

Inventory Routing Problem with Simultaneous Pickup and Delivery of Returnable Transport Items with Consideration of Renting & Repairing

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Abstract—Companies share their returnable transport items (RTIs) among the different partners of a closed-loop supply chain due to following main reasons: reducing environmental impact, related regulations and potential for operational benefits. RTI management is mostly implemented as one can reuse the RTI multiple times; and this solution becomes cheaper than to buy a new one-way package every time the company needs to transport its product. In this paper, we have developed a new RTI management planning approach with consideration of renting and repairing and compared it with the generic models in use nowadays with only option to buy new RTIs to full fill any variation in demands. We have also used a variation of Clarke and Wright savings heuristic to develop routes for delivery of products packed in RTIs and pickup of empty RTIs from a set of customers present at various distances from the depot. We consider a producer, located at a depot, who has to distribute his products packed in RTIs to a set of customers. The producer takes charge of the collection of empty RTIs for reuse in the next production cycle. Penalty is charged if RTIs are damaged by the customer. During any uncertainty (shortage of RTIs) the producer either rents or buys RTIs from a RTI manufacturer/provider depending on the uncertainty. This research will address a simultaneous pickup and delivery inventory-routing problem (SPDIRP) over a planning horizon.

Keywords—Returnable Transport Item; Closed-loop supply chain; Inventory Routing Problem; Renting and Repairing of RTIs; Simultaneous pickup and delivery; Multiple homogeneous vehicle routing; Clarke and wright savings algorithm

I. INTRODUCTION

Closed-loop supply chains are a special type of supply chains that consider the return flow of used materials in addition to the downstream flow of products e.g. Guide and van Wassenhove (2006). Efficiently managing product returns in addition to the flow of final products may both reduce cost and contribute to improving the sustainability of the supply chain by reducing waste.

Following the first United Nations Conference on the Human Environment in 1972 and other summits on the subject, the paradigm of corporate environmental responsibility has taken on increasing importance among managers' top concerns. Companies are constantly looking for new innovative solutions to green their supply chains Sarkis (2006). However, from the perspective of Guide and Van Wassenhove (2009), environmental improvements

cannot be a business goal by themselves rather improvements of this nature make sense with additional economic value. According to the Sustainable Packaging Coalition (2011) one of the criteria necessary in achieving sustainable packaging is the effective recovery of packaging at the end of their useful life, followed by subsequent reuse in industrial or biological cycles. To do so, end-of-life recovery systems must be designed to create closed-loop material chains. One of the means developed to achieve this goal makes use of returnable transport items (RTI). RTIs consist of all means to assemble goods for transportation, storage, handling and product protection in a supply chain that returned by end consumer for further usage (IC-RTI, 2003). Returnable transport items (RTIs) are a special type of reusable packaging materials, such as pallets, trays, boxes, or crates, and they represent an important corporate asset in many industries today. Using RTIs for transporting products along the stages of a supply chain can lead to many benefits, including reduced packaging material and waste, improved protection and security of products, more efficient handling and cube utilization, better opportunities for out-sourcing, pooling and standardization, and lower CO₂ emissions across the life cycle of the packaging material see Hekkert et al.(2000); Hellström,(2009); Hellström and Johansson,(2010); Malecki and Reimche,(2011). However, the management of RTIs is an essential component in the performance of the entire supply chain. Indeed, a breakdown in the supply of RTIs would impact the overall flow of manufactured products; for instance, such a breakdown would lead to increased delivery times to customers, induced backlogging and storage costs. Returnable Transport Items (RTIs) are used for moving or transporting goods. RTIs are typically classified according to size, weight, application, or material and are often managed as exchangeable items rather than as individual assets. According to packaging and ownership the RTIs are classified as:

1. Primary packaging: This includes all the material that is used to primarily hold the actual good or commodity and is returned and reused. This type form a miniscule amount of total RTI industry by volume and gross, commonly prevalent in returnable glass bottling as is the case in aerated carbonated soft drinks for example – Pepsi, Coca-Cola etc.

2. Secondary Packaging: Returnable secondary packaging includes totes, plastic crates, and other durable containers used for transporting goods.
3. Tertiary packaging: Returnable load carriers include pallets, and rolling materials such as roll containers, dolly, garment racks, etc.

Inventory Routing Problems- IRPs are an extension of the VRP (Vehicle Routing Problem) in which routing, delivery scheduling and inventory policy decisions have to be synchronized and taken jointly. Ghiani et al. (2004) define IRP as “deciding which customers to visit during each period (e.g. one day) of a given time horizon (e.g. one week) and how much to deliver to each one of them”. Bertazzi et al. (2008) explain that “the inventory component arises because customers consume products overtime and have a limited storage capacity. It adds a time dimension to the traditional special dimension of routing problems”. This time dimension increases the complexity of routing decisions. Indeed, when determining the quantity to be delivered to a customer, truck and storage capacity have to be taken into consideration. Furthermore, IRPs have to address longer planning horizons, compared with VRP which are usually busy with in a single day. IRP arises when vendor-managed inventory routing (VMI) is being used; that is, when decisions about deliveries (i.e., timing, sizing and routes) are determined by the supplier and not by the customer, as a result of mutual agreement. As such, there are no customer orders. A supplier develops a distribution strategy that minimizes inventory holding costs and saves on distribution costs as he or she can coordinate pickups and deliveries to various customers. Buyers also benefit by not allocating efforts to inventory control.

