

Investigation into the Low Amplitude Steady Vibration of a Steel Frame Model

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Abstract - The main object of this investigation is to determine the mode shapes and frequencies of the structure theoretically and experimentally to compare and analyze the measured and predicted values. At first, static and dynamic tests are made on samples of elements of the model in order to determine the properties of these elements and to validate the accuracy of the measurements taken on the model. Vibration records of the model subject to a low amplitude harmonic force are obtained with an automatic instrumentation connected to a microcomputer. The first few resonant frequencies and corresponding mode shapes are determined by performing consecutive tests at different levels of the model.

The specific damping capacity at each floor level is determined from the experimental results.

Preceding the experimental program, the theoretical values of the vibration modes and frequencies of the structure are calculated with the finite element program.

Finally, comparison between various theoretical and experimental values are studied and the correct resonant frequencies and mode shapes are assigned to the structure for further vibration studies

Keywords: Structure – Steel – Low Amplitude – Vibration – Construction – Finite Element Method.

INTRODUCTION

In the analysis of linear structures, it is convenient to distinguish between the static and dynamic components of the applied loading to evaluate the response to each type of loading separately, and then to superpose the two response components to obtain the total effect.

In practice, the purpose of using scaled models (plastic, aluminum, steel material...) is to investigate static and dynamic behavior of any structure which enables the designer to achieve an optimum design before the construction of the full size structure. Therefore, before any complex structure is built, simple tests can be made on a scale model to determine resonant frequencies, mode shapes, static and dynamic displacements and stresses for various input forces. Thus, informations are reviewed from several investigations to show that an understanding of material properties, model design and fabrication techniques, and instrumentation capabilities is required before a structural model test should be planned [1, 2, 3].

In fact, for dynamic testing purposes, major improvements have been seen in the availability, ease of operation, and reliability of basic instrumentation. Systems based on transducers, magnetic tape recorders, digital computers

processing have made it possible to handle immensely large quantities of data.

In addition, the development of specialized devices makes it possible to monitor dynamic tests much more effectively. A major step forward has been the development of computers that make possible to carry out many elaborate structural response calculations during tests.

It is intended, in this paper, to study the experimental behavior of a framed steel model subjected to harmonic excitation with low amplitude.

A brief description of the model fabrication and the instrumentation used for the tests is given.

Tests at different levels of the structure have been carried out in order to determine the dynamic characteristics which define the real behavior of the structure.

Dynamic structural model

A dynamic structural model is defined as a physical system that will duplicate the dynamic characteristics of a prototype to the extent that measurements can be made on the model to predict the behavior of the prototype. The model does not have necessarily to be a scaled duplicate, but it is necessary to make sure that the model characteristics are valid representation of how the full sized prototype will perform.

When the model is constructed of material different from the prototype, their frequency relationship is as stated by the following equation [4]:

$$f_m \frac{L_m}{L_p} = f_p \frac{C_m}{C_p} \quad (1)$$

Where:

L_m and L_p represent the model and prototype dimensions; f_m and f_p represent the model and prototype frequencies; C_m and C_p represent the longitudinal velocity of sound in the model and in the prototype.

For steel prototype, C_p is a constant, but C_m for the visco-elastic material of the model is a function of Section frequency and temperature.

The model constructed for this study does not necessarily duplicate the dynamic characteristics of a real building prototype. However, it is especially made for the purpose of studying the dynamical behavior of framed structure.

The dimensions and size of the elements are chosen and scaled according to a standard structural building, in order to make the model be had more or less as a real structure.

In order to arrive at the necessary dimensions of the model, steps have been followed:

a) the first step is to consider a four story full sized steel, building with the following dimensions:

Longitudinal dimension: 8m

Transversal dimension: 7m

Total height: 14m with 3.50m floor height.

b) The second step is to scale the dimensions of the building and the applied loads at 1/15, in order to find the assigned dimensions to the experimental model.

The characteristics of the mild steel used for structural elements (columns and plates) are:

- mass density: 7850kg/m^3
- Young's modulus: $E=2.0\text{E}11\text{N/m}^2$
- Poisson's ratio: 0.3
- Thickness of plates:
 - 1st and 2nd floors: 8mm
 - 3rd and 4th floors: 6mm.
- Section of columns: solid section 12x12mm

A block of reinforced concrete is built up to support the model frame and make it rigid at base.

