International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 3 Issue 4, April - 2014

Jacobian Analysis of Limited DOF Parallel Manipulator using Wrench and Reciprocal Screw Principle

Hassen Nigatu[1]
Department of Mechanical & Vehicle Engineering
Adama Science & Technology University, Adama, Ethiopia

Ajit Pal Singh[2]
Manufacturing Section
Production Engineering Department
College of Engineering
Defence University, Bishoftu, Ethiopia

P. Prabhu[3]
Research Scholar, Department of Engineering & Technology
Himalayan University
Arunachal Pradesh, India.

***Abstract*—This paper presents a new methodology of formulating Jacobian matrix for limited degree of freedom (DOF) parallel kinematic machine (PKM), which is a very important tool to relate the end-effector velocity with the joint rate velocity. Even if it is believed by many researchers that Jacobian matrix is critical to generating the trajectories of the prescribed geometry in the end-effector space, it was cumbersome to formulate in simple and descriptive form by partial differentiation. In this work screw mathematics is used to formulate the Jacobian matrix in simple and integrated form under a unified framework and it is proved that the resulted Jacobian matrix is 6×6 which provides clear information about the architecture and singularity condition of the manipulator. Obtaining Jacobian matrix in unambiguous way is very crucial step to formulate and solve velocity, acceleration and motor torque equations with less computational burden. The 3PRS parallel kinematic machine is selected as an example to demonstrate the methodology. The numerical solution is obtained using MATLAB**.

Keywords*—*Jacobian analysis; limited DOF; end-effector, joint space

I. INTRODUCTION

The development of limited-DOF (degree of freedom) PKMs (parallel kinematic machines) has been hot research area due to their advantage in terms of simple structure, lower cost and easy to control comparing with 6-DOF parallel manipulators. The appearance and application of limited DOF PKMs with coupling of translation and rotation provides an option for a bottleneck problem of the manufacture and assembly for large components in aircraft and automobile industries. As an outstanding representation of the limited-DOF PKMs, the 3PRS (prismatic-revolute-spherical) manipulator has been applied to many areas because of compact architecture and excellent kinematic and dynamic performances. For example, the famous sprint-Z3 head made by DS Technology Company in Germany [1, 2].

It is well known that the Jacobian of 6-DOF general purpose parallel manipulator is 6×6 matrix, relating the end-effector linear and angular velocities to the six input joint rates. However, it is not clear as to what is the best way to express the Jacobian of limited-DOF parallel manipulator. A smart approach is to drive an input-output velocity relationship from which the Jacobian matrix of such a manipulator can be formulated [3, 5]. While this approach is valid for general purpose planar and spatial parallel manipulator for which the connectivity of each serial chain(limb) is equals to the mobility of the moving platform, it is not necessarily true for parallel manipulators with less than 6-DOF. For example, this approach leads to 3×3 Jacobian matrix for the 3-UPU parallel manipulator assembled for pure translation [4, 6]. However, such a 3×3 Jacobian matrix cannot predict all possible singularities and cannot provide full information about the architecture.

In what follows, the researcher develops a methodology for the Jacobian analysis of limited DOF parallel kinematic machines. The Jacobian matrix so derived provides full information about both architecture and constraint singularities. The 3-PRS parallel manipulator [5, 7] is used to illustrate the methodology.

II. JACOBIAN ANALYSIS

In this section generalized Jacobian for limited-DOF parallel manipulator is developed. A limited DOF manipulator possesses F-DOF, where F is between 0 and 6. Also it is considered that the moving platform is constrained by F number of limbs and each limb is driven by one actuator. Then apply the theory of reciprocal screw to find the Jacobian matrix that can relate the joint rate and end-effector velocity. In this regard, the instantaneous twist

$$\$_p = \sum_{j=1}^{c_i} \theta_{j,i} \hat{\$}_{j,i} \tag{1}$$

Where c_i is joint connectivity $\theta_{j,i}$ is intensity and $\hat{\$}_{j,i}$ represents a unit screw associated with j^{th} joint of the i^{th} limb.