As the global attention on environmental problems originating from industrial activities increases, reverse logistics operations that aim to reduce waste production and energy consumption gain more importance worldwide. In this regard, new legislations force companies to regulate their waste management. Reverse logistics activities involve recycling and reusing operations that require both distribution and collection services. The design of transportation systems involving bi-directional flow of goods has been a vital task for companies to minimize transportation costs. A significant increase on the transportation costs of the companies is likely to occur when pickup and delivery operations are to be carried out on separated transportation systems. Also, this will bring about extra energy consumption which is more considerable than the benefit of these operations to the environment. Hence, all these issues make the integration of pickup and delivery operations necessary. The vehicle routing problems with pickups and deliveries arise from these requirements and the purpose of these problems is to combine the bi-directional flow of goods effectively and efficiently. There exist three variants of the problem in the literature, which are Vehicle Routing Problem with Backhauls (VRPB), Vehicle Routing Problem with Mixed Pickup and Delivery (VRPMPD) and Vehicle Routing Problem with Simultaneous Pickup and Delivery (VRPSPD), respectively. In the VRPB, vehicles can only service backhauls (customers having pickup demand) after visiting all customers with delivery

demand, called as line hauls. Imposing this type of precedence constraint makes the reorganization of the pickup and delivery loads on the vehicles easier. Moreover, accepting all pickup goods after servicing the delivery demands simplifies the control of the vehicle load. The VRPMPD relaxes the precedence constraint of the VRPB, and customer demands can be satisfied in any order. Making such a relaxation on the problem constraint can reduce the transportation cost but results in a fluctuating load on the vehicle. The VRPB and the VRPMPD model transportation systems in which each customer has either a pickup or a delivery demand.

However, in numerous distribution and collection systems, the customers require both a pickup and a delivery demand. Additionally, they want to be serviced once by the same vehicle. The VRPSPD covers such transportation systems where customers have both types of demands and require to be serviced exactly once. Therefore, the VRPSPD can be defined as a generalized version of the VRPMPD and a solution approach constructed for the VRPSPD can be directly applied to the VRPMPD. One of the most well-known real life examples to the VRPSPD occurs in soft drink industry where the operations of delivering full bottles and collecting empty ones are performed simultaneously by the same vehicle. The problem can also be encountered at grocery stores where empty pallets or containers are reused for transportation. Another example to the problem occurs on the collection of used products at the end of their lifecycles for remanufacturing operations.

In this paper a RTI management and routing method, with simultaneous pickup and delivery which cater to the variable demands of a number of customers for each time period has been developed. The management methods takes into consideration the facility from which the producer can rent RTIs to satisfy the changes in demand and also setting up a repair unit where the damaged (and repairable) RTIs can be repaired and transferred to the depot in the next period and also a facility from where producer can purchase new RTIs. The costs of operation for the model which considers renting and repairing of RTIs have been compared with another model that only has an option to buy new RTIs to cater to the variation in demands which is similar to the models in use nowadays. The inventory routing is for simultaneous pickup and delivery or simultaneous pickup and delivery inventory routing problem (SPDIRP) in which a batch of homogeneous vehicles deliver and cater the demand for the customers for particular time with delivery and also pick up the empty RTIs left at customer from the previous time period. A variation of Clarke and Wright savings algorithm with quantity constraints has been used to develop the route for pickup and delivery.

II. LITERATURE REVIEW

“Decision support models for managing returnable transport items in supply chains: A systematic literature review” by Christoph H. Glock, 2015 gives us the basic idea about the uses of RTIs, its definition and the summary of research carried out in this field till now. “The evolution of

closed-loop supply chain research” by Guide, V.D.R., Van Wassenhove, L.N., 2009 focuses on the field of closed-loop supply chains with a strong business perspective, i.e., profitable value recovery from returned products. It recounts the evolution of research in this growing area over the past 15 years. “The inventory-routing problem of returnable transport items with time windows and simultaneous pickup and delivery in closed-loop supply chains” by Galina Iassinovskaia, Sabine Limbourg, Fouad Riane (2016) details about a model for inventory routing of RTIs for a closed loop supply chain but like many other papers considered to form an efficient method for management of RTIs considers the returned RTI by the customers always in usable condition for the next period which is not so in practical situation as during customer usage the RTIs tend to get damaged. Ray et al. (2006) studied the cost per pallet trip of two alter- native pallet systems: purchased and rental pallets (RTIs).The authors first conducted an empirical study to collect data on the cost of using both types of systems. Subsequently, they approximated the cost of operating the systems analytically and verified their approximation in a simulation model. Carrano et al. (2015) developed a model for assessing the environmental impact of three different pallet management strategies, namely single-use expandable pallets, reusable (purchased) pallets, and reusable (leased/pooled) pallets. We found that the majority of works contained in our sample assumed that the sender owns the RTIs used in the supply chain. Ray et al. (2006), Carrano et al. (2015) are the only works we are aware of that studied scenarios where the supply chain members have the opportunity to rent or lease RTIs. This is surprising given the fact that RTIs are often rented or leased in practice, and that large companies have evolved over the years that specialize on renting out RTIs. Finally it was found that the papers reviewed only considered two conditions of the RTIs that are rented and damaged but in practical case RTIs pass through a range of conditions which include the conditions in which the RTIs are only slightly damaged and it is more economical to repair the RTI than just discard it. Hence in the model in this research work, a facility of repairing the RTIs with the producer has been introduced. The producer also has the option to rent the RTIs from RTI renting facility and to purchase the new RTIs to replace the damaged ones. Thus this variety of options has been included to adjust to the practicalities of the business involving the use of RTIs most efficiently and create a model that can deal with these variabilities while giving the lowest cost possible to the producer. Mustafa Avci and Seyda Topalogu in their paper have explained different types of vehicle routing and why vehicle routing problem with simultaneous pickup and delivery (VRPSPD) is most efficient and most preferred in case of products having RTIs in their transportation and packaging as is the case in soft drinks and grocery industries. Anders Segerstedt in his paper presents a variant of the Clarke and Wright's saving method that is suitable for introducing the vehicle routing problem and the importance of efficient vehicle routing. The method uses only the first pair of calculated savings and uses these also when searching for complements or additions to an already decided route. Milan Stanojević, Bogdana Stanojević, Mirko Vujošević have introduced a new way of merging routes and the

