tilde{\Gamma}_p(c)\left\{\prod_{j=1}^{n} \tilde{\Gamma}_p(a_j - \rho_j)\tilde{\Gamma}_p(b_j - \rho_j)\tilde{\Gamma}_p(a_j)\right\}}{\prod_{j=1}^{n}\left\{\tilde{\Gamma}_p(a_j)\tilde{\Gamma}_p(b_j)\right\}\tilde{\Gamma}_p(c - \rho_1 - \ldots - \rho_n)} \qquad \ldots(2.3)$$

for $\operatorname{Re}\left(a_j - \rho_j, b_j - \rho_j, \rho_j, c - \rho_1 - \ldots - \rho_n\right) > p - 1;\ j = 1, \ldots, n.$

2.4 $F_C^{(n)} = F_C^{(n)}\left(a, b, c_1, \ldots, c_n; -\tilde{X}_1, \ldots, -\tilde{X}_n\right)$

$$M\left(F_C^{(n)}\right) = \int_{\tilde{X}_1 > 0} \cdots \int_{\tilde{X}_n > 0} \left|\tilde{X}_1\right|^{\rho_1 - p} \cdots \left|\tilde{X}_n\right|^{\rho_n - p} \times F_C^{(n)}\left(a, b, c_1, \ldots, c_n; -\tilde{X}_1, \ldots, -\tilde{X}_n\right) d\tilde{X}_1 \ldots d\tilde{X}_n$$

$$= \frac{\left\{\prod_{j=1}^{n}\tilde{\Gamma}_p(c_j)\right\}\prod_{j=1}^{n}\tilde{\Gamma}_p(a-\rho_1-\ldots-\rho_n)\tilde{\Gamma}_p(b-\rho_1-\ldots-\rho_n)}{\tilde{\Gamma}_p(a)\tilde{\Gamma}_p(b)\left\{\prod_{j=1}^{n}\tilde{\Gamma}_p(c_j-\rho_j)\right\}}\left\{\prod_{j=1}^{n}\tilde{\Gamma}_p(\rho_j)\right\} \qquad \ldots(2.4)$$

for $\mathrm{Re}(c_j-\rho_j,\rho_j,a-\rho_1-\ldots-\rho_n,b-\rho_1-\ldots-\rho_n)>p-1;\ j=1,\ldots,n.$

2.5 $F_D^{(n)} = F_D^{(n)}(a,b_1,\ldots,b_n;c;-\tilde{X}_1,\ldots,-\tilde{X}_n)$

$$M\left(F_D^{(n)}\right) = \int_{\tilde{X}_1>0}\ldots\int_{\tilde{X}_n>0}|\tilde{X}_1|^{\rho_1-p}\ldots|\tilde{X}_n|^{\rho_n-p}\times$$

$$F_D^{(n)}(a,b_1,\ldots,b_n;c;-\tilde{X}_1,\ldots,-\tilde{X}_n)d\tilde{X}_1\ldots d\tilde{X}_n$$

$$= \frac{\tilde{\Gamma}_p(c)\prod_{j=1}^{n}\left\{\tilde{\Gamma}_p(b_j-\rho_j)\tilde{\Gamma}_p(\rho_j)\right\}\tilde{\Gamma}_p(a-\rho_1-\ldots-\rho_n)}{\tilde{\Gamma}_p(a)\prod_{j=1}^{n}\left\{\tilde{\Gamma}_p(b_j)\right\}\tilde{\Gamma}_p(c-\rho_1-\ldots-\rho_n)} \qquad \ldots(2.5)$$

for $\mathrm{Re}(b_j-\rho_j,\rho_j,a-\rho_1-\ldots-\rho_n,c-\rho_1-\ldots-\rho_n)>p-1;\ j=1,\ldots,n.$

3. KAMPÉ DE FÉRIET’S FUNCTION OF MATRIX ARGUMENTS

Theorem 3.1

$$F_{1:1;1}^{1:2;2}\left[\begin{matrix}\alpha:\beta,\lambda;\beta',\lambda';\\ \gamma:\mu;\mu';\end{matrix}\quad -\tilde{X},-\tilde{Y}\right] = \frac{\tilde{\Gamma}_p(\mu)\tilde{\Gamma}_p(\mu')}{\tilde{\Gamma}_p(\lambda)\tilde{\Gamma}_p(\mu-\lambda)\tilde{\Gamma}_p(\lambda')\tilde{\Gamma}_p(\mu'-\lambda')}\times$$

$$\int_0^I\int_0^I|\tilde{U}|^{\lambda-p}|\tilde{V}|^{\lambda'-p}|I-\tilde{U}|^{\mu-\lambda-p}|I-\tilde{V}|^{\mu'-\lambda'-p}\times F_1\left(\alpha,\beta,\beta';\gamma;-\tilde{U}^{1/2}\tilde{X}\tilde{U}^{1/2},-\tilde{V}^{1/2}\tilde{Y}\tilde{V}^{1/2}\right)d\tilde{U}d\tilde{V}$$