All the details are given in figure 1 and table1 and table2:

Table 1: Elements of the model

C O L U M N S A N D P L A T E S (M I I d s t e e l)						
TYPE	Elements	Dimension (mm)		Thickness (mm)	Quantity	
		a	b			
COLUMNS		1072	200	12x12	6	
PLATES		54	456	6	2	
		54	456	8	2	
R E I F O R C E M E N T (M I I d s t e e l)						
// to X axis		630	150	Diameter (mm)	4	
// to Y axis		630	150	Diameter (mm)	4	

Table2: Composition of the concrete used for the base of the model

ELEMENT	WEIGHT (kg)
1. coarse aggregate	114.24
2. sand	68.34
3. cement	44.88
4. water	18.84
Strength of the concrete after 7 days	47.5N/mm ²

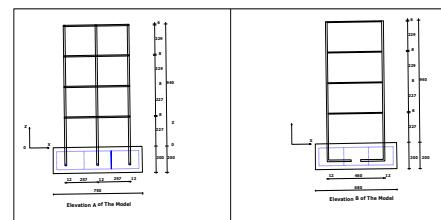
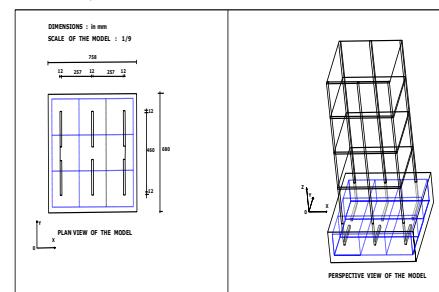


Figure 1: Model characteristics
Instrumentation

The dynamic response of the model is recorded with an automatic instrumentation system connected to a microcomputer. The block diagram showing the connections between the different instruments necessary for the tests is shown in figure 2 and figure 3.

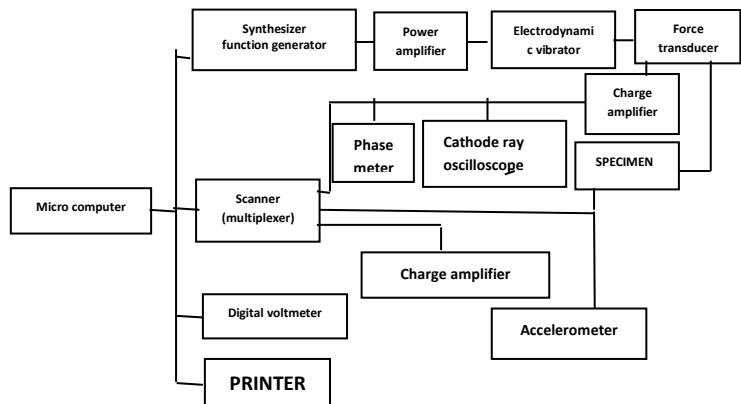


Figure 2: Block diagram of instrumentation



Figure 3: Instrumentation and experimental model

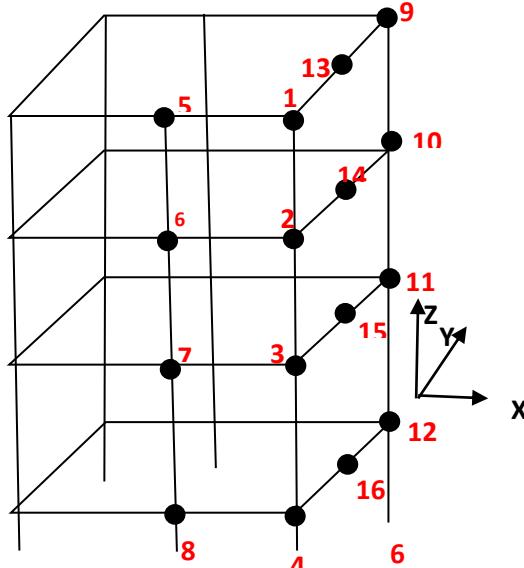


Figure 4: Force input location

$$\text{Gradient} = \frac{\Delta f}{\Delta p} = \frac{f_2 - f_1}{p_2 - p_1} = \dots$$

$$= \frac{f_n - f_{n-1}}{p_n - p_{n-1}} = \frac{L^3}{48EI}$$

(2)

Where:

n: number of measurements.