corresponding formula for calculating savings. They have also apply the enhanced merging to develop a new heuristic – Extended Savings Algorithm (ESA) that dynamically recalculates savings during iterations. Computational results show that, on average ESA gives better solutions than the original savings algorithm. They have Implemented randomization of some steps of their heuristic and obtained even better results which compete with more complex and well known heuristics. The ESA is further used to generate good routes as part of a set-covering-based algorithm for the Capacitated VRP (CVRP). H. Paessens in his paper has given a survey concerning the savings method for the vehicle routing problem. Results for several methods and data sets are compared. Furthermore, modifications of the savings method are presented which show less CPU time and reduced storage requirements. Therefore, the savings method can be implemented on microcomputers.

III. PROBLEM DESCRIPTION

In this research work, the case of a two stage supply chain composed of a main producer and multiple customers is considered. The producer manufactures and distributes/delivers its products to customers using RTIs. The producer also takes charge of the collection of empty RTIs for reuse in the next production cycle. Penalty is charged if RTIs are damaged by the customer. During any uncertainty (shortage of RTIs) the producer either rents or buys RTIs from a RTI manufacturer/provider depending on the uncertainty. In the other model to cater to the uncertainty in demands the option of buying new RTIs is only considered which is generally used nowadays.

Each partner (i.e. producer or customers) has two main storage areas one dedicated to empty RTIs while the other serves for loaded RTIs storage. Each of these stocks is characterized by both initial levels and maximum storage capacity. The delivery of loaded RTIs and pickup of empty RTIs happen simultaneously. Routes are formed, which utilize the batch of homogeneous vehicles available, depending on the demands of customers during that period and also depending on the distance travelled to serve the customers.

This problem belongs to the family of vendor-managed inventory systems: a supplier develops a distribution strategy that minimizes the inventory holding costs and saves on distribution costs by being able to better coordinate pickups and deliveries to various customers. As deliveries and returns are performed by a homogeneous fleet of vehicles that can carry simultaneously empty and loaded RTIs, it is required to solve a simultaneous pickup and delivery inventory-routing problem (SPDIRP) over a planning horizon.

A quality check department at the depot (producer) classifies the RTIs as:

1. Undamaged RTIs: These are the RTIs which are undamaged and do not require any repairing. They can be reused further. Only maintenance is required. After maintenance they are transferred to the empty inventory at the depot.

2. Damaged and Repairable RTIs: These are RTIs that are damaged but can be repaired in the repair unit of the producer and then can be reused further. They are transferred to the repair unit at depot.
3. Damaged and Unrepairable RTIs: These are RTIs which are damaged and can't be repaired. They can't be reused further and are disposed in dispose unit.

As the number of undamaged RTIs, number of damaged RTIs and number of repairable RTIs is uncertain; the producer rents/buys RTIs from a RTI provider/manufacturer if there is a shortage of RTIs during any period.

The supply chain network of the considered problem is presented with the travelling cycle of RTIs in figure 1. L stands for Loaded RTIs and E stands for Empty RTIs. At the beginning of a period, if there is a shortage of empty RTIs then the RTIs are rented. These empty RTIs are filled completely with products and are loaded into the vehicles. A vehicle leaves the depot with loaded RTIs in it. As the vehicle reaches a customer location it delivers the quantity demanded and take back the empty RTIs of the previous period from the customer. This goes on for each customer and then the vehicle arrives back at the depot with empty RTIs. At the end of the period all the empty RTIs go through a quality check. Damaged RTIs are transferred to Dispose Unit, repairable RTIs are transferred to Repair Unit and undamaged RTIs after maintenance are transferred to Empty Inventory. As total number of empty RTIs in the system must be equal to total empty RTIs at the beginning of the planning horizon, new RTIs equal to the number of damaged RTIs are bought at the end of each period and transferred into the empty inventory. The repairable RTIs transferred to the repair unit at the end of a period are repaired during the next period and are transferred to the empty inventory at the end of next period. The undamaged RTIs undergo maintenance and are transferred to the empty inventory at the end of the period. All the RTIs present in the empty inventory at the end of a period can be reused in the next period.