The twist of the moving platform is defined as $\$p = \left[\omega^T v^T\right]^T$

Where, ω is angular velocity of the moving platform and v is the linear velocity of a point in the moving platform which is instantaneously coincident with the origin of reference frame in which the screws are expressed.

The C_i numbers of screw on a limb forms n^{th} systems for which a one system of reciprocal screw exists. This reciprocal screw forms a Jacobian of constraint and it is reciprocal to all the joint screw of the i^{th} limb from a (6-C_i) reciprocal screw system. Since this reciprocal screw can be identified let $\hat{\$}_{r,j,i}\$_p$ denote the j^{th} reciprocal screw of the i^{th} limb. Therefore taking the orthogonal product both sides of Eq. (1) with each of reciprocal screw gives

$$\$_r{}^T{}_{j,i}\$_p = 0 \text{ for } j=1,2,3\text{--}6\text{-}Ci \qquad (2)$$

Writing equation to once for each limb produces the following constraint matrix.

$$Jc = \begin{bmatrix} \hat{\$}^T{}_{r,1,1} \\ \hat{\$}^T{}_{r,1,2} \\ . \\ . \\ \hat{\$}^T{}_{r,6-CF,F} \end{bmatrix}$$

Each row in Jc represents a unit wrench of constraints imposed by the joint of a limb and it is known that its rank should be equal to $6-Ci$ to properly constrain the moving platform. Then the generalized Jacobian for 3 DOF manipulator will be

$$J = \begin{bmatrix} J_a \\ J_c \end{bmatrix}, J_a = \begin{bmatrix} \hat{\$}^T wa,g_1,1 / \hat{\$}^T ta,g_1,1 \\ \hat{\$}^T wa,g_2,2 / \hat{\$}^T ta,g_2,2 \\ . \\ . \\ . \\ \hat{\$}^T wa,gf,f / \hat{\$}^T ta,gf,f \end{bmatrix}, J_c = \begin{bmatrix} J_{c,1} \\ J_{c,2} \\ . \\ . \\ . \\ J_{c,i} \end{bmatrix},$$

$$J_{c,i} = \begin{bmatrix} \hat{\$}^T wc,1,i / \hat{\$}^T wc,1,i, \hat{\$}^T tc,1,i \\ \hat{\$}^T wc,2,i / \hat{\$}^T wc,2,i, \hat{\$}^T tc,2,i \\ . \\ . \\ . \\ \hat{\$}^T wc,6-n,i / \hat{\$}^T wc,6-n,i, \hat{\$}^T tc,6-n,i \end{bmatrix}$$

(3)

Where, $\hat{\$}^T wa,g_1,1$ is the wrench of actuation, $\hat{\$}^T ta,g_1,1$ is twist screw of actuation, $\hat{\$}^T wc,1,i$ is the wrench of constraints and $\hat{\$}^T tc,1,i$ twist screw of the constraint if each limb and generally J_c and J_a are Jacobian of constraint and Jacobian of actuation respectively.

III. METHODOLOGY

In this section the 3DOF parallel manipulator is used to demonstrate the methodology (specifically named 3PRS high speed machine).

In this manipulator there are three limbs which connect the moving platform at point Bi and to the fixed base at point Ai. Also each link in a limb is connected by three joints prismatic, revolute and spherical respectively as shown in the following Fig. 1.

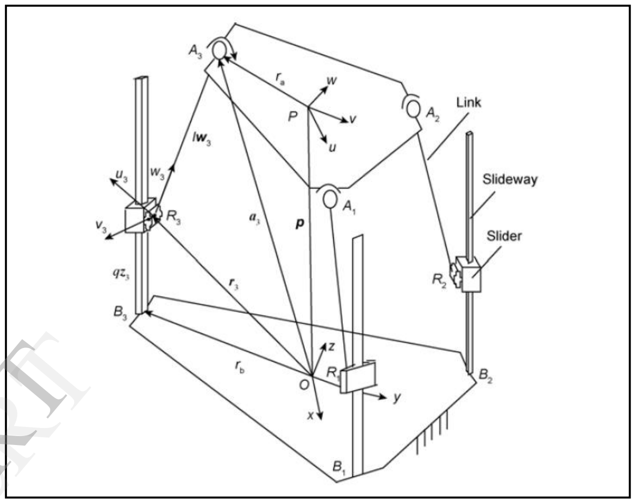

Fig. 1. Schematic diagram of 3PRS parallel robot.