The values of the elastic modulus are shown in table 4.

	Section (mm ²)	Length (mm)	2 nd Mt area (mm ⁴)	Deflection (m)	Weight (kg)	
Columns	$b \times h = 12 \times 12$	L=885	$\frac{bh^3}{12} = 1728$	$\frac{pL^3}{48EI}$	1	7854
Slabs	$b \times h = 6 \times 24$	L=570	$\frac{bh^3}{12} = 428$	$\frac{pL^3}{48EI}$	0.644	7846

Table 3: Characteristics of the plates and columns.

Static and dynamic properties of plates and columns samples

Before the model can be used with confidence for the evaluation of its vibration characteristics, the accuracy and validity of the measurements must be established by means of static and dynamic tests on samples of structural elements, such as plates, beams and columns.

For static tests, concentrated loads are applied to the elements in the vertical direction by means of small weights which are required for getting accurate results. The deflections are measured with a mechanical deflection (fig.3). To obtain accurate static data, deflections should be measured several times for the same loading conditions. The static modulus of elasticity of samples is determined.

For dynamic tests, the dynamic modulus of elasticity and internal damping are interest and in particular the effects of vibration frequency on these properties are important. Internal damping is of concern because a high value of this quantity may distort the vibration and make the excitation difficult. The three lowest frequencies are determined for the samples. Internal material damping is obtained by measuring the width of the resonant frequency curve or by measuring the log decrement of forced vibration after the harmonic excitation is applied, as recorded on the magnetic oscilloscope.

▪ STATIC TESTS

The characteristics of the steel samples are given in table 3.

According to the consecutive loads applied to the structural samples are the corresponding measured deflections (table 4).

The elastic modulus is calculated using:

LOADING (Newton) p	C O L U M N S		S L A B S	
	Deflection (mm)	Gradient	Deflection (mm)	Gradient
10	0.40	0.043	0.44	0.048
20	0.83	0.040	0.92	0.046
30	1.23	0.040	1.38	0.045
40	1.63	0.038	1.83	0.046
50	2.02	0.038	2.29	0.045
60	2.40	0.040	2.74	0.046
70	2.80	0.042	3.20	0.045
80	3.22	0.045	3.65	0.048
90	3.67	0.040	4.13	0.045
100	4.07	0.043	4.58	0.044
110	4.50	0.040	5.02	0.045
120	4.90	0.043	5.47	0.045
130	5.33	0.041	5.92	0.046
140	5.74	0.041	6.38	0.045
150	6.15	0.041	6.83	0.043
160	6.56		7.26	
Average gradient	0.041			0.045
Static elastic modulus (N/m ²)	2.0383E11			2.0032E11

▪ DYNAMIC TESTS

► Columns

The three lowest resonant frequencies and the corresponding damping capacities are determined for the column from a series of dynamic tests. Thus, a longitudinal harmonic excitation is applied to the simply supported column element.

From equation 3 given below for the longitudinal vibration of uniform beams the three resonant frequencies are calculated for the element and compared with the experimental values.

$$w_n = \frac{n\pi}{L} \sqrt{\frac{E}{\rho}} [5] \quad (3)$$

Where:

w_n : circular frequency

L: length of the bar

E: elastic modulus (from static tests)

ρ : density of the material

The three lowest resonant frequencies are calculated using

eq(3) therefore;

1stmode:

$$w_1 = \frac{\pi}{L} \sqrt{\frac{E}{\rho}} \Rightarrow f_1 = \frac{1}{2L} \sqrt{\frac{E}{\rho}}$$

2ndmode: (4)

$$w_2 = \frac{2\pi}{L} \sqrt{\frac{E}{\rho}} \Rightarrow f_2 = \frac{1}{L} \sqrt{\frac{E}{\rho}}$$

3rdmode :

$$w_3 = \frac{3\pi}{L} \sqrt{\frac{E}{\rho}} \Rightarrow f_3 = \frac{1}{2L} \sqrt{\frac{E}{\rho}}$$

On the other hand, from a series of dynamic tests on the element subjected to a longitudinal harmonic excitation, the three lowest resonant frequencies and the corresponding specific damping capacities are obtained.