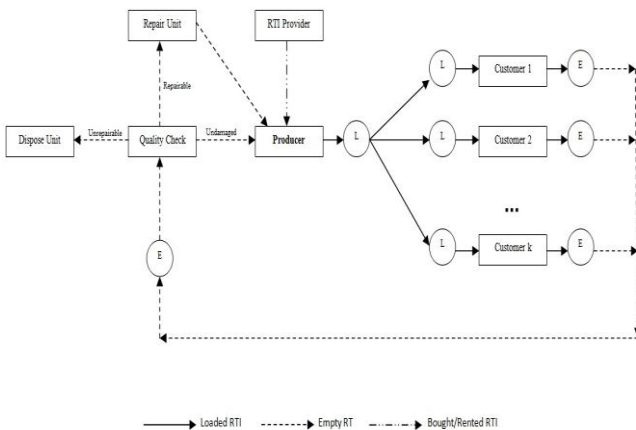


Fig. 1. RTIs flow in a model with repair and renting facilities

IV. MATHEMATICAL FORMULATION

A. MODEL 1: SPDIRP considering Renting and Repairing

The SPDIRP problem is defined on a directed graph $G = (N, A)$, where N is the set of nodes indexed by $i, j \in \{0, \dots, n\}$ and $A = \{(i, j) : i, j \in N, i \neq j\}$ is the arc set. Node 0 represents the producer location (depot) and the set $N_0 = N \setminus \{0\}$ denotes the customer locations. Each customer i has a demand U_{it} at period t . Moreover, each customer and the producer incur unit inventory holding costs per period ($\forall i \in N$), h_i^L for the loaded RTI and h_i^E for the empty RTI, with inventory capacities C_i^L for the loaded RTI and C_i^E for the empty RTI. Inventories are not allowed to exceed the holding capacity and must be positive. The length of the planning horizon is p with discrete time periods $t \in T = \{1, \dots, p\}$. We also have a batch of vehicles $v \in V = \{1, \dots, k\}$ which each have a capacity Q for RTIs loaded with products and we have also assumed that an empty RTI have $1/4^{\text{th}}$ the volume of loaded RTI hence capacity in terms of empty RTIs comes out to be $4Q$, this factor can be varied from industry to industry depending on the size of RTIs used in proportion to product packed in RTI. This is specified as the vehicles have to carry a mixed batch of loaded and empty RTI throughout its route. The number of vehicle each travel on different routes to serve customers which are determined by savings algorithm and demands of customer, the route r contains sets of (i, j) nodes which determine customer locations; $r \in R \{1, \dots, k\}$. α is taken as fixed cost per km; β is variable cost depending upon the weight carried in the vehicle per km; W_L is the weight of loaded RTI while W_E is the weight of the empty RTI. A distance d_{ij} is associated for all $(i, j) \in A$. The producer is assumed to have sufficient inventory and capacity to perform all of the pickups and deliveries during the planning horizon. For any time period t , the cost to buy a new RTI is b ; the maintenance cost per RTI is c , the cost to rent a RTI is r and the repairing cost per RTI is \acute{r} . At the beginning of the planning horizon, the producer knows current inventory levels: I_{i0}^L of the loaded RTI and I_{i0}^E of the empty RTIs, and receives information on the demand of each customer i for each period t . Decision variables used in formulation are a set of binary variables y_{ijrt} equal to 1 if and only if arc (i, j) is used on the route of vehicle v in time period t as determined by savings algorithm.

The objective of the problem is to minimize the total cost of the system while satisfying the inventory level constraints for each customer in each period. Some of the main assumptions are: 1. Every customer can only be visited exactly once in each time period. 2. All the products delivered to a customer in a period are sold in that period itself. 3. All the RTIs are returned on time by the customers. 4. Damaged but repairable RTIs take 1 period to be repaired. 5. All RTIs delivered to the customers are undamaged. 6. Demand is in the multiples of RTIs i.e. all the RTIs are fully filled by the products. 7. Customer is available during the entire period to receive the RTIs. 8. The RTIs rented at the beginning of a period are returned back at the end of the next period when the customers give them back. 9. An empty RTI have $1/4^{\text{th}}$ the volume of loaded RTI.

The integer variables are listed as follows:

- U_{it} : demand of customer i in period t ;
- I_{it}^L : inventory level of loaded RTI at node i at the end of period t ;
- I_{it}^E : inventory level of empty RTI at node i at the end of period t ;
- I_{00}^L : inventory level of loaded RTI at the depot in the beginning of first planning horizon;
- I_{00}^E : inventory level of empty RTI at the depot in the beginning of first planning horizon;
- I_{i0}^L : inventory level of loaded RTI of a customer in the beginning of first planning horizon;
- I_{i0}^E : inventory level of empty RTI of a customer in the beginning of first planning horizon;
- Du_{it} : damaged (unrepairable) RTIs returned by a customer in a period t ;
- Dr_{it} : repairable (damaged but repairable) RTIs returned by a customer in a period t ;
- Um_{it} : undamaged RTIs returned by a customer in a period t ;
- P_t : RTIs to be filled with products produced at the depot at the beginning of period t ;
- Re_t : RTIs rented by the producer at the beginning of period t ;
- B_t : RTIs bought by the producer at the end of period t ;
- h_0^L : unit inventory holding cost per period of loaded RTI incurred by the producer;
- h_0^E : unit inventory holding cost per period of empty RTI incurred by the producer;
- h_i^L : unit inventory holding cost per period of loaded RTI incurred by a customer;
- h_i^E : unit inventory holding cost per period of empty RTI incurred by a customer;
- d_{ij} : distance between node i and j
- x_{ijt} : number of loaded RTI quantity transported from node i to node j in period t ;
- z_{ijt} : number of empty RTI quantity transported from node i to node j in period t ;
- G_r : total number (sum) of empty and loaded RTIs in a vehicle for a route r ;

