Inventory model with linear inventory-level dependent demand rate and variable holding cost

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Abstract

Inventory models in which the demand rate depends on the inventory level are based on the common real-life observation that greater product availability tends to stimulate more sales. Previous models incorporating stock-level dependent demand rate assume that the holding cost is constant for the entire inventory cycle. In this model we will discuss a stock-level dependent demand rate and a storage-time dependent holding cost. The holding cost per unit of the item per unit time is assumed to be an increasing function of the time spent in storage. Two time-dependent holding cost step functions are considered: Retroactive holding cost increase, and incremental holding cost increase. Procedures are developed for determining the optimal order quantity and the optimal cycle time for both cost structures.

Keywords: Inventory models, Stock-dependent demand, Variable holding cost, Optimization

1. Introduction

In traditional inventory models, the demand rate is assumed to be a given constant. Various inventory models have been developed for dealing with varying and stochastic demand. All these models implicitly assume that the demand rate is independent, i.e. an external parameter not influenced by the internal inventory policy. In real life, however, it is frequently observed that demand for a particular product can indeed be influenced by internal factors such as price and availability. The change in the demand in response to inventory or marketing decisions is commonly referred to as demand elasticity.

Most models that consider demand variation in response to item availability (i.e. inventory level) assume that the holding cost is constant for the entire inventory cycle. This paper presents an inventory model with a linear stock-level dependent demand rate and a variable holding cost. In this model, the holding cost is an increasing step function of the time spent in storage. Two types of time-dependent holding cost increase functions are considered: Retroactive increase, and incremental increase.

This structure is representative of many real-life situations in which the storage times can be classified into different ranges, each with its distinctive unit holding cost. This is particularly true in the storage of deteriorating and perishable items such as food products. The longer these food products are kept in storage, the more sophisticated the storage facilities and services needed, and therefore, the higher the holding cost. For example, three different holding cost rates may apply to short-term, medium-term, and long-term food storage.

2. Problem definition and scope

The main objective of this paper is to determine the optimum (i.e. minimum cost) inventory policy for an inventory system with inventory-level dependent demand rate and a time-dependent holding cost. Assuming the demand rate to be inventory-level dependent means the demand is higher for greater inventory levels. Assuming the holding cost per unit of the item per unit time to be time dependent means the unit holding cost is higher for longer storage periods. The model that will be developed for the inventory system is based on allowing unit holding cost values to vary across different storage period. Variable unit holding costs are considered in the model in determining the optimal inventory policy.

The holding cost per unit is assumed to increase only when the storage time exceeds specified discrete values. In other words, the holding cost per unit per unit time is an increasing step function of the storage time. Two types of holding cost step functions are considered: Retroactive increase, and incremental increase. In retroactive increase, the unit holding cost rate of the last storage period is applied to all storage periods. In incremental increase, the rate of each period, including the last period, is applied only to units stored in that particular period.

2.1. Notation

q (t) The quantity on-hand at time t

R Constant demand rate

N Number of distinct time periods with different holding cost rates

t Time

t_i End time of period i, where $i=1,2,3\ldots.n$ $t_0 = 0$, and $t_n = \infty$

A Ordering cost per order

h_i Holding cost of the item in period i

h(t) Holding cost of an item at time t, $h(t) = h_i$ if $t_{i-1} \le t \le t_i$

T Cycle time

β Demand parameter indicating elasticity in relation to the inventory level

2.2. Assumption and limitations

1. The demand rate R is linearly increasing function of the inventory level q.

2. The holding cost is varying as an increasing step function of time in storage.

3. Replenishments are instantaneous.

4. Shortages are not allowed.

5. A single item is considered.

6. The demand rate R is linear function of the inventory level q which is expressed as
$R(q) = R + \beta q(t)$, $R > 0, 0 < \beta < 1, q(t) \ge 0.$

2.3 Inventory Model

Our main objective is to minimize the TIC per unit time, which includes two components: The ordering cost, and the holding cost. Since one order is made per cycle, the ordering cost per unit time is simply $\frac{A}{T}$ and the total holding cost as follows.

$TIC = Total\ Inventory\ Cost$

$$= \frac{A}{T} + \frac{1}{T}\int_0^T \mathrm{h(t)}q(t)d \qquad (2.3.1)$$

Since the demand rate is equal to the rate of inventory level decrease, we can describe inventory level $q(t)$ by the following differential equation:

$$\frac{dq(t)}{dt} = -R(t) = -\left(R + \beta q(t)\right) \qquad (2.3.2)$$

$$\therefore \frac{dq(t)}{R + \beta q(t)} = -dt$$

$$\therefore \int_0^t \frac{dq(t)}{R + \beta q(t)} = -\int_0^t dt\ ;$$

$$where\ \ 0 \le t \le T.$$

$$\therefore \frac{1}{\beta}\left[\ln\left(R + \beta q(t)\right)\right]_0^t = -t$$

Here we have

$q(0) = Q = Initial\ Inventory$

$$\ln\left(R + \beta q(t)\right) - \ln(R + \beta Q) = \beta t$$

Hence,

$$q(t) = \left(\frac{R}{\beta} + Q\right)e^{-\beta t} - \frac{\alpha}{\beta}. \qquad (2.3.3)$$