...(3.1)

for $\mathrm{Re}(\lambda,\lambda',\mu-\lambda,\mu'-\lambda')>p-1$

Proof: From definition (2.1) we deduce that,

$$M(F_{1:1;1}^{1:2;2}) = \int_{\tilde{X}>0}\int_{\tilde{Y}>0}|\tilde{X}|^{\rho_1-p}|\tilde{Y}|^{\rho_2-p}\times$$

$$F_{1:1;1}^{1:2;2}\left[\begin{matrix}\alpha:\beta,\lambda;\beta',\lambda';\\ \gamma:\mu;\mu';\end{matrix}\quad -\tilde{X},-\tilde{Y}\right]d\tilde{X}d\tilde{Y}$$

$$= \frac{\tilde{\Gamma}_p(\gamma)\tilde{\Gamma}_p(\mu)\tilde{\Gamma}_p(\mu')\tilde{\Gamma}_p(\alpha-\rho_1-\rho_2)\tilde{\Gamma}_p(\beta-\rho_1)\tilde{\Gamma}_p(\lambda-\rho_1)}{\tilde{\Gamma}_p(\alpha)\tilde{\Gamma}_p(\beta)\tilde{\Gamma}_p(\lambda)\tilde{\Gamma}_p(\beta')\tilde{\Gamma}_p(\lambda')\tilde{\Gamma}_p(\gamma-\rho_1-\rho_2)\tilde{\Gamma}_p(\mu-\rho_1)}\times$$

$$\frac{\tilde{\Gamma}_p(\beta'-\rho_2)}{\tilde{\Gamma}_p(\mu'-\rho_2)}\tilde{\Gamma}_p(\lambda'-\rho_2)\tilde{\Gamma}_p(\rho_1)\tilde{\Gamma}_p(\rho_2) \quad \ldots(3.2)$$

for $\mathrm{Re}(\rho_1,\rho_2,\alpha-\rho_1-\rho_2,\beta-\rho_1,\lambda-\rho_1,\beta'-\rho_2,\lambda'-\rho_2,\gamma-\rho_1-\rho_2,\mu-\rho_1,\mu'-\rho_2)>p-1$

Now taking the M-transform of the right side of eq.(3.1) with respect to the variables $\tilde{X},\tilde{Y}$ and the parameters ρ_1,ρ_2 respectively, we have,

$$\int_{\tilde{X}>0}\int_{\tilde{Y}>0}|\tilde{X}|^{\rho_1-p}|\tilde{Y}|^{\rho_2-p}F_1\left(\alpha,\beta,\beta';\gamma;-\tilde{U}^{1/2}\tilde{X}\tilde{U}^{1/2},-\tilde{V}^{1/2}\tilde{Y}\tilde{V}^{1/2}\right)d\tilde{X}d\tilde{Y} \quad \ldots(3.3)$$

Applying the transformations,

$$\tilde{X}_1=\tilde{U}^{1/2}\tilde{X}\tilde{U}^{1/2},\ \tilde{Y}_1=\tilde{V}^{1/2}\tilde{X}\tilde{V}^{1/2}$$

(implying thereby $d\tilde{X}_1=|\tilde{U}|^p d\tilde{X}, d\tilde{Y}_1=|\tilde{V}|^p d\tilde{Y}$, and $|\tilde{X}_1|=|\tilde{U}||\tilde{X}|, |\tilde{Y}_1|=|\tilde{V}||\tilde{Y}|$) in the expression (3.3) and then making use of Mathai's definition of M-transform of an Appell's function F_1 we get,

$$|\tilde{U}|^{-\rho_1}|\tilde{V}|^{-\rho_2}\frac{\tilde{\Gamma}_p(\gamma)\tilde{\Gamma}_p(\rho_1)\tilde{\Gamma}_p(\rho_2)\tilde{\Gamma}_p(\alpha-\rho_1-\rho_2)}{\tilde{\Gamma}_p(\alpha)\tilde{\Gamma}_p(\beta)\tilde{\Gamma}_p(\beta')\tilde{\Gamma}_p(\gamma-\rho_1-\rho_2)}\tilde{\Gamma}_p(\beta-\rho_1)\tilde{\Gamma}_p(\beta'-\rho_2) \quad \ldots(3.4)$$

Substituting this expression on the right side of eq. (3.1) and integrating out the variables $\tilde{U}$ and $\tilde{V}$ in the resulting expression by using a type-1 Beta integral we finally obtain $M\left(F_{1:1;1}^{1:2;2}\right)$ as given by eq. (3.2)