The Simultaneous Pickup and Delivery Inventory Routing Problem (SPDIRP) in a closed-loop is then formulated as follows:

$$\min \sum_{i \in N} \sum_{j \in N} \sum_{t \in T} [\alpha \sum_{r \in R} y_{ijt} + \beta (W_L X_{ijt} + W_E Z_{ijt})] d_{ij} + \sum_{i \in N} \sum_{t \in T} (h_i^L I_{it}^L + h_i^E I_{it}^E) + \sum_{i \in N} \sum_{t \in T} c(U_{mit}) + 2r Re_t + b B_t + \sum_{i \in N} \sum_{t \in T} f(Dr_{it-1})$$

Subject to:

1. If $\sum_{i \in N} U_{it} > I_{0t-1}^L$, $P_t = \sum_{i \in N} U_{it} - I_{0t-1}^L$ else $P_t = 0 \quad \forall t \in T$.
2. If $P_t > I_{0t-1}^L$, $Re_t = P_t - I_{0t-1}^L$ else $Re_t = 0 \quad \forall t \in T$.
3. $I_{it}^L = I_{it-1}^L - U_{it}, \forall i \in N_0, \forall t \in T$.
4. $I_{it}^E = I_{it-1}^E + U_{it}, \forall i \in N_0, \forall t \in T$.

5. $(Um)_{it} = U_{it-1} + Du_{it} - Dr_{it}, \forall t \in T$.
6. $I_{0t}^L = I_{0t-1}^L + P_t - \sum_{i \in N} U_{it}, \forall t \in T$.
7. $I_{0t}^E = I_{0t-1}^E - P_t + Re_t + \sum_{i \in N} (Um_{it} + Du_{it} + Dr_{it-1}) - Re_t, \forall t \in T$.
8. $I_{0t}^E \geq 0, \forall t \in T$.
9. $0 \leq I_{it}^L \leq c_i^L, \forall i \in N, t \in T$.
10. $0 \leq I_{it}^E \leq c_i^E, \forall i \in N, t \in T$.
11. $I_{00}^E = I_{0t}^L + I_{0t}^E + \sum_{i \in N} U_{it} + \sum_{i \in N} Dr_{it} - Re_t \quad \forall t \in T$.
12. $B_t = \sum_{i \in N} Du_{it}, \forall t \in T$.
13. $U_{it}, U_{it}, d_{ij}, I_{it}^L, I_{it}^E, Du_{it}, Dr_{it}, Um_{it}, P_t, Re_t, B_t, X_{ijt}, Z_{ijt} \in Z^+ \quad \forall (i, j) \in A, t \in T$
14. At the beginning of the route $G_r = \sum_{(i,j) \in r} u_{ijt} \leq Q \quad \forall t \in T \text{ \& } \forall r \in R$
15. During the route $G_r = \{G_r - u_{ijt} + (1/4) u_{ij(t-1)}\} \leq Q \quad \forall (i,j) \in r \text{ \& } \forall r \in R$

The objective function is minimizing the total cost. The first sum corresponds to the transportation cost for route 1 or vehicle 1 for the whole time period. The second sum corresponds to the inventory costs of empty and loaded RTIs at both customer locations and the depot, the third sum is the maintenance cost of the undamaged RTIs, the fourth sum represents renting cost of the RTIs, the fifth sum represents the cost to buy new RTIs, and the last sum represents the cost of repairing of repairable RTIs. The RTIs rented at the beginning of a period are returned back at the end of the next period when the customer gives them back so the fourth sum contains $2r$ as the RTIs are rented for 2 periods. Also the number of RTIs repaired during a period is equal to the number of repairable RTIs returned at the end of previous period so for the last sum repairing cost f is multiplied by the term Dr_{it-1} . Constraints (1) give the number of RTIs to be filled with products produced at the depot at the beginning of period t . Constraints (2) give the number of RTIs to be rented at the beginning of a period. Constraints (3) state the loaded inventory conservation condition over successive periods for a customer. In the same way, constraints (4) state the empty inventory conservation condition over successive periods for a customer: they take into consideration the assumption that all the products delivered to a customer in a period are sold in that period itself. Equation (5) gives the number of undamaged RTIs returned by a customer i in a period t . Constraints (6) ensure inventory conservation conditions for the loading of RTIs over successive periods at the depot. In the same way, constraints (7) ensure inventory conservation conditions for the empty RTIs over successive periods at the depot: they take into consideration the assumption that damaged but repairable RTIs take 1 period to be repaired. Constraints (8), (9) and (10) define the bounds on the

inventory of loaded and empty RTIs held by each customer and the producer throughout all periods. Constraints (11) states that the total number of empty RTIs at the end of every period must be constant and equal to the total number of empty RTIs at the depot in the beginning of the first planning horizon. Constraints (11) lead us to Equation (12) which gives us the total number of RTIs to be bought at the end of a period. Constraints (13) define non-negativity on the variables. Constraint (14) ensures that for a route the sum of total loaded RTIs to be delivered during that time period should be not more than the capacity of the vehicle while the constraint (15) ensures that at every simultaneous pickup and delivery throughout the route does not result in the quantity of loaded and empty RTIs inside the vehicle ever exceeding the capacity of the vehicle.

