Iterative Codeword Construction Based Papr Reduction Of Coded Ofdm Signals

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ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) allows transmission of data with high data rates over broadband radio channels without the need for powerful equalization. However, OFDM is sensitive to non-linearity's in the system due to the high Peak to Average Power Ratio (PAPR)in the transmitted signal

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In this paper, we propose a simple method for reducing the PAPR in the OFDM signals using the well known BCH codes. The method relies on an iterative procedure to select a codeword that has a low PAPR from a translation of the code to another partition in a larger BCH code. The information about the chosen partition is loaded in the redundancy of the sent codeword and, therefore, no sub camers are reserved for transmitting this information. The cost payed for this PAPR reduction is reduced error correction capability. Using this method, a reduction of the PAPR of up to 6 dB can be achieved with moderate complexity.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a very promising technique for wireless communications which has been widely investigated in the recent years, [1] .This technique offers the possibility of transmission of high data rates over broadband radio channels subject to multipath fading without the need for powerful equalization. The OFDM technique is already in use in many practical systems such as audio and video data transmission. There are, however, certain problems inherent in this technique that require to be solved. One of the most important problems in OFDM is the high Peak to Average Power Ratio (PAPR), of the transmitted signal. The high peak power in the signals requires amplifiers with very high dynamic range. These amplifiers are both very expensive

and have non-linearity's in their characteristics. The nonlinearities in the circuits result with distortion and lowering the average total power in relation to the average power of the signal.

A number of proposals were made to solve the high PAPR problem in OFDM transmission. Among these methods we find clipping with filtering, coding, selected mapping and pulse shaping, [2],[3],[4],[5],[6].

Clipping with filtering is the simplest solution to the PAPR problem. Unfortunately, it results with a high distortion in the clipped signals. Maybe the most significant results of PAPR reduction with coding are those presented in [4] where a PAPR of at most 3 dB is ensured using the Go-lay complementary sequences. However, the Reed-Muller code used in this method has a very low rate which makes the method inflexible to implementations that require higher rate at the cost of degradation in the PAPR properties. We propose here a new method for PAPR reduction in OFDM transmission that relies on using a BCH code for both error correction and PAPR reduction. The method can be applied to all cyclic codes with a slight improvement.

2. SYSTEM MODEL

In this paper, we consider an OFDM modulation with a total of n sub-carriers. The data after the encoder is arranged as a vector $c = (c_1 c_2 \ldots c_n)$. Each symbol ci is an element of $GF(2^m)$. Alternatively, it can be said that ci contains m bits. Each symbol in c is then mapped to some signal in a preset modulation scheme (e.g., BPSK, QPSK or QAM) to get the complex vector $X = (X_1 X_{2 \ldots} X_n,)$. Let us denote the function used for mapping each symbol in c to the complex plane $\mathbf{C}$ by φ, i.e.

$$\varphi: GF(2^m) \longrightarrow c$$
$$x_{i=}(c_i),$$

where it is assumed that cp is a modulation function to one of the above mentioned modulation schemes. The vector X undergoes Inverse Fast Fourier Transform (IFFT) to acquire the transmitted signal in time domain. The equivalent n-sampled low-pass of the OFDM transmitted signal can be written as a vector z = $(x1, x2,. . . , 2,)$ such that each symbol $x(1)$ in the vector is:

$$X_i = \sum_{i=1}^{n} X_i\, e^{j\, 2\pi i l} \quad …….(2)$$

Where j = $\sqrt{-1}$

The OFDM symbol x undergoes distortion in the transmission channel. We denote the signal at the receiver by y. This signal undergoes OFDM demodulation, i.e., Fast Fourier Transform (FFT) to acquire a demodulated signal $Y = (YI , Y2 , . . . , Yn)$. Each one of the Complex valued symbols Y, is mapped to a $GF\,(2^m)$ symbol to acquire the n vector $z1$ over $GF\,(2^m)$. The vector v is decoded to acquire an estimation of the sent codeword which we denote by c.