Each limb connects the fixed base to the moving platform by a prismatic joint followed by revolute joint and spherical joint. A linear actuator drives each of the three prismatic joints. The connectivity of each limb is equal to five. Therefore the instantaneous twist, $\$_p$ of the moving platform can be expressed as a linear combination of 5 instantaneous screws.

$$\$p = \dot{d}_{1,i}\hat{\$}_{1,i} + \dot{\theta}_{2,i}\hat{\$}_{2,i} + \dot{\theta}_{3,i}\hat{\$}_{3,i} + \dot{\theta}_{4,i}\hat{\$}_{4,i} + \dot{\theta}_{5,i}\hat{\$}_{5,i} \qquad (4)$$

Where,

$$\hat{\$}_{1,i} = \begin{bmatrix} s_1 \\ 0 \end{bmatrix}, \hat{\$}_{2,i} = \begin{bmatrix} (b_i - l_i) \times s_2 \\ s_2 \end{bmatrix}, \hat{\$}_{3,i} = \begin{bmatrix} b_i \times s_3 \\ s_3 \end{bmatrix},$$

$$\hat{\$}_{4,i} = \begin{bmatrix} b_i \times s_4 \\ s_4 \end{bmatrix}, \text{ and } \hat{\$}_{5,i} = \begin{bmatrix} b_i \times s_5 \\ s_5 \end{bmatrix}$$

Where s_i is a unit screw along the i^{th} direction of each limb. These five screw forms a five system for which a one system of reciprocal screw exists. This reciprocal screw lies on the intersection of the two planes the first plane is perpendicular to the prismatic joint and the second plane is containing both the revolute and spherical joint. Fig. 2 shows the reciprocal screw which is orthogonal to all instantaneous screws except s_1 and parallel to s_2.

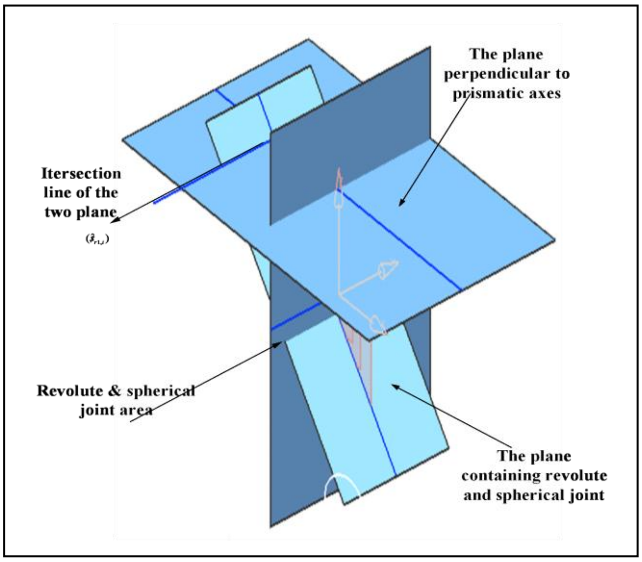

Fig. 2. Representation of the unit wrench screw.

$$\$_{r,1,i} = \begin{bmatrix} b_i \times s_{2,i} \\ s_{2,i} \end{bmatrix} \qquad (5)$$

Taking the orthogonal product of both sides of equation one we found J_c.