A program is written to read the experimental data and give the results of the tests carried out of the sample.

► PLATES

A transversal dynamic force is applied to the simply supported sample of plate. The following equation for the transverse vibration of a uniform beam gives the values of the lowest resonant frequencies:

$$w_n = \frac{a}{L^2} \sqrt{\frac{EI}{A\rho}} \text{ rad/sec} \quad (5)$$

Where:

$a = 22.4$ for 1st mode

$a = 61.7$ for 2nd mode

$a = 121.0$ for 3rd mode

L: length of the sample

E: elastic modulus (from static tests)

ρ : density of the material

Experimental results

The various model measurements are taken with a constant force input at a given frequency range, and the resonant frequency is the frequency corresponding to the maximum receptance of the structure in that range. For a given test, each resonant frequency is checked twice, by changing the force input value. The resonance test is carried out for sixteen force locations: figure 4:

- eight locations in the X direction of the model;
- eight locations in the Y direction of the model.

For each test (i.e each force location), six resonant frequencies are found within the range 10 – 110 Hz. Table 4 gives the summary of the experimental resonant frequencies with the corresponding maximum receptances for the sixteen tests.

The results in table 4 show that the values of the 2nd, 4th, 5th, and 6th frequencies obtained are predominantly constant irrespective of the number and direction of test. The first and third resonant frequencies, however, can be seen to differ slightly. The explanation may lie in that the mode shape obtained due to the test in a particular direction exhibits a major portion of the displacement in that direction, with possibly slight displacement in the perpendicular direction..

Table 4: Results of experimental tests

TEST N°		1 st resonant frequency		2 nd resonant frequency		3 rd resonant frequency		4 th resonant frequency		5 th resonant frequency		6 th resonant frequency	
		Freq. in Hertz	Max. receptance m/N	Freq. in Hertz	Max. receptance m/N	Freq. in Hertz	Max.receptance m/N	Freq. in Hertz	Max. receptance m/N	Freq. in Hertz	Max.receptance m/N	Freq. in Hertz	Max. receptance m/N
Y DIRECTION	1	18.1	5.25E-4	33.8	1.53E-5	52.1	1.08E-4	71.5	1.56E-6	101.8	6.68E-6
	2	11.2	2.97E-4	18.0	3.57E-4	33.6	52.2	1.52E-6	71.0	3.16E-6	102.0	8.10E-6
	3	11.2	1.13E-4	18.0	9.67E-5	33.6	7.62E-5	51.9	6.97E-5	102.0	6.22E-6
	4	11.3	2.52E-5	18.0	2.31E-5	33.8	4.44E-5	51.6	2.59E-5	71.5	1.94E-6	102.0	3.58E-6
	5	11.3	1.25E-4	33.5	7.07E-5	52.3	4.71E-6	71.9	1.43E-6	101.8	4.90E-6
	6	11.2	1.57E-4	18.0	1.50E-5	33.9	1.53E-5	72.0	2.59E-6	101.8	1.65E-5
	7	11.3	6.33E-5	18.2	1.69E-5	33.8	4.72E-5	52.2	3.83E-6	79.1	2.57E-7	101.4	6.18E-6
	8	11.3	5.25E-5	33.8	52.2	2.16E-6	72.0	1.39E-6	101.8	5.24E-6
X DIRECTION	9	12.2	4.18E-4	18.2	2.86E-4	36.0	6.79E-5	51.8	5.43E-5	71.8	2.54E-6	101.5	3.78E-6
	10	12.3	2.93E-4	18.2	3.09E-4	36.0	9.41E-6	52.2	1.05E-5	71.7	3.59E-6	101.6	7.39E-6
	11	12.0	1.77E-4	18.2	2.23E-4	35.8	5.15E-5	51.8	5.06E-5	72.5	6.73E-7	101.5	6.65E-5
	12	12.0	7.94E-5	18.0	1.25E-4	35.7	4.05E-5	51.8	3.92E-5	71.8	4.46E-6	101.6	3.38E-6
	13	12.2	4.02E-4	18.3	2.65E-5	36.0	3.94E-6	52.2	2.72E-6	72.0	2.21E-6	101.5	2.41E-6
	14	12.1	1.86E-4	18.2	1.94E-5	36.0	9.73E-6	52.2	1.60E-6	71.6	3.83E-6	101.4	6.16E-6
	15	12.0	1.78E-4	18.2	2.41E-5	35.7	5.01E-5	52.2	4.39E-6	72.5	4.91E-7	101.4	5.80E-6
	16	12.0	6.51E-5	18.3	8.46E-6	35.8	4.38E-5	52.2	7.85E-7	71.8	9.15E-6	101.4	5.23E-6