The discrete time PAPR of an OFDM signal of (2) can be defined as follows:

$$PAPR(x) = \frac{max|\,x_i|^2}{E\{|x_i|^2\}} \quad …………..(3)$$

Where $E\{•\}$ is the expectation of a sequence of symbols. In communication applications the objective is to decrease the complementary cumulative distribution of the PAPR exceeding a threshold, PAPRt defined as follows:

$$P_{PAPR} = P_r(PAPR > PAPR_t). \quad ……………(4)$$

The PAPR reduction method presented in this paper always tries to return a codeword that has a PAPR less than PAPRt. For this we will use (4) as a measure of performance of the method.

3. ITERATIVE CODEWORD CONSTRUCTION FOR PAPR REDUCTION

We now present the algorithm for constructing a low PAPR codeword. The algorithm is described for *primitive, narrow sense [7,* p. 203] BCH codes but can easily be extended to
all cyclic codes. We first begin with some definitions. Let C1 be a primitive, narrow sense BCH code with parameters $[n, k_1 , d_1]$ over the finite field $GF(2^m)$.

The size of the finite field should be equal to the number of signals in the modulation used. i..e., for QPSK modulation, the finite field should be $GF(2^2)$, for 8PSK the field should be $GF(2^3)$ and so on. Let $C2$ be another BCH code over $GF(2^m)$ with parameters $[n, k2, d2]$ such that :

$$C_1 \subset C_2$$

It is clear that $d2 < dl$. Let S be a set of codeword's chosen as follows :

$$S = \{\, s \in C_2/C_1 \mid PAPR(\varphi(s<)) \; t \,\} \quad ……..(5)$$

Where, t is a preset threshold. i.e., S is the set of code words in C2 but not in C1 and having the required PAPR characteristics. We then create the matrix:

$$S = (s_1^T. s_2^T … … … s_n^T), …………….(6)$$
$$s_i \in S, \qquad \forall_i$$

such that. the of rows S are pair wise independent. Acquiring the matrix S in practice can be done by finding only one codeword in $C2$ that fulfills the requirements in (5) using, for example, computer search, and then, generating the matrix S by augmenting all independent cyclic shifts of this vector. That is to say, the i:th row, s, is the same codeword after cyclically shifting it i times. The risk is that the size of the acquired matrix might be smaller than that acquired by using **(6).** The rows in S can be viewed as a way to partition the larger code C2 into different partitions each of which is a translated version of the smaller code C1. More about the properties of S will be given below in 3.2.
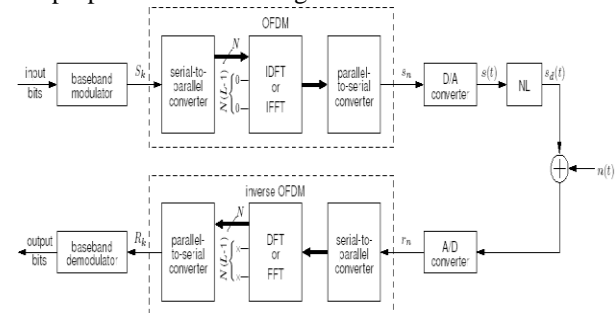

Fig 1: Block diagram of PAPR in OFDM

3.1. Encoding

We now explain the encoding algorithm. The optimal Method for performing this PAPR reduction in OFDM is shown in Figure 1. In the optimal case, the algorithm adds the original codeword c to all the rows of the matrix to obtain all the possible translations and then chooses the one with lowest PAPR. However, this method is very time consuming

and requires many calculations. However, if we are only interested to keep the PAPR of the OFDM signal below a certain threshold *t,* the complexity can be reduced considerably by using an iterative method instead.

Here, the iterative method proceeds as follows: Generate the signal *20* and check its PAPR against the threshold *t*. If PAPR(x_0) > *t*, then, generate the signal x_1 and check its PAPR against the threshold *t*. The steps above are repeated until the obtained OFDM signal *xi* satisfies the PAPR condition or when all the signals x_i , *i* ϵ { 1 , .. , *1* } were generated. In the latter case the algorithm returns the OFDM signal with lowest PAPR even though it does not satisfy the PAPR threshold.