$\$_p \otimes \$_{r,1,i} = 0$, since the constraint wrench is reciprocal to all screw, the right hand side equation will be set to zero. The constraint Jacobian of the manipulator along the revolute joint axis will be found where its row represents a unit wrench constraining the degree of freedom of the moving platform. Since, 3DOF manipulator is a capable of three independent DOF, but moves all in six, it is expected that the Jacobian of constraint would be composed of three rows.

$$J_c = \begin{bmatrix} s_{2,1}^{\ T} & (b_1 \times s_{2,1})^T \\ s_{2,2}^{\ T} & (b_2 \times s_{2,2})^T \\ s_{2,3}^{\ T} & (b_3 \times s_{2,3})^T \end{bmatrix} \qquad (6)$$

This matrix represents the constraint imposed by the revolute joint. Then we look for the reciprocal screw for each limb which forms two systems. An additional basis screw which is reciprocal to the passive joint of the i^{th} limb can be identified as zero pitch screw passing through the center of spherical joint. This reciprocal screw represents wrench of actuation and it is normal to the previous one system. This can be represented as follow,

$$\hat{\$}_{r2,i} = \begin{bmatrix} bi \times s_{3,i} \\ s_{3,i} \end{bmatrix} \qquad (7)$$

Take the inner product of this reciprocal screw with for both sides of the twist screw (1) gives

$$\$p \otimes \hat{\$}_{r2,i} = \dot{d}_i(\hat{\$}_{1,i} \otimes \hat{\$}_{r2,i})$$

$$J_x = \begin{bmatrix} s_{3,1}^{\ T} & (b_1 \times s_{3,1})^T \\ s_{3,2}^{\ T} & (b_2 \times s_{3,2})^T \\ s_{3,3}^{\ T} & (b_3 \times s_{3,3})^T \end{bmatrix} \qquad (8)$$

Again, from the right hand side of the inner product we obtain j_q.

$$\alpha + \beta = \chi . \$p \otimes \hat{\$}_{r2,i} = \dot{d}_i(\hat{\$}_{1,i} \otimes \hat{\$}_{r2,i})$$

$$\begin{bmatrix} s_1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} b_i \times s_3 \\ s_3 \end{bmatrix}$$

$$J_q = \begin{bmatrix} s_{3,1}^{\ T} s_{1,1} & 0 & 0 \\ 0 & s_{3,2}^{\ T} s_{1,2} & 0 \\ 0 & 0 & s_{3,3}^{\ T} s_{1,3} \end{bmatrix}$$

Since this mechanism is not outer driving manipulator j_q will not be identity matrix.

$$J_a = \begin{bmatrix} J_x / J_q \end{bmatrix}$$

$$J_a = \begin{bmatrix} \dfrac{s_{3,1}^{\ T}}{s_{3,1}^{\ T} s_{1,1}} & \dfrac{(b_1 \times s_{3,1})^T}{s_{3,1}^{\ T} s_{1,1}} \\ \dfrac{s_{3,1}^{\ T} s_{1,2}}{s_{3,2}^{\ T} s_{1,2}} & \dfrac{(b_2 \times s_{3,2})^T}{s_{3,2}^{\ T} s_{1,2}} \\ \dfrac{s_{3,3}^{\ T} s_{1,3}}{s_{3,3}^{\ T} s_{1,3}} & \dfrac{(b_3 \times s_{3,3})^T}{s_{3,3}^{\ T} s_{1,3}} \end{bmatrix} \qquad (9)$$

and we have seen that J is $J = \begin{bmatrix} J_a \\ J_c \end{bmatrix}$. The generalized Jacobian for this manipulator will be 6 x 6.

By combining the Jacobian of actuation and Jacobian of constraint we formulate the general Jacobian matrix as shown above which is very important to solve the velocity of the active joint.

This 6×6 dimensional matrix characterizes the instantaneous motion of the moving platform. The upper sub-matrix transforms the linear and angular velocity of the moving platform to the actuated joint rate. The constraint singularity will be analyzed by evaluating the rank of Jc where as the architectural singularity occurs when the determinant of the overall Jacobian is equals to zero.

$$
J = \begin{bmatrix}
\dfrac{s_{3,1}{}^T}{s_{3,1}{}^T s_{1,1}} & \dfrac{(b_1 \times s_{3,1})^T}{s_{3,1}{}^T s_{1,1}} \\
\dfrac{s_{3,1}{}^T s_{1,2}}{s_{3,2}{}^T s_{1,1}} & \dfrac{(b_2 \times s_{3,2})^T}{s_{3,2}{}^T s_{1,2}} \\
\dfrac{s_{3,3}{}^T s_{1,3}}{s_{3,3}{}^T s_{1,3}} & \dfrac{(b_3 \times s_{3,3})^T}{s_{3,3}{}^T s_{1,3}} \\
s_{2,1}{}^T & (b_1 \times s_{2,1})^T \\
s_{2,2}{}^T & (b_2 \times s_{2,2})^T \\
s_{2,3}{}^T & (b_3 \times s_{2,3})^T
\end{bmatrix}
$$