4. LAURICELLA FUNCTIONS OF MATRIX ARGUMENTS

Theorem 4.1

$$F_A^{(n)}=F_A^{(n)}\left(\alpha,\beta_1,\ldots,\beta_n;\gamma_1,\ldots,\gamma_n;-\tilde{X}_1,\ldots,-\tilde{X}_n\right)$$

$$=\frac{1}{\tilde{\Gamma}_p(\beta_n)}\int_{\tilde{T}>0}e^{-tr(\tilde{T})}|\tilde{T}|^{\beta_n-p}\Psi_A^{(n)}\left(\alpha,\beta_1,\ldots,\beta_{n-1};\gamma_1,\ldots,\gamma_n;-\tilde{X}_1,\ldots,-\tilde{X}_{n-1},-\tilde{T}^{1/2}\tilde{X}_n\tilde{T}^{1/2}\right)d\tilde{T}$$

$$\ldots(4.1)$$

for $\mathrm{Re}(\beta_n)>p-1$

Proof: Taking the M-transform of the right side of eq. (4.1) with respect to the variables $\tilde{X}_1,\ldots,\tilde{X}_n$ and the parameters $\rho_1,\ldots,\rho_n$ respectively, we get

$$\int_{\tilde{X}_1>0}\cdots\int_{X_n>0}\left|\tilde{X}_1\right|^{\rho_1-p}\cdots\left|\tilde{X}_n\right|^{\rho_n-p}\Psi_A^{(n)}\left(\alpha,\beta_1,\ldots,\beta_{n-1};\gamma_1-\gamma_n;-\tilde{X}_1,\ldots,\tilde{X}_{n-1},-\tilde{T}^{1/2}\tilde{X}\tilde{T}^{1/2}\right)d\tilde{X}_1\ldots d\tilde{X}_n$$

…(4.2)

which on applying the transformation

$$\tilde{Y}_n=\tilde{T}^{1/2}\tilde{X}\tilde{T}^{1/2}\ \text{(with } d\tilde{Y}_n=\left|\tilde{T}\right|^p d\tilde{X}_n \text{ and } \left|\tilde{Y}_n\right|=\left|\tilde{T}\right|\left|\tilde{X}_n\right|\text{)}$$

$$\left|\tilde{T}\right|^{-\rho_n}\frac{\tilde{\Gamma}_p(\alpha-\rho_1\ldots-\rho_n)}{\tilde{\Gamma}_p(\alpha)}\frac{\left\{\prod_{i=1}^{n-1}\tilde{\Gamma}_p(\beta_i-\rho_i)\right\}\left\{\prod_{j=1}^{n}\tilde{\Gamma}_p(\gamma_j)\Gamma_p(\rho_j)\right\}}{\left\{\prod_{i=1}^{n-1}\tilde{\Gamma}_p(\beta_i)\right\}\left\{\prod_{j=1}^{n}\tilde{\Gamma}_p(\gamma_j-\rho_j)\right\}}\qquad\ldots(4.3)$$

Substituting this expression on the right side of eq. (4.1) and then integrating out $\tilde{T}$ in the resulting expression by using a Gamma integral produces $M\left(F_A^{(n)}\right)$ as given by eq. (2.2).

Theorem 4.3

$$F_B^{(n)}\left(\alpha_1,\ldots,\alpha_n,\beta_1,\ldots,\beta_n;\gamma;-\tilde{X}_1,\ldots,-\tilde{X}_n\right)$$

$$=\frac{1}{\tilde{\Gamma}_p(\beta_n)}\int_{\tilde{T}>0}e^{-tr(\tilde{T})}\left|\tilde{T}\right|^{\beta_n-p}\Xi_1^{(n)}\left(\alpha_1,\ldots,\alpha_n,\beta_1,\ldots,\beta_{n-1};\gamma;-\tilde{X}_1,\ldots,-\tilde{X}_{n-1},-\tilde{T}^{1/2}\tilde{X}_n\tilde{T}^{1/2}\right)d\tilde{T}$$

…(4.4)

for $\mathrm{Re}(\beta_n)>p-1$

Theorem 4.5

$$F_C^{(n)}\left(\alpha,\beta;\gamma_1,\ldots,\gamma_n;-\tilde{X}_1,\ldots,-\tilde{X}_n\right)$$

$$=\frac{1}{\tilde{\Gamma}_p(\alpha)}\int_{\tilde{T}>0}e^{-tr(\tilde{T})}\left|\tilde{T}\right|^{\alpha-p}\Psi_2^{(n)}\left(\beta;\gamma_1,\ldots,\gamma_n;-\tilde{T}^{1/2}\tilde{X}_1\tilde{T}^{1/2},\ldots,-\tilde{T}^{1/2}\tilde{X}_n\tilde{T}^{1/2}\right)d\tilde{T}\qquad\ldots(4.5)$$

for $\mathrm{Re}(\alpha)>p-1$

Theorem 4.6

$$F_D^{(n)}\left(\alpha,\beta_1,\ldots,\beta_n;\gamma;-\tilde{X}_1,\ldots,-\tilde{X}_n\right)$$

$$=\frac{1}{\tilde{\Gamma}_p(\beta_n)}\int_{\tilde{T}>0}e^{-tr(\tilde{T})}\left|\tilde{T}\right|^{\beta_n-p}\Phi_D^{(n)}\left(\alpha,\beta_1,\ldots,\beta_{n-1};\gamma;-\tilde{X}_1,\ldots,-\tilde{X}_{n-1},-\tilde{T}^{1/2}\tilde{X}_n\tilde{T}^{1/2}\right)d\tilde{T}\qquad\ldots(4.6)$$

for $\mathrm{Re}(\beta_n)>p-1$

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