It is clear that if we let *t* = 0, then, the number of iterations becomes *l*+ 1 and the iterative method becomes equivalent to Figure 1

3.2. Detection and decoding

The detection process relies on the fact that the codeword *c'* obtained by adding the original codeword *c* to one of the rows in the matrix *S,* is always a codeword in the larger code C2. This is due to the fact that all the rows in *S* are codeword's in C2 and that $C_1 \subset C_2$. It is, however, important that the rows of *S* translate the smaller code C1 to different Partitions of of the larger code *C2*. It is possible to show that for BCH codes the code C1 partitions Ca to$(2)^{m(k2-k1)}$ where 2^m, as explained above, is the size of the alphabet of the BCH code. We use one of the properties **of** BCH codes regarding the Mattson-Solomon transform **[7,** pp. 239-2481]of a codeword. The Mattson-Solomon Transform is also known as the Discrete Fourier Transform, we will refer to it, however, as the Mattson-Solomon Transform to avoid confusing this part of the detection with the Fourier Transform used in demodulation of the OFDM symbol to obtain the sent message.

Let *A* = (A0, A1,. . . **,** An-l) be the Mattson-Solomon transform (MST), of the codeword *c* $\in$ C1. Then:

$$A_1, A_2, \ldots , A_{d1-1} = 0 \ldots,\ldots (7)$$

as shown in [7, p. 243]. Let *A'* = (A_1', A_1', ... A_{n-1}') be the (MST) of *c'* ϵ C2. Then:

$$A_1', A_2' \ldots A_{d2-1}' = 0 \ldots (8)$$

Since C1 $\subset$ C2, then, if

$$A_{d2}' , A_{d2+1}' , \ldots\ldots\ldots A_{d1-1}' = 0,$$

We will know that *c'* is a codeword in C1. If, on the other hand:

$$A_{d2}' , A_{d2+1}' , \ldots\ldots\ldots A_{d1-1}' \neq 0,$$

then they can be used as a pointer to which row in the matrix S was added to the original codeword *c.* The decoding will commence as follows: Apply FFT on the received symbol y to obtain *Y* and remap the symbols of *Y* to obtain the n-long vector *w* over *GF* (2^m). Obtain the (MST) for the vector *w*. If the number of consecutive zeros is less than d2 - 1, then, decode the vector using a Bounded Minimum Distance (BMD) decoder, e.g., a Berlekamp-Massey decoder [8], otherwise leave it as it is. Apply the (MST) on the result again and use

$A_{d2}' , A_{d2+1}' , \ldots\ldots A_{d1-1}'$ to point out which row in *S* was used. Subtract it from the decoded codeword to obtain the estimated codeword C$^-$.

It can easily be shown that by choosing the rows in such a way to translate the code C1 to different partitions in C_2, we will be certain that

A_{d2}' , A_{d2+1}' , ... A_{d1-1}' will point to the correct row in *S.* It is also possible to show that choosing the rows of *S* to be cyclic shifts of the same codeword, the same objective is achieved.

It can be shown that as long as the number of errors added by the channels is less than [d2 - 1]/2, then the detection and decoding process will be error-free, i.e., C$^-$ will be equal C.

4. NUMERICAL RESULTS

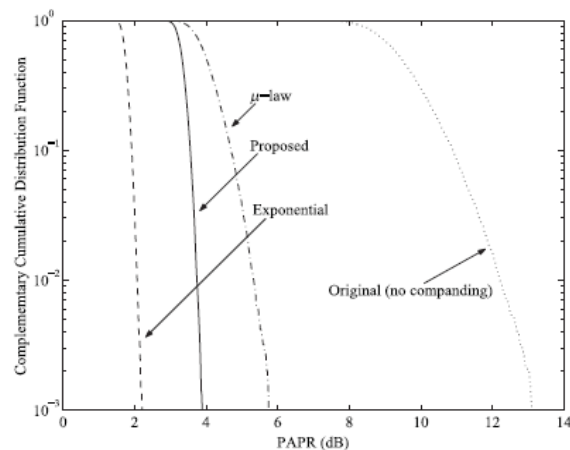The PAPR reduction algorithm was studied for many Choices of code rates.