Equation (10) tells us how we use this Jacobian to relate the moving platform with joint rate velocity.

$$\dot{q} = J\$p \qquad (10)$$

Where $\dot{q}$ is the joint rate velocity which can be represented as follows in matrix form

$$\begin{bmatrix} d_1 \\ d_2 \\ d_{\substack{1\\3}} \end{bmatrix} \qquad (11)$$

and $\$_p$ is the twist screw which contains the linear and angular velocity of the moving platform. It can also expressed mathematically as

$$\$_p = \begin{bmatrix} v_p \\ \omega_p \end{bmatrix} \qquad (12)$$

Use the generalized Jacobian and expanding (10) will let to solve the active joint velocity in simple and unified form.

$$
\begin{bmatrix}
\dfrac{s_{3,1}{}^T}{s_{3,1}{}^T s_{1,1}} & \dfrac{(b_1 \times s_{3,1})^T}{s_{3,1}{}^T s_{1,1}} \\
\dfrac{s_{3,1}{}^T s_{1,2}}{s_{3,2}{}^T s_{1,1}} & \dfrac{(b_2 \times s_{3,2})^T}{s_{3,2}{}^T s_{1,2}} \\
\dfrac{s_{3,3}{}^T s_{1,3}}{s_{3,3}{}^T s_{1,3}} & \dfrac{(b_3 \times s_{3,3})^T}{s_{3,3}{}^T s_{1,3}} \\
s_{2,1}{}^T & (b_1 \times s_{2,1})^T \\
s_{2,2}{}^T & (b_2 \times s_{2,2})^T \\
s_{2,3}{}^T & (b_3 \times s_{2,3})^T
\end{bmatrix}
\begin{bmatrix} v_0 \\ \omega_n \end{bmatrix}
=
\begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (13)
$$

IV. Numerical Solution

All numerical solutions shown are the sub and generalized Jacobian matrix of limited DOF parallel industrial robot and ranks of sub-matrices which tells us architectural and constraint singularity conditions.

$$
J_a = \begin{bmatrix}
0.0009 & -0.4251 & 1.0000 & 0.0195 & 0.0000 & -0.0000 \\
0.3691 & 0.2126 & 1.0000 & -0.0078 & 0.0144 & -0.0002 \\
-0.3671 & 0.2125 & 1.0000 & -0.0078 & -0.0144 & 0.0002
\end{bmatrix}
$$

$$
J_c = \begin{bmatrix}
-0.5000 & -0.8660 & 0 & 0.0053 & -0.0031 & 0.0084 \\
-0.5000 & 0.8660 & 0 & 0.0027 & 0.0015 & -0.0177 \\
1.0000 & 0 & 0 & 0 & -0.0031 & 0.0084
\end{bmatrix}
$$

$$
J = \begin{bmatrix}
0.0000 & 0 & 1.0000 & -0.0958 & -0.2165 & 0.0000 \\
0.0000 & 0 & 1.0000 & -0.0958 & 0.2165 & 0.0000 \\
0 & -0.1685 & 1.0000 & 0.2186 & -0.0000 & -0.0000 \\
-0.9470 & -1.6402 & 0 & -0.1318 & 0.0761 & -0.4458 \\
-0.9470 & 1.6402 & 0 & 0.1318 & 0.0761 & -0.4458 \\
1.9206 & -0.0000 & 0 & 0.0000 & 0.3086 & -0.3678
\end{bmatrix} \qquad (10)
$$

$$
J_q = \begin{bmatrix}
0.9203 & 0 & 0 \\
0 & 0.9200 & 0 \\
0 & 0 & 0.9206
\end{bmatrix}
$$

Rank (Jc)=3 and rank (Ja)=3 (11)

V. CONCLUSIONS

In this paper it is shown that the Jacobian of small degree of freedom parallel kinematic machine can be driven using reciprocal screw which is not common for such application.