Now the period T can be evaluated by putting $q(T) = 0$

$$T = \frac{1}{\beta}\ln\left(1 + \frac{\beta Q}{R}\right) \qquad (2.3.4)$$

So,

$$Q = \frac{\alpha}{\beta}\left(e^{\beta T} - 1\right) \qquad (2.3.5)$$

2.4 Retroactive holding cost Increase

As stated earlier, the holding cost is assumed to be an increasing function of storage time, i.e. $h_1 < h_2 < h_3 \ldots\ldots < h_n$. In this case uniform holding is used and the holding cost of the last storage period is applies retroactively to all previous periods. Thus, if the cycle ends in the period *e* then the holding cost rate h_e is applied to all periods i = 1, 2, 3…e. In this case, the TIC per unit time can be expressed as

$$TIC = \frac{A}{T} + \frac{h_i}{T}\int_0^T q(t)dt\ ; \qquad (2.4.1)$$

$$where\ t_{i-1} \le T \le\ t_i$$

Substituting the value of $q(t)$ from (2.3.3) we get;

$$TIC = \frac{A}{T} + \frac{h_i}{T}\int_0^T \left(\left(\frac{R}{\beta} + Q\right)e^{-\beta t} - \frac{\alpha}{\beta}\right)dt$$

$$= \frac{A}{T} + \frac{h_i}{T}\left[\left(\frac{R}{\beta} + Q\right)\left(\frac{1}{\beta}\right)\left(1 - e^{-\beta T}\right) - \frac{\alpha}{\beta}T\right]$$

Now substituting value of T from (2.3.4) we get;

$$TIC = \frac{A\beta}{\ln\left(1 + \frac{\beta Q}{R}\right)}$$

$$+ \frac{h_i\beta}{\ln\left(1 + \frac{\beta Q}{R}\right)}\left[\begin{array}{c}\left(\frac{R}{\beta} + Q\right)\left(\frac{1}{\beta}\right)\left(1 - e^{-\ln\left(1 + \frac{\beta Q}{R}\right)}\right) \\ -\frac{\alpha}{\beta}\left(\frac{1}{\beta}\ln\left(1 + \frac{\beta Q}{R}\right)\right)\end{array}\right]$$

We will get the value of Q on setting $\frac{dTIC}{dQ} = 0.$

$$\frac{-A\beta\left[\frac{1}{\left(1+\frac{\beta Q}{R}\right)}\right]\frac{\beta}{R}}{\left[\ln\left(1+\frac{\beta Q}{R}\right)\right]^2} + \frac{h_i\left[\ln\left(1+\frac{\beta Q}{R}\right)\right] - h_i Q\left[\frac{1}{\left(1+\frac{\beta Q}{R}\right)}\right]\frac{\beta}{R}}{\left[\ln\left(1+\frac{\beta Q}{R}\right)\right]^2} = 0$$

On solving the above equation for Q we will get value of required order quantity Q.

2.5 Stepwise Incremental holding cost Increase

The holding cost is now assumed to be an increasing step function of storage time. According to this function the holding cost rates h_1 applied to period 1, rate h_2 applied to period 2 and so on. Now the total inventory cost obtained as follows.

$$TIC = \frac{A}{T} + \frac{h_1}{T}\int_0^{t_1} q(t)dt + \frac{h_2}{T}\int_{t_1}^{t_2} q(t)dt + \dots\dots\dots + \frac{h_e}{T}\int_{t_{e-1}}^{t_e} q(t)dt. \quad (2.5.1)$$

Substituting the value of $q(t)$ from (2.3.3), we obtain:

$$TIC = \frac{A}{T} + \sum_{i=1}^{e}\frac{h_i}{T}\int_{t_{i-1}}^{t_i}\left[\left(\frac{R}{\beta}+Q\right)e^{-\beta t} - \frac{\alpha}{\beta}\right]dt$$

$$= \frac{A}{T} + \sum_{i=1}^{e}\frac{h_i}{T}\left[\left(\frac{R}{\beta}+Q\right)\left[\frac{e^{-\beta t_i} - e^{-\beta t_{i-1}}}{-\beta}\right] - \frac{\alpha}{\beta}(t_i - t_{i-1})\right]$$

Substituting the value of T from (2.3.4), we obtain:

$$TIC = \frac{A}{T} + \sum_{i=1}^{e}\frac{h_i\beta}{\ln\left(1+\frac{\beta Q}{R}\right)} K \text{ Where } K = \left[\left(\frac{R}{\beta}+Q\right)\left[\frac{e^{-\beta t_i} - e^{-\beta t_{i-1}}}{-\beta}\right] - \frac{\alpha}{\beta}(t_i - t_{i-1})\right]$$

To find the quantity Q, we set the derivative of TIC with respect to Q equal to zero.

2.6 Conclusions and suggestions

A model has been presented of an inventory system with stock-dependent demand, in which the holding cost is a step function of storage time. Two types of holding cost variation in terms of storage time have been considered: retroactive increase, and stepwise incremental increase. Based on the formulas developed, it can be concluded that both the optimal order quantity and the cycle time decrease when the holding cost increases. The model presented in this study provides a basis for several possible extensions. For future research, this model can be extended to accommodate planned shortages, variable ordering costs, and non-instantaneous receipt of orders. Another extension possibility would be to consider the holding cost as a decreasing step function of storage time. The case of the increasing holding cost considered in this paper applies to company-owned storage facilities, and particularly to perishable items that require extra care if stored for longer periods. A decreasing holding cost step function is applicable to rented storage facilities, where lower rent rates are normally obtained for longer-term leases.

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