B. MODEL 2: SPDIRP without considering Renting and Repairing

Now a second approach is considered where renting of RTIs is not possible. Also there is no repair unit. With the same assumptions and variables mentioned in the above model the new objective function becomes:

$$\min \sum_{i \in N} \sum_{j \in N} \sum_{t \in T} [\alpha \sum_{r \in R} Y_{ijt} + \beta (W_L X_{ijt} + W_E Z_{ijt})] d_{ij} + \sum_{i \in N} \sum_{t \in T} (h_i^L I_{it}^L + h_i^E I_{it}^E) + \sum_{i \in N} \sum_{t \in T} c(U_{mit}) + bB_t$$

Subject to:

1. If $\sum_{i \in N} U_{it} > I_{0t-1}^L$, $P_t = \sum_{i \in N} U_{it} - I_{0t-1}^L$ else $P_t = 0$ $\forall t \in T$.
- 2a. If $P_t > I_{0t-1}^L$, $B_t = P_t - I_{0t-1}^E$ else $B_t = 0$ $\forall t \in T$.
3. $I_{it}^L = I_{it-1}^L$, $\forall i \in N_0, \forall t \in T$.
4. $I_{it}^E = I_{it-1}^E + U_{it}$, $\forall i \in N_0, \forall t \in T$.
- 5a. $(Um)_{it} = U_{it-1} + Du_{it}$, $\forall t \in T$.
6. $I_{0t}^L = I_{0t-1}^L + P_t - \sum_{i \in N} U_{it}$, $\forall t \in T$.
- 7a. $I_{0t}^E = I_{0t-1}^E - P_t + B_t + \sum_{i \in N} Um_{it}$, $\forall t \in T$
8. $I_{0t}^E \geq 0$, $\forall t \in T$.
9. $0 \leq I_{it}^L \leq c_i^L$, $\forall i \in N, t \in T$.
10. $0 \leq I_{it}^E \leq c_i^E$, $\forall i \in N, t \in T$.
13. $U_{it}, d_{ij}, I_{it}^L, I_{it}^E, Du_{it}, Dr_{it}, Um_{it}, P_t, Re_t, B_t, X_{ijt}, Z_{ijt} \in Z^+$ $\forall (i, j) \in A, t \in T$
14. At the beginning of the route $G_r = \sum_{(i,j) \in r} u_{ijt} \leq Q$ $\forall t \in T$ & $\forall r \in R$
15. During the route $G_r = \{G_r - u_{ijt} + (1/4) u_{ij(t-1)}\} \leq Q$ $\forall (i, j) \in r$ & $\forall r \in R$

The objective function for model 2 is subjected to all the constraints of model 1 except constraints (11 & 12). Also constraints (2, 5 & 7) have been modified to constraints (2a, 5a & 7a). Constraints (2a) give the number of RTIs to be bought at the beginning of a period. Equation (5a) gives the number of undamaged RTIs returned by a customer i in a period t. Constraints (7a) ensure new inventory conservation conditions for the empty RTIs over successive periods at the depot.

V. COMPUTATIONAL EXPERIMENTS

A. Clarke and wright's savings algorithm

Clarke and Wright's savings algorithm heuristic uses the triangular inequality i.e. the sum of two sides of a triangle is always greater than or equal to the third side. It applies this theorem to calculate the savings factor between the two nodes or two customer locations. The savings factor is essentially the savings of distance that is obtained if we serve those two pair of nodes in one route i.e. one after the other from the depot to the nodes and then back to the depot over the distance that is obtained if we serve those two nodes individually i.e. from depot to node 1 then back to depot than to node 2 and back to depot. In this case the total distance travelled is:

$$D_{12} = D_{01} + D_{10} + D_{01} + D_{20}$$

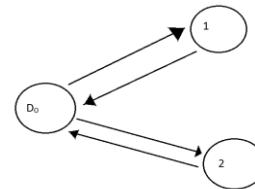


Fig. 2. Serving two nodes individually after returning to depot

For the other case;

$$D_{12} = D_{01} + D_{12} + D_{20}$$

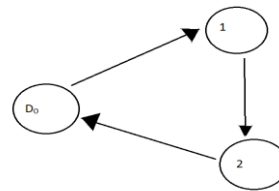


Fig. 3. Serving two nodes in one go

The difference between the two cases is the savings factor:

$$S_{12} = (D_{01} + D_{10} + D_{01} + D_{20}) - (D_{01} + D_{12} + D_{20})$$

$$S_{12} = (D_{01} + D_{02}) - D_{12}$$

Hence, savings factor between pair of nodes (i,j) is:

$$S_{ij} = (D_{0i} + D_{0j}) - D_{ij}$$

If we are given a distance matrix between various nodes to apply savings heuristic we first need to calculate the savings factor between different pair of nodes and then arrange these savings in descending order. To determine the route we first pick up the pair of nodes with maximum savings add then to route and then so on until we have covered all the nodes to get the route. To implement the savings algorithm for our case to find the routes and the associated costs we have

constructed a C++ program which can be downloaded from <https://1drv.ms/f/s!AjeEoeaXpblIcScqFQJGMf1kKTI>.

The procedure followed for the same is given in figure 4:

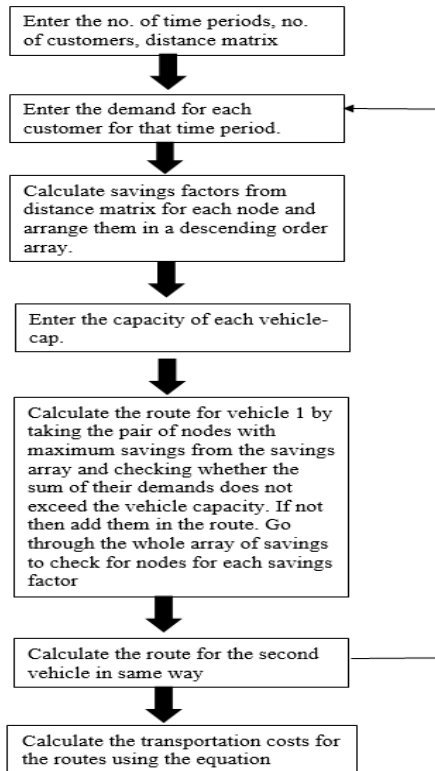


Fig. 4. Flowchart for implementation of savings algorithm

B. Instance Generation

Considering different demands for customers for each period total 15 instances were generated for 7 customers and the inventory management constraints as well as the transportation constraints and Clarke and Wright savings algorithm heuristic were applied on them to get the total costs for whole supply chain management i.e. the inventory management as well as inventory routing for 3 planning horizons considering 1 planning horizon = 5 time periods.