The generalized Jacobian (J) is 6 x 6 matrix consists of two sub-matrices, one associated with the constraint imposed by the joint and the other associated with actuator movement. A manipulator is said to be an architecture singularity when the actuation sub-matrix becomes rank deficient or the overall Jacobian becomes rank deficient while the constraint sub-matrix have full rank. On the other hand the manipulator will be constraint singularity when the constraint sub-matrix becomes rank deficient. Therefore, conclude that one can get information about the manipulator simply like architectural and constraint singularities from the Jacobian matrix by obtaining it in unified and simple form.

REFERENCES

[1] The DS Technologie Company. http://www.ds-technologie, 2008.

[2] N. Hennes, Ecospeed: An innovative machining concept for high performance 5-axis-machining of large structural component in aircraft engineering, Proceedings of the 3rd Chemnitz Parallel Kinematic Seminar, Chemnitz, Germany, 2002, pp. 763-774.

[3] D. Zlatanov, R. G. Fenton, and B. Benhabib, Singularity analysis of mechanism and robots via a velocity-equation model of the instantaneous kinematics, Proc. of the IEEE International Conference on Robotics and Automation, San Diego, CA, 1994, pp. 986-991.

[4] L.W. Tsai, Kinematics of a Three-DOF Platform with Three Extensible Limbs. Recent Advances in Robot Kinematics, J. Lenarcic and V. Parenti-Castelli (eds.), Kluwer Academic Publishers, 1996, pp. 401-410.

[5] Hassen Nigatu and A. P. Singh and S. Seid, Acceleration Analysis of 3DOF Parallel Manipulators, Global Journal of Researches in Engineering: A, Mechanical and Mechanics Engineering, Global Journals Inc. USA, Version 1.0, vol. 12, no. 7, 2012, pp. 30-41.

[6] L. W. Tsai and S. A Joshi, Kinematics and optimization of a spatial 3-UPU parallel manipulator, ASME J. Mech. Des., vol. 122, no. 4, 2000, pp. 439-446.

[7] H. Nigatu, Dynamic formulation of 3 PRS PKM based on screw theory and newton-Euler's approach, Proc. of National Conference on Science Technology and Innovation for Prosperity of Ethiopia, 16th-18th May, 2012, pp. 92-101.

[8] L. W. Tsai and R. Stamper, A parallel manipulator with only translational degrees of freedom, Proc. 1996 ASME Design Engineering Technical Conferences, Irvine, CA, 96-DETC/MECH-1152, 1996.

[9] L. W. Tsai, G. C. Walsh and R. Stamper, Kinematics of a novel three DOF translational Platform, Proc. of the IEEE International Conference on Robotics and Automation, Minneapolis, MN, 1996, pp. 3446-3451.

[10] P. H. Yang, K. J. Waldron and D. E. Orin, Kinematics of a three degree-of-freedom motion platform for a low-cost driving simulator, Advances in Robot Kinematics, J. Lenarcic and V. Parenti-Castelli (eds.), Kluwer Academic Publishers, 1996, pp. 89-98.

[11] D. Zlatanov, I. Bonev and C. Gosselin, Constraint singularities. News Letter on: http://www.parallemic.org, July, 2001.

[12] D. Zlatanov, R. G. Fenton and B. Benhabib, Singularity analysis of mechanism and robots via a velocity-equation model of the instantaneous kinematics, Proc. of the IEEE International Conference on Robotics and Automation, San Diego, CA, 1994, pp. 986-991.

[13] F. C. Park and J. W. Kim, Singularity analysis of closed kinematic chains, ASME J. Mech. Des. vol. 121., no.1, 1999, pp. 32-38.

[14] M. G. Mohamed and J. Duffy, A direct determination of the instantaneous kinematics of fully parallel robot manipulators, ASME J. Mech., Transm., Autom. Des. 107, 1985, pp. 226-229.

[15] L. W. Tsai, Robot Analysis: The Mechanics of Serial and Parallel Manipulators, John Wiley & Sons, New York, 1999.