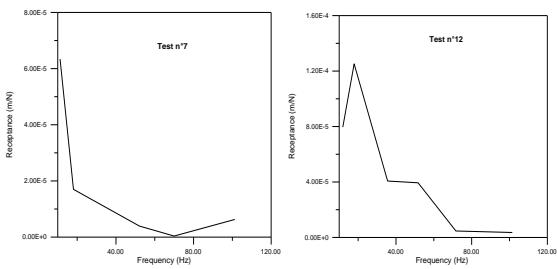


Figure 5: Resonant frequencies

On the other hand, in some tests, the resonant frequencies do not appear at all, as indicated by the broken lines in table 4. This may be due to the non linear behavior of the structure in certain modes of vibration.

Figures 5 show roughly the set of the resonant frequency curves found from four different tests in the range 10 – 110 Hz.

The tests are carried out again for sixteen locations figures 1 et 2 in order to determine the mode shapes corresponding to the experimental resonant frequencies previously found. For each test location and for each resonant frequency, the tests are carried out with a constant force. Some tests have revealed that in some locations of the vibrator, it is possible to excite clearly some of the desired modes. This may be partly due to the fact that the vibrator is located too close to one of the nodal points of the desired mode and partly due to interference from other modes (Table 6 and figure 6).

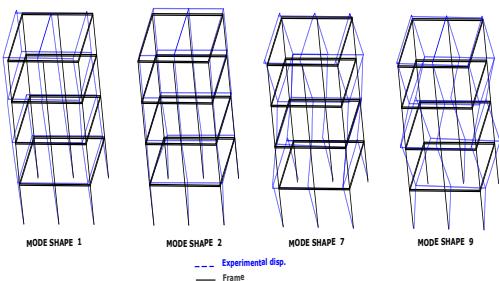


Figure 6: Experimental mode shapes

The main problems in gaining sufficient information from forced vibration tests are:

1. to locate the vibration exciter far enough away from nodal points of the desired modes;
2. to locate the vibration exciter such that interference between the response of modes that have frequencies close together is eliminated.

In order to locate approximately the mode that could be excited at a given frequency and force input, the accelerometer is placed consecutively at 30 different nodes of the model for each test as shown in figure 1. The duration of placing the accelerometer at each mode is about 15 seconds which is thought to be sufficient to record the response at each node.

For each node, the phase angle is recorded at the same time with the amplitude of vibration in order to determine at which frequencies the translational modes are excited and at which frequencies the tensional modes are excited.

Experimental results of damping capacity

Observations on the dynamic behaviour of the experimental model show that the specific damping capacity is affected by the level of excitation and the resonant frequency used to calculate that damping.