Two C++ programs were formulated:

- 1) Inventory management and cost calculations: This program took the constraints provided above and helped to manage inventory of RTIs at depot according to the various demands of each period and to calculate the costs. The inputs needed were:
 - a) Total demand for each period
 - b) Total no. of damaged RTIs returned in each period
 - c) Total no. of repairable RTIs returned in each period
 - d) The loaded and empty inventory values at the beginning of the first planning horizon

- 2) Routing and transportation costs: In second program a variation of Clarke and Wright's savings algorithm was coded which analyzed the distance matrix and calculated the savings factor between the nodes and routed the vehicle so as to utilize the maximum savings pair of nodes or customer locations with the considerations of demand of the customers and capacity of the vehicle. The inputs needed were:
 - a) The total number of customers
 - b) Distance between different nodes/customers.
 - c) Demand at of each customer for that time period
 - d) Capacity of vehicle (We have considered homogeneous fleet of 2 vehicles).

C. Calculation of inventory

At the beginning of the planning horizon i.e. at $t = 0$; the inventory level at the depot are: $I_{00}^L = 0, I_{00}^E = 30$

Also for the constants in the expression:

(Rs.- Indian rupee (INR))

$$h_0^L = \text{Rs.1/ day item} \quad f = \text{Rs. 5/item}$$

$$h_0^E = \text{Rs. 0.5/ day item} \quad V = 2 \text{ vehicles}$$

$$h_1^L = \text{Rs. 2 / day item} \quad \alpha = \text{Rs. 10/km}$$

$$h_1^E = \text{Rs. 1 / day item} \quad \beta = \text{Rs. 0.1/km item}$$

$$c = \text{Rs. 1.5/item} \quad W_L = 20\text{kg}$$

$$r = \text{Rs. 10/item} \quad W_E = 1\text{kg}$$

$$b = \text{Rs. 200/item}$$

$$Q = 30 \text{ loaded RTIs or } 120 \text{ empty RTIs}$$

The demand of a customer was randomly selected from 1-15 RTIs and was entered for each customer in every period. For Model 1, the damaged RTIs and repairable RTIs returned by each customer in each period were entered randomly. For Model 2, the undamaged RTIs returned were kept same as in Model 1 but the damaged RTIs included both the repairable and non-repairable RTIs as repairing them was not possible. The specifics of each instance and costs with the program code for computation can be downloaded from the given link: <https://1drv.ms/f/s!AjeEoeaXpblIbSrCAiQfsLMq-Wo>.

D. Calculation of transportation costs

The distance matrix for seven customers is given Table 1:

Table I. Distance matrix between customers

	0	1	2	3	4	5	6	7
0	0	24	64	57	25	34	45	36
1	24	0	43	79	33	22	33	60
2	64	43	0	122	75	58	30	98
3	57	79	122	0	53	76	101	31
4	25	33	75	53	0	23	65	46
5	34	22	58	76	23	0	56	66
6	45	33	30	101	65	56	0	74
7	36	60	98	31	46	66	74	0

This distance matrix was used to calculate the savings factors between pair of nodes (i,j) like (1,2) , (1,3) , (1,4).....and so on.

By use of formula: $S_{ij} = (D_{0i} + D_{0j}) - D_{ij}$

Saving factor in descending order:

$S_{26} = 79$

$S_{37} = 62$

$S_{12} = 45$

$S_{25} = 40$

$S_{15} = 36$

$S_{16} = 36$

and so on.

Finally the savings factor matrix formed with descending with descending order of savings is: $S = \{79, 62, 45, 40, 36, 36, 29, 23, 16, 15, 15, 14, 7, 5, 4, 2, 2, 1, 0\}$

The demand for each of the seven periods for each customer is given in table 2:

Table II. Customer demands for each period

Customer \ Time period	1	2	3	4	5	6	7	Total Demand
1	8	5	6	9	4	3	5	40
2	9	9	5	3	8	6	6	46
3	15	5	3	6	6	4	5	44
4	12	10	9	6	3	6	7	53
5	8	8	5	2	11	10	3	47
6	8	6	5	4	9	3	5	40
7	6	7	9	9	6	5	10	52
8	8	8	6	5	5	5	3	40
9	9	5	11	8	7	6	3	49

10	5	3	14	5	3	6	5	41
11	8	7	8	6	5	4	5	43
12	7	7	5	11	6	9	5	50
13	12	2	0	5	6	10	6	41
14	8	8	5	11	3	6	7	48
15	9	3	5	9	8	7	6	47

The route for each of the two vehicles for every period was developed considering the maximum savings as well as the demand and vehicle capacity constraints. After getting the routes, the transportation costs for each period were calculated.

VI. RESULTS

After the computation of costs for various demands for each time period for seven customers for each of the model i.e. with renting and repairing and without it, the following results were obtained.