Fig 2: Performance simulation of PAPR (db) and Complementary cumulative Distributive Function.

The Mattson-Solomon Transform, however, can only be performed on vectors of length $2^p - 1$, where p is an integer. Therefore, the extended symbol in the vector is removed before . In Figure 2 we studied the case when C1 is the [64, 27, 18] extended BCH code over $GF(2^2)$ and C2 is the [64,30,16] extended BCH code over $GF(2^2)$ also The modulation method for the sub-carriers is QPSK. Using extended BCH codes is required in order to obtain a number of sub-carriers equal to 64. applying the (MST). Two target thresholds were investigated for these codes, 5.4 dB and **7** dB. The iterative algorithm satisfies both required thresholds. Needless to say, the higher the target threshold the fewer the number of required iterations to satisfy this threshold. For a threshold equal to 5.4 dB, the average number of extra IFFT operations required was found to be about 1.5 and for a threshold equal to **7** dB, the average number of extra IFFT operations required was found to be about 0.35. In order to investigate the effect of limiting the total number of iterations allowed, we chose C1 to be the [64, 24, 16] extended binary BCH code and *Ca* to be the [64, 30,14] extended binary BCH code. The modulation method for the sub-camers is BPSK. The target threshold was chosen to be 4.7 dB. The maximum number of iterations was changed between 8 and 64 iterations.
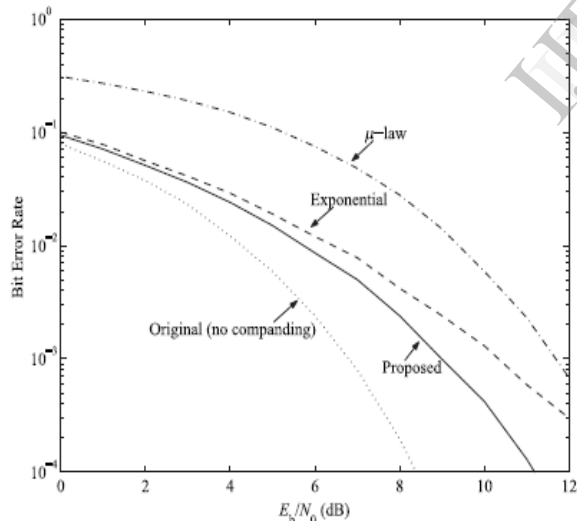

Fig. 3. Bit error rate vs. SNR for original and companded   signals in AWGN channel

The results are shown in Figure 3 below. It can be seen that limiting the maximum number of iterations to at most 8 iterations degrades the performance of the algorithm to about 1 dB away from the target at $P_{PAPR} = 10^{-3}$

5. PAPR Reduction Methods

PAPR reduction methods have been studied for many years and significant number of methods has been developed. These methods are discussed below:

- *Clipping*: Clipping naturally happens in the transmitter if power back-off is not enough. Clipping leads to a clipping noise and out-of-band radiation. Filtering after clipping can reduce out-of-band radiation, but at the same time it can cause "peak regrowth". Repeated clipping and filtering can be applied to reduce peak regrowth in expense of complexity. Several methods for mitigation of the clipping noise at the receiver were proposed: for example reconstructing of the clipped sample, based on another samples in the oversampled signal.

- *Coding:* Coding methods include Golay complementary sequences [**Error! Reference source not found.**], block coding scheme [**Error! Reference source not found.**], complementary block codes (CBC) [**Error! Reference source not found.**], modified complementary block codes (MCBC) [**Error! Reference source not found.**] etc. An application of the Go lay Complementary sequences is limited by the fact that they cannot be used with *M*-QAM modulation. Simple scheme, proposed in [**Error! Reference source not found.**], relies on lookup tables containing sequences with lower PAPR. This method doesn't attempt to utilize those sequences for error correction/detection. CBC utilizes complement bits that are constructed from the subset of the information bits. MCBC is a modification of CBC suitable for large number of sub-carriers. Coding methods have low complexity but PAPR reduction is achieved in expense of redundancy causing data rate loss.