Table 5: Values of damping capacity from different tests

Test number		First res. freq. 11.2 Hz	Second res. freq. 18.2 Hz	Third res. freq. 33.8Hz	Fourth res. freq. 52.2 Hz	Fifth res. freq. 71.5 Hz	Sixth res. freq. 101.5 Hz
Y DIRECTION	1	0.035	0.037	0.024	0.112	0.024	
	2	0.112	0.070	0.060	0.105	0.062	
	3	0.112	0.071	0.037	0.073		0.062
	4	0.112	0.069	0.037	0.049	0.110	0.031
	5	0.111		0.037	0.012	0.105	0.025
	6	0.056	0.069	0.037		0.111	0.019
	7	0.111	0.051	0.037	0.024	0.112	0.025
	8	0.111		0.056	0.036	0.110	0.025
	12.2 Hz	18.2 Hz	36.0 Hz	52.2 Hz	71.5 Hz	101.5 Hz	
	9	0.052	0.069	0.024		0.070	0.019
	10	0.051	0.069	0.024		0.066	0.012
	11	0.105	0.069	0.049		0.104	0.019
	12	0.105	0.035	0.024		0.070	0.037
	13	0.051	0.034	0.012		0.088	0.019
	14	0.105	0.073	0.024		0.105	0.025
	15	0.105	0.035	0.024		0.052	0.025
	16	0.105	0.035	0.024		0.088	0.012

Table 5 shows that the damping is relatively high for the first and fifth resonant frequencies; the damping values vary between 0.052 and 0.112. Then they start to decrease for the second and third resonant frequencies where they vary between 0.035 and 0.070. Finally, the fourth and sixth resonant frequencies give the lowest damping values, which are acceptable for defining the dynamic behaviour of the model in terms of damping capacity. These values vary between 0.012 and 0.062 depending on the level of excitation.

Mathematical model

The steel model is made of two different elements which are the columns and plates.

Among the three commonly used element geometries for the thick plate (figure 7), the 17 node element is chosen. Four different mathematical models are considered for analysis of the dynamic behavior of the structure with different finite element discretization.

Frequencies and mode shapes are determined for the following mathematical models:

- Model with 118 nodes, 8 plate elements and 24 beam elements
- Model with 190 nodes, 16 plate elements and 24 beam elements
- Model with 214 nodes, 8 plate elements and 120 beam elements
- Model with 286 nodes, 16 plate elements and 120 beam elements

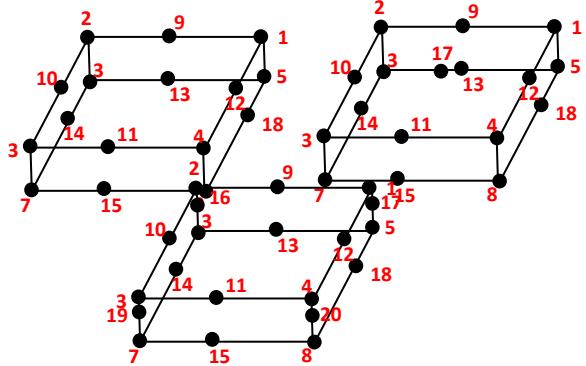


Figure 7: Commonly used element geometries (18 Node element, 17 Node element and 20 Node element)

For each model, the structure is assumed with:

- rigid joints at the base
- Pinned joints at the base.

The highest frequency found for each model is the frequency which corresponds to the fourth tensional mode. All the frequencies beyond that mode are not taken into account in this analysis. Table 6 gives the summary of the frequency results of the one mathematical model for the selected mode shapes.

Table 6: Results obtained for the model with 118 nodes.

CASE	MODE NUMBER	CIRCULAR FREQUENCY (rad/sec)	FREQUENCY (Hz)	PERIOD (sec)	TOLERANCE
RIGID BASE	1	115.60	18.39	0.0544	0.3406E-15
	2	115.60	18.39	0.0544	0.3405E-15
	3	138.20	22.00	0.0454	0.3331E-15
	4	331.60	52.78	0.0190	0.2210E-09
	5	332.00	52.83	0.0189	0.1453E-14
	6	396.80	63.15	0.0158	0.4410E-12
	7	505.10	80.38	0.0124	0.1729E-07
	8	506.70	80.64	0.0124	0.2817E-12
	9	604.90	96.27	0.0104	0.1625E-09
	10	613.60	97.66	0.0102	0.3007E-05
	11	619.40	98.58	0.0101	0.4389E-11
	12	670.30	106.70	0.0094	0.3014E-08
	13	675.60	107.50	0.0093	0.2833E-08
	14	677.80	107.90	0.0093	0.6798E-08
	15	682.30	108.60	0.0092	0.1055E-05
	16	738.50	117.50	0.0085	0.2135E-15
PINNED BASE	1	74.76	11.90	0.0840	0.00
	2	74.76	11.90	0.0840	0.1627E-15
	3	89.34	14.22	0.0703	0.2279E-15
	4	277.20	44.12	0.0227	0.0
	5	277.40	44.15	0.0226	0.1891E-15
	6	331.70	52.80	0.0189	0.1322E-15
	7	475.00	75.61	0.0132	0.3869E-15
	8	476.30	75.80	0.0132	0.2566E-15
	9	568.90	90.54	0.0111	0.1349E-15
	10	606.00	96.44	0.0104	0.4029E-06
	11	611.00	97.25	0.0103	0.3897E-15
	12	670.20	106.70	0.0094	0.1613E-09
	13	675.30	107.50	0.0093	0.2361E-07
	14	677.30	107.80	0.0093	0.1458E-08
	15	681.60	108.50	0.0092	0.2812E-04
	16	728.70	116.00	0.0086	0.4385E-15