The Inventory Management Costs for each time period are listed in Table 3:

Table III. Inventory management costs for each period

Time Period	MODEL1 Costs with renting and repair (Rs.)	MODEL2 Costs without renting and repair (Rs.)
1	240	2040
2	2404.5	9286
3	1492.5	4902
4	2027	4907
5	2364	4115
6	903.5	1298
7	1435	4714
8	1070.5	1912
9	1686	2701
10	1444.5	3105
11	1399	2299
12	1784	4502
13	1779	3107
14	1376.5	3104
15	1673	3927
Total	23079	55919

The variation in costs of the two cases is contrasted in the graph given below:

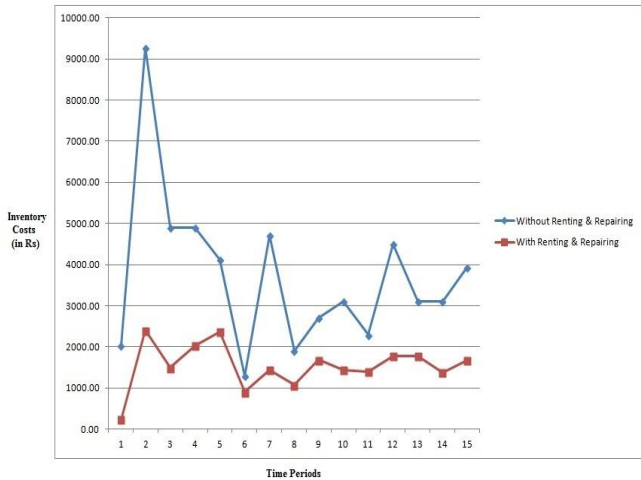


Fig. 5. Graph between Time period vs. Inventory management costs

It is clear from the graph that for each time period the system without renting and repairing (Model 2) costs much more for inventory management. There is also a high variation in costs between every consecutive time period. Hence it can be said that in this model the operator would have to spend a considerably higher or lower amount for each time period with respect to the mean of the 15 time periods. The Inventory Management Costs for each planning horizon are listed in Table 4:

Table IV. Inventory management costs for each planning horizon

Planning horizon	MODEL1 Costs (in Rs.) with renting and repair (C1)	MODEL2 Costs (in Rs.) without renting and repair (C2)	Difference (in Rs.) between C2-C1	Cost reduced (in %) w.r.t C2
1	8528.00	25250.00	16722.00	66.22%
2	6539.50	13730.00	7190.50	55.37%
3	8011.50	16939.00	8927.50	52.70%
Total	23079.00	55919.00	32840.00	58.72%

After the calculations of the inventory management costs for both the models it was found that the total costs for model 1(with renting and repairing) over the 15 time periods was Rs. 23079.00 while that for model 2 (without renting and repairing) was Rs. 55919.00. So, there was a difference of Rs. 32840.00 or the costs calculated for model 2 exceeded costs calculated for model 1 by **58.72%** as the option of renting and repairing was not available.

The vehicle routes, developed using Savings Algorithm, and their respective transportation costs are listed in Table 5.

Table V. Vehicle routes and transportation costs

Time period	Vehicle Routes (for vehicle 1 & 2)	Transportation cost (Rs.)
1	0-2-6-3-7-1-0 0-5-4-0	15598.00
2	0-2-6-3-7-4-0 0-1-5-0	13867.00
3	0-2-6-3-7-5-4-0 0-1-0	17310.40
4	0-2-6-1-0 0-3-7-5-4-0	14538.60
5	0-2-6-3-7-4-0 0-1-5-0	13052.70
6	0-2-6-3-7-1-0 0-5-4-0	15678.50
7	0-2-6-1-5-0 0-3-7-4-0	14036.00
8	0-2-6-3-7-1-0 0-5-4-0	15599.40
9	0-2-6-3-7-0 0-1-5-4-0	13539.90
10	0-2-6-3-7-0 0-1-5-4-0	14398.90
11	0-2-6-3-7-5-0 0-1-4-0	15598.90
12	0-2-6-3-7-0 0-1-5-4-0	13136.40
13	0-2-6-3-7-1-0 0-5-4-0	17149.20
14	0-2-6-3-7-5-0 0-1-4-0	15182.90
15	0-2-6-3-7-1-0 0-5-4-0	17503.80

It was found that the transportation costs for both the models were same as they only depend on customer demand and the route travelled.

VII. CONCLUSIONS

Two mathematical models for inventory management of RTIs at depot or producer were formulated to solve the simultaneous pickup and delivery inventory routing problem (SPDIRP) considering a case 7 customers over 15 time periods. SPDIRP considering Renting and Repairing (Model 1) considered renting and repairing of RTIs to tackle the variation in demands of the customers while SPDIRP without considering Renting and Repairing (Model 2) only considered buying of RTIs to tackle the variation in demands of the customers. The total inventory costs over the 15 time periods for model 1 were Rs. 23079.00 while those for model 2 were Rs 55919.00. Thus the costs of model 2 exceeded the costs of model 1 by **58.72%**. So, it can be concluded that considering renting and repairing of RTIs is more efficient and beneficial.

Also a variation of Clarke and Wright’s savings algorithm heuristic was used, for inventory routing with the constraints of vehicle capacity and customer demands, to develop vehicle routes to simultaneously pick up empty RTIs and deliver loaded RTIs to the customers in an efficient manner, minimizing the transportation costs. Therefore inventory management as well as routing of RTIs can be done effectively by using the model developed in this work (Model

1), which will be very helpful in a number of industries that rely on RTIs for packaging. This model will enable them to create an efficient and green closed loop supply chain network with considerably low environmental impact.

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