- *Partial Transmit Sequences (PTS):* a set of sub-carriers of an OFDM symbol is divided into non-overlapping sub-blocks [**Error! Reference source not found.**]. Each sub-block undergoes zero-padding and IDFT resulting in *p(k), k=1…V, called* PTS. Peak value optimization is performed over linear combination of PTSs: $\sum_{k=1}^{V} p(k)b(k)$, where *b(k)* is optimization parameter. The optimization

parameter is often limited to four rotation factors: $b(k) \in \{\pm 1 \pm j\}$.

- *Selected mapping (SLM)* [**Error! Reference source not found.**]: a set of sub-carriers of an OFDM symbol is multiplied sub-carrier wise by *U* rotation vectors *b*.Then all the rotated U data blocks are transformed into the time-domain by IDFT and then the vector with the lowest PAPR is selected for transmission.

- *Interleaving* [**Error! Reference source not found.**]: The same data block is interleaved by *K* different interweavers. *K* IDFTs of the original data block and modified data blocks are calculated. PAPR of *K* blocks is calculated. The block with minimum PAPR is transmitted.

- *Tone Reservation (TR)* [**Error! Reference source not found.**]: *L* sub-carriers are reserved for peak reduction purposes. The values of the signals to insert on peak reduction sub-carriers are computed by suitable Linear Programming algorithm.

- *Tone Injection (TI)* [**Error! Reference source not found.**]: TI maps one constellation point of the original constellation (for example QPSK) to several constellation points of the expanded constellation (for example 16QAM). PAPR redaction is achieved by choosing constellation points of the expanded constellation.

- *Active Constellation Extension (ACE)* [**Error! Reference source not found.**]: ACE modifies original constellation by moving nominal constellation points located on the outer constellation boundaries in the directions that don't decrease Euclidean distances between constellation points.

- *Nonlinear Companding Transform (NCT)* [**Error! Reference source not found.**, **Error! Reference source not found.**]: NCT compand original OFDM signal using strict monotone increasing function. Companded signal can be recovered by the inverse function at the receiver.

6. CONCLUSIONS

The new PAPR reduction algorithm shown in this paper is a flexible and rather simple method that can be used for PAPR reduction of coded OFDM modulation. In the explanation and investigation of the algorithm, extended BCH codes were used. However, any cyclic code can be used since the properties required of the codes used are common for all cyclic codes. The algorithm requires performing the Mattson-Solomon Transform on the vector to detect which partition the received codeword belongs to. However, the Berlekamp-Massey decoder requires performing the MST on the received vector to acquire its syndrome in order to decode the vector. Therefore, no extra circuitry requires to be added to the receiver. The proposed algorithm was able to reduce the PAPR more than 5 dB at a moderate increase in complexity. The penalty paid was a slight decrease in the error correction capability. The method has some similarities with selected mapping [5], especially regarding the iterated IFFT operation. The new method, however, is fundamentally different from selected mapping since it relies on choosing different partitions of a large code in search for a codeword with low PAPR. The method also uses a simple way to inform the receiver of which partition was used since this side information is embedded in the redundancy of the sent codeword at the cost of degraded error correction capability.

The flexibility of the algorithm is clear from the fact that there is no restriction on the code rate used. Therefore, the code rate and minimum distance are chosen for each application for purposes like bit rate and error correction capability and not for PAPR reduction purposes. Viewing the characteristics of the algorithm we feel that it can have its place in implementations requiring PAPR reduction without the distortion accompanying clipping with filtering [2] or in implementations requiring higher data rates than that allowed by using Go lay complementary sequences [4] at a somewhat higher PAPR.

7. REFERENCES

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