Among the four mathematical models, the one whose values most converge with the experimental results is chosen for comparison (model having 118 nodes).

From figure 8, it can be seen that most of the deflected shapes of the experimental selected mode seem to be convergent with the deflected shapes of the corresponding analytical modes.

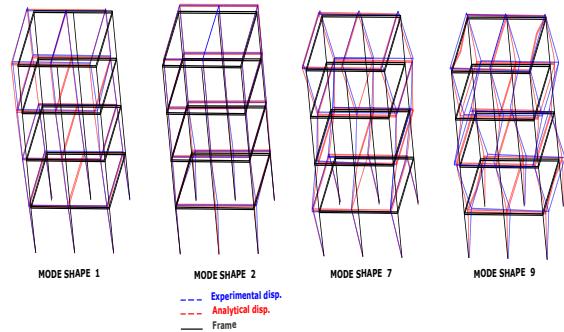


Figure 7: Experimental and analytical mode shapes [6]

CONCLUSION

The major objective of this study has been to evaluate the dynamic characteristics of the experimental model, such as frequencies and mode shapes.

From an extensive series of resonances tests on the model, it has been found that an extremely accurate speed control of the vibration exciter is needed to fully define the response of the structure. Thus, observation of the dynamic behavior of the model during the tests has shown that the resonant frequencies and mode shapes may be affected by the location of the excitation.

The final experimental frequencies of the model are: 11.2Hz, 12.2 Hz, 18.2 Hz, 33.8 Hz, 36.0 Hz, 52.2 Hz, 71.2 Hz, and 101.5 Hz.

The experimental results have also shown that the specific damping capacity is affected by the level and direction of the excitation. The damping capacity is also frequency dependant, i.e the higher the frequency, the lower the damping value, except for the frequency 71.5 Hz where the damping is slightly higher than expected.

The highest values of damping have varied between 7% and 11.2%.

The lowest values have varied between 5% and 1.2%.

On the other hand, the analytical study of the model using the iteration method which has not taken into account the torsional modes, has shown that the resonant frequencies calculated for the translational modes converge well with the corresponding frequencies calculated for the finite element model.

The mathematical analysis has given more detailed results of the frequencies and mode shapes, i.e translational modes, torsional modes and slab vibration modes. However the frequency results are seen to differ slightly from one mathematical model to another, depending on the number of beam and plate elements.

Furthermore, for the three lowest modes of vibration, the frequencies of the pinned frame at base are found to be almost 35% lower than the frequencies of the rigid frame at base. The difference in percentage decreases from mode to another until the values become identical in the fourth modes.

The theoretical and experimental results are compared together. From these results it can be seen that some of the measured frequencies such as 11.2 Hz, 12.2 Hz, 52.2 Hz, 71.2 Hz and 101.5 Hz converge much better with the frequencies calculated for the model having 118 nodes and pinned base. The structure, therefore, behaves as pinned at the base for these frequencies.

Further research

Further extension of this investigation may include an experimental and analytical dynamic study of the model by:

- using vertical bracing system between floors;
- using horizontal bracing at base level between columns to make the structure behave rigidly at the base;
- using more than one vibrator at the same time at different locations, to excite the model and therefore determine possibly the missing modes and frequencies found by analytical work;
- taking into account the vibration of